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Desired Attitude Angles Design Based on Optimization for Side Window Detection of Kinetic Interceptor*

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Abstract: In this paper the characters of pitch angle, yaw angle and roll angle of Kinetic Interceptor (KI) with side window during the terminal guidance process are discussed. In order to guarantee that KI is able to track target missile with high speed continually, a novel method to design the desired attitude angles is presented. Given an ellipse side window, the constraints from visibility, angle of attack and the angle of sideslip on attitude angles are analyzed and illustrated in inequalities. The existence of the solution to these inequalities is confirmed through analyzing a special solution. An object function is designed and a single-objective nonlinear optimization model is thereby formulated. The solutions of this optimization problem, which could be obtained through solving the optimization model, are regarded as the desired attitude angles. The initial value chosen to solve the problem by numerical iterative algorithm is also discussed. The validity of the analysis and the effectiveness of the method are confirmed by simulation results.

Key Words: Kinetic interceptor, Attitude tracking, Desired attitude angles, Nonlinear optimization

1 INTRODUCTION

The unexpected aero-optical effects arise on the head of Kinetic Interceptor due to its high speed in aerosphere [1]. As a result, the measurement accuracy of angles of line-of-sight (LOS) is lost badly. In order to avoid this negative impact, the side window detection technique was adopted [2], while a new problem follows. The vision scope of the imaging infrared seeker is limited by the side window. It is difficult to design the reference attitude angles (also called the desired attitude angles in this paper) in this situation since the main task of attitude control is to ensure that the moving target missile is in the vision scope of seeker during the whole intercepting process [3]. Besides, the angle of attack is often regulated according to dynamic force desired, and the angle of sideslip is always expected to be as small as possible so as to reduce the lateral force during the intercepting process. It is well known that both angle of attack and angle of sideslip are nonlinear functions with respect to attitude angles and trajectory angles [4], so the constraints on the two angles can be converted into the constraints on three attitude angles. Base on the above analysis, it is necessary to take all the constraints from visibility, the angle of attack and the angle of sideslip into account when designing desired attitude angles for KI with side optical widow.

However, many researchers focus on designing new guidance law [5-8]. The desired attitude angles are obtained through solving the designed guidance law in these methods. But these methods only based on solving guidance law are no longer applicable because the visibility of target missile with high speed is not guaranteed without considering the special characteristics of side window.

In this paper, a novel approach to design desired attitude angles for KI with side window is investigated. Given an ellipse side window, the constraints from visibility, the angle of attack and the angle of sideslip on attitude angles are analyzed and described in inequalities. The existence of the solution to the group of inequalities is proved through analyzing a special solution when LOS just goes cross the center of the window. According to the practical control requirements, the desired attitude angles are specified and an object function is designed. By this method, the task is converted into a single-object nonlinear optimization problem. This paper is organized as follows: Section 2 introduces the problem in detail. Section 3 designs the object function. And the initial values chosen to solve the problem by numerical iterative algorithm are also discussed. Section 4 presents the simulation tests and related results. Section 5 gives some conclusions.

2 PROBLEM STATEMENT

The related notations of the problem are given first and then eleven inequalities used to specify the problem are designed. The characteristics of the constraints and the solution to the inequalities are also analyzed in this section.

2.1 Notation

Two basic characters are supposed for the KI investigated in this paper.

(1) A strapdown seeker is equipped and the azimuth and elevation of detecting range of seeker is limited by the side window

(2) The side window is approximately elliptic and the long axis of the ellipse is in the longitudinal symmetrical plane of KI.

The detailed definition of inertial coordinate frame, body coordinate frame and other physical parameters in this

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paper are given by [4] and the notations are given as follows:

$Axyz$ The earth coordinate frame

$Ox_1y_1z_1$ The body coordinate frame

$Cx_wy_wz_w$ The side window coordinate frame

A The origin of $Axyz$

O The mass center of KI and origin of $Ox_1y_1z_1$

C The center of window and origin of $Cx_wy_wz_w$

F_1, F_2 The Focus of ellipse side window

H The Seeker

B The Head of KI

T The target missile

HT Line-of-sight (LOS)

M Intersection point of LOS and side widow

D Image of point C in the plane Ox_1z_1

ϑ, ψ, γ Pitch angle, yaw angle and roll angle of KI

$\vartheta_1, \psi_1, \gamma_1$ Attitude angles of KI at a specified moment

$\vartheta_r, \psi_r, \gamma_r$ The desired attitude angles of KI

α, β Angle of attack and angle of sideslip of KI

θ, ψ_v Flight path angle of KI

a The length of long axis of elliptical window d_1, d_2 The distance between M and F_1, F_2

$|\;|$ The length of line segment

x_{ij}, y_{ij}, z_{ij} The coordinates of point i in j coordinate frame

σ Miss distance of intercepting mission

The side window coordinate frame (see Fig. 1) is defined as $Cx_wy_wz_w$, where x_w is along the long axis of side window and the direction pointing to head of KI is defined to be positive. y_w is perpendicular to the window plane and the direction pointing upward is defined to be positive. z_w is defined according to Right-Hand Rule.

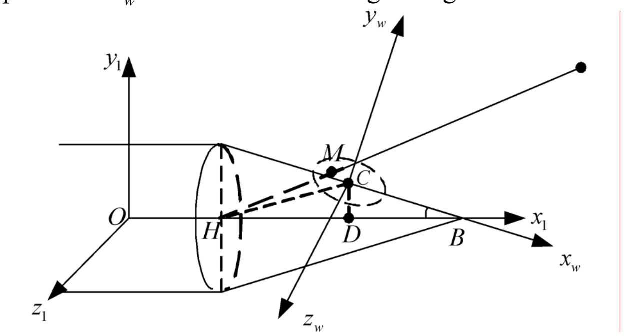

Fig. 1 the position of target in body coordinate frame

Obviously, the transform matrix between $Ox_1y_1z_1$ and $Cx_wy_wz_w$ can be easily calculated based on the knowledge of the structure parameters of KI and is a constant matrix during the whole process of terminal guidance. The purpose of defining the side window coordinate frame is to offer a convenient method to calculate the three attitude angles according to the azimuth and elevation of LOS measured by seeker.

In order to make it easier to analyze the relative position relation between KI and target, we translate point A to point O (see Fig. 2). The case that the three points H, C, T are collinear is illustrated in Fig.2.

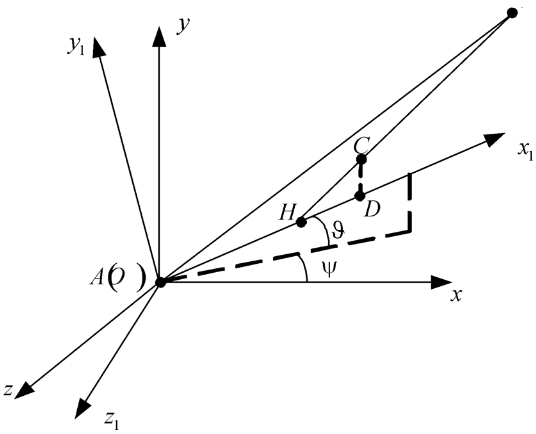

Fig. 2 Relative position of KI and target

2.2 Formulation of the Constraints

For some certain relative position between KI and target missile, d_1 and d_2 are nonlinear functions with respect to ϑ, ψ, γ and can be rewritten as $d_1(\vartheta, \psi, \gamma)$ and $d_2(\vartheta, \psi, \gamma)$. In that case, the requirement for visibility of target is equivalent to the requirement that point M should be inside the window (see Fig. 3). Hence, the constraint is described by:

$$g_1(\vartheta, \psi, \gamma) = d_1(\vartheta, \psi, \gamma) + d_2(\vartheta, \psi, \gamma) - 2a < 0 \quad (1)$$

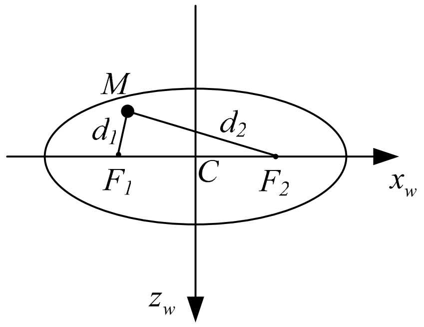

Fig. 3 the Position of M in the elliptic window

For some certain θ, ψ_v, α and β are nonlinear functions with respect to ϑ, ψ, γ [4] given as follows:

$$\beta(\vartheta, \psi, \gamma) = \arcsin\left(\cos\theta\left(\cos\gamma\sin(\psi - \psi_v) + \sin\vartheta\sin\gamma\cos(\psi - \psi_v)\right) - \sin\theta\cos\vartheta\sin\gamma\right)$$

$$\alpha(\vartheta, \psi, \gamma) = \arcsin\left(\frac{\cos\theta\left(\sin\vartheta\cos\gamma\cos(\psi - \psi_v) - \sin\gamma\sin(\psi - \psi_v)\right) - \sin\theta\cos\vartheta\cos\gamma}{\cos\beta}\right)$$

The constraints from angle of attack and angle of sideslip are given by:

$$g_2(\vartheta, \psi, \gamma) = \alpha(\vartheta, \psi, \gamma) - \alpha_{\max} \leqslant 0 \quad (2)$$

$$g_3(\vartheta, \psi, \gamma) = \alpha_{\min} - \alpha(\vartheta, \psi, \gamma) \leqslant 0 \quad (3)$$

$$g_4(\vartheta, \psi, \gamma) = \beta(\vartheta, \psi, \gamma) - \beta_{\max} \leqslant 0 \quad (4)$$

$$g_5(\vartheta, \psi, \gamma) = \beta_{\min} - \beta(\vartheta, \psi, \gamma) \leqslant 0 \quad (5)$$

Where $\alpha_{\min}, \alpha_{\max}$ denote the minimum value and the maximum value of angle of attack respectively and $\beta_{\min}, \beta_{\max}$ denote the minimum value and the maximum value of angle of sideslip respectively. The four parameters are designed according to practical control requirements. Usually, the angle of sideslip is expected to be as small as possible in practical flight process. For this purpose, we can make both $\beta_{\min}$ and $\beta_{\max}$ near zeros. Sometimes the angle of attack is expected to be a specified value, which is calculated according to guidance law, rather than to be designed in a range. For this purpose, we can let $\alpha_{\min} = \alpha_{\max} = \alpha_{\exp}$, where $\alpha_{\exp}$ denotes the specified value of angle of attack. It is not necessary to change the form of the constraint given by(2)~(3) in that case.

According to the definitions of pitch angle, yaw angle and roll angle [4], the three angles must satisfy at any time

$$g_6(\vartheta,\psi,\gamma) = \vartheta - \frac{\pi}{2} \leqslant 0 \quad (6)$$

$$g_7(\vartheta,\psi,\gamma) = -\frac{\pi}{2} - \vartheta \leqslant 0 \quad (7)$$

$$g_8(\vartheta,\psi,\gamma) = \psi - \frac{\pi}{2} \leqslant 0 \quad (8)$$

$$g_9(\vartheta,\psi,\gamma) = -\frac{\pi}{2} - \psi \leqslant 0 \quad (9)$$

$$g_{10}(\vartheta,\psi,\gamma) = \gamma - \pi \leqslant 0 \quad (10)$$

$$g_{11}(\vartheta,\psi,\gamma) = -\pi - \gamma \leqslant 0 \quad (11)$$

2.3 Analysis of These Constraints

It is well known that the main task of attitude control for KI during the terminal guidance phase is to ensure the target missile with high speed is always in the vision scope of seeker, the control requirement described by (1)must be satisfied all the time. Hence, it is a hard constraint.

In most cases, the requirements described by (2) ~ (5) are just the expectations of attitude control and trajectory control and are not indispensable. Hence, they are considered to be soft constraints (except for the case when the angle of attack is hoped to be a specified value).

The requirements described by(6) ~ (11)are obtained based on the definitions of the three attitude angles. They have no impact on the existence of the solution of the inequalities given by (1) ~ (5)and may have impact on the number of the solutions when the problem is resoluble.

2.4 A Special Solution to the Inequalities

In order to check the solution to the inequalities given by (1) ~ (11)exists, we will analyze the special case when the LOS just goes across the center of the window, i.e., the three points H,C,T are collinear (see Fig.2). Based on geometry of ellipse, $d_1(\vartheta,\psi,\gamma) + d_2(\vartheta,\psi,\gamma)$ reaches the minimum value in that case. The three attitude angles are obtained through two rotations of body coordinate frame.

Firstly, rotate $Ox_1y_1z_1$ along axis Oy until T is inside the longitudinal symmetrical plane of KI and we have

$$\psi = ac\tan\left(-\frac{z_{T0}}{x_{T0}}\right) \quad (12)$$

Then, rotate plane Ox_1y_1 along axis Oz_1 until T is on line HC. We suppose the velocity vector of the target missile is in the longitudinal symmetric plane of KI at the moment. So tracking the target without rolling is realized and we have

$$\gamma = 0 \quad (13)$$

Based on H,C,T are collinear, we have

$$\frac{y_{T1} - y_{H1}}{x_{T1} - x_{H1}} = \frac{y_{C1} - y_{H1}}{x_{C1} - x_{H1}} \quad (14)$$

where

$$[x_{T1}\ y_{T1}\ z_{T1}] = [x_{T0}\ y_{T0}\ z_{T0}] \times L^T(\gamma,\vartheta,\psi) \quad (15)$$

In the above equation (15), $L(\gamma,\vartheta,\psi)$ given in [4] is the transform matrix between $Ox_1y_1z_1$ and $Axyz$.

Combining(12)~(15)yields

$$\vartheta = ac\sin\left(\frac{|CD|\times|OH|}{r}\right) - ac\sin\left(\frac{|CD|\times\sqrt{(x_{T0})^2+(z_{T0})^2} - |HD|\times y_{T0}}{r}\right) \quad (16)$$

where

$$r = \sqrt{(|CD|\times\sqrt{(x_{r0})^2+(z_{r0})^2} - |HD|\times y_{r0})^2 \times (|CD|\times y_{r0} + |HD|\times\sqrt{(x_{r0})^2+(z_{r0})^2})^2} \quad (17)$$

Because the positions of H,C,T in $Ox_1y_1z_1$ is determined by the structure arrangement of KI,$|HD|$,$|OH|$, |CD| are constants and can be easily calculated based on the structure parameters. x_{T0}, y_{T0}, z_{T0} can be calculated based on navigation information including the relative distance, flight time and relative velocity etc[9].

2.5 The Specified Problem

In order to offer a convenience to understand the problem, a specified description with respect to a certain moment in the intercepting process is given here.

Given the actual $\vartheta_1, \psi_1, \gamma_1$ measured at time t_1 and the four nonlinear functions $d_1(\vartheta_1,\psi_1,\gamma_1)$, $d_2(\vartheta_1,\psi_1,\gamma_1)$, $\alpha(\vartheta_1,\psi_1,\gamma_1)$, $\beta(\vartheta_1,\psi_1,\gamma_1)$ which satisfy the inequalities given by $(1) \sim (11)$, the question is how to design the desired $\vartheta_r, \psi_r, \gamma_r$ for the next control time $t_1 + \Delta t$, so that the four following objects will be satisfied at time $t_1 + \Delta t$:

- The four functions $d_1(\vartheta_r,\psi_r,\gamma_r)$, $d_2(\vartheta_r,\psi_r,\gamma_r)$, $\alpha(\vartheta_r,\psi_r,\gamma_r)$, $\beta(\vartheta_r,\psi_r,\gamma_r)$ will satisfy the constraints given by inequalities(1)~(11).
- In order to obtain the maximal visibility margin, we need that the point M is as close to point C as possible.

- In order to reduce overload, the angle of sideslip $\beta(\vartheta_r, \psi_r, \gamma_r)$ is as small as possible.
- In order to reduce overload (or to satisfy the requirement for fit aerodynamic force), the angle of attack $\alpha(\vartheta_r, \psi_r, \gamma_r)$ is as small as possible (or is equal to the specified value $\alpha_{\exp}$).

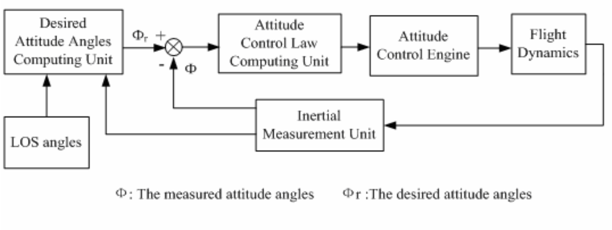

Fig. 4 Block diagram of KI attitude control system

Remark 1:

(1) This paper use "visibility margin" to illuminates the possibility of visibility. Its meaning is similar to the concept of robustness in control theory. We use it here to express the possibility of M keeping inside the window in spite of maneuver of target missile or any small disturbance.

(2) For intercepting moving target, the problem should be solved online and the solution is a sequence of desired attitude angles during the whole process of terminal guidance.

(3) The special characters of the reference attitude angles for KI with slide window and the feasible methods to calculate them are the two main concerns in this paper. Hence, we don't care other problems here, such as the form of the attitude control loop and the implementation of the command (see fig.4).

Based on the qualitative description of desired attitude angles, this question can be converted to an optimization problem with respect to nonlinear constraints given by inequalities(1)~(11). We can get a sequence of the desired attitude angles through solving the optimization problem online.

3 MAIN RESULTS

Generally speaking, the problem described in section 2 has more than one feasible solution. However, the optimal one is our primary concern. How to define the best solution to the problem is an open question. A method based on optimization is introduced in this section.

3.1 A Single-Object Optimization Model

In order to satisfy the requirements for visibility, small angle of attack and small angle of sideslip with respect to the inequalities given by(1)~(11), a single-object nonlinear optimization model is given by

$$\begin{cases} \min\limits_{\vartheta,\psi,\gamma} f(\vartheta,\psi,\gamma) = w_1\left(\dfrac{d_1(\vartheta,\psi,\gamma)+d_2(\vartheta,\psi,\gamma)}{2a}\right)^2 \\ \qquad + w_2\left(\dfrac{\alpha(\vartheta,\psi,\gamma)}{\alpha_{\max}}\right)^2 + w_3\left(\dfrac{\beta(\vartheta,\psi,\gamma)}{\beta_{\max}}\right)^2 \\ s.t.\quad g_i(\vartheta,\psi,\gamma) \leqslant 0, \qquad i = 1,2\cdots 11 \end{cases} \tag{18}$$

where

- $w_1, w_2, w_3 \in (0,1)$, satisfying $w_1+w_2+w_3=1$, denote the weight coefficients of the three parts of the object function respectively.
- The three parts $d_1(\vartheta,\psi,\gamma)+d_2(\vartheta,\psi,\gamma)$, $\alpha(\vartheta,\psi,\gamma)$, $\beta(\vartheta,\psi,\gamma)$ are normalized by being divided by $2a, \alpha_{\max}, \beta_{\max}$.
- The expressions of $g_i(\vartheta,\psi,\gamma) \leqslant 0,\ i = 1,2\cdots 11$ are given by the inequalities(1)~(11), in which (1) is the necessary condition for continual tracking and can't be modified during the whole intercepting process.

Remark 2:

(1) Based on the analysis for inequalities (2)~(5)in section 2.3, the values of $\alpha_{\max}, \beta_{\max}$ can be designed according to the specified requirements in practical application.

(2) The solution of the problem described in(18)is considered as the desired attitude angles at time $t_1 + \Delta t$ rather than the desired attitude angles at time t_1 .

3.2 The Initial Values of The Problem

Solving the optimization problem online is dynamic and should be implemented during the whole process of terminal guidance. As for the initial attitude angles, there are two different meanings here.

- The attitude angles at the beginning of terminal guidance. They are determined by many factors. However, the requirement that the moving target should be in the vision scope of infrared seeker must be satisfied.
- The initial values used in iterative algorithms to solve the optimization problem.

4 SIMULATIONS

In order to contrast, two groups of experiments with the same simulation conditions except for the method to calculate the reference attitude angles are implemented.

Method 1: Calculate the reference attitude angles through solving typical guidance law.

Method 2: Calculate the reference attitude angles through solving the single-object optimization problem presented in this paper.

In Method 2, we choose the actual attitude angles at t_1 as the initial values when iterative algorithms are used to calculate the reference angles at $t_1 + \Delta t$.

In order to ensure the target is in the vision scope of seeker at beginning of terminal guidance, the attitude angles when H, C, T are collinear are adopted at the moment in the simulations. Both the two methods use Matlab to implement simulations. Under this condition, local exhaustive algorithm is used to generate 40000 groups of simulated data with proportional spacing in the feasible range of flight path angle and position coordinates of KI. The other invariable simulation parameters are given in table1

Table1 The Invariable Initial Parameters

Relative distance	60km
Meeting angle	27°
Velocity of KI	1400 m/s
Velocity of target	2000 m/s
The position of target	$(109.161, 73.625, -15.000)$ km
The number of groups	40000

It is well known that intercepting accuracy is one of the most important indexes to evaluate the performance of KI. Hence, the missing distance is our primary concern. The statistical characteristics of the 40000 groups of data are described in Fig.5-7.

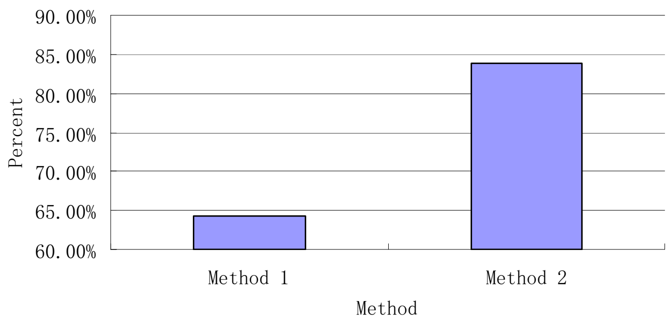

Fig. 5 The statistical results for $\sigma \leqslant 1m$

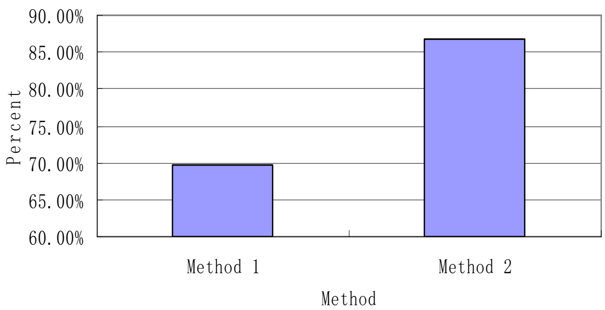

Fig. 6 The statistical results for $\sigma \leqslant 10m$

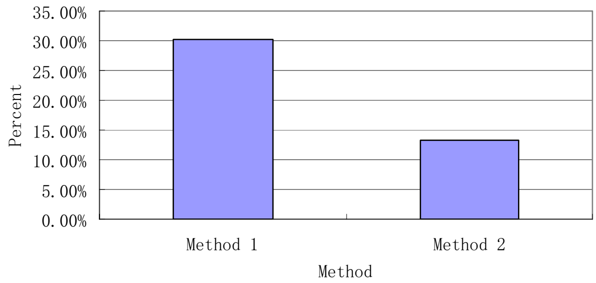

Fig. 7 The statistical results for $\sigma > 10m$

For simulations without any noises, the main reason for big miss distance is that the seeker can't detect the target any longer during the process of terminal guidance. Hence, the degree of the intercepting accuracy reflects the stability of LOS in the vision scope of seeker to a great extent. In addition, three conclusions are made based on the simulation results. (1) It is necessary to take the constraint from side window into account when calculating the reference attitude angles for attitude control of KI. (2) The novel method based on optimization presented in this paper is an efficient way to ensure the visibility of target in the intercepting process and is able to improve the intercepting accuracy of this KI. (3) In order to obtain the same accuracy of intercepting mission, the method designed in this paper has greater range to choose initial condition for terminal guidance than others methods without considering the characteristic of side window.

5 CONCLUSION

In this paper we present a novel method to design the reference attitude angles for a KI with side window during process of terminal guidance. The main purpose is to ensure the visibility of target missile with high speed. The constraints from both the angle of attack and the angle of sideslip are also considered in the method. Simulation results indicate that it is necessary to take the requirements for visibility into account when designing. The approach introduced in this paper can guarantee that the target is in the vision scope of seeker effectively. Furthermore, it is a new way to improve the performance of this kind of interceptors. For practical implementation, it is necessary to consider some key factors, such as the performance of actual attitude control engines and the time consumed to solve the optimization problem online continuously. These problems need to be investigated in further study.

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