Adaptive Compensation for Robust Tracking of Uncertain Dynamic Delay Systems

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Abstract To overcome the drawbacks of the high-gain feedback term, a controller with an adaptive compensation term is developed for robust tracking of uncertain dynamic delay systems. It is proved that the introduction of the adaptive compensation term will not affect the stability of the original closed-loop system. In this way, it will be practically flexible to decide whether to include the adaptive compensation term depending on the tracking performance requirements and controller gain constraints. The effectiveness of the proposed controller is demonstrated by numerical examples.

Key words Model following, robust tracking, time-delay systems, periodic signals

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Recently, various methods are proposed for a class of uncertain dynamic systems to track dynamic outputs of finite-dimensional systems. In [1−2], linear feedback controllers that guaranteed uniform ultimate boundedness of the tracking error were proposed. The error bound can be reduced to an arbitrarily small value by increasing the controller gain. In [3], a nonlinear switching controller was developed to guarantee that the tracking error was reduced to zero asymptotically by increasing the maximum value of control effort. Following the idea of [3−4] proposed an improved adaptive law with σ-modification for robust tracking of dynamical signals. However, the resulting controller is still a high-gain feedback controller. Reference [5] designed an H_∞ filter-based tracking controller for a class of linear time-delay systems subjected to parametric uncertainties. By assuming some matching conditions for uncertainties and the disturbance being an $\mathcal{L}_2$ function, this controller ensures that the closed-loop system is H_∞ asymptotically stable. Whereas, all the controllers above essentially rely on increasing controller gains to achieve good tracking performance, which is not always feasible in practice. The drawbacks of the high-gain feedback solutions are related to the fact that they may saturate the actuators, excite high-frequency modes, etc.

In order to overcome these drawbacks, this paper proposes a combined controller that introduces an adaptive compensation term into the original controller. The adaptive compensation term plays the role as integrators in PID controllers. In comparison with integrators, the adaptive compensation term can compensate for not only the constant component but also the periodic components in unknown dynamics. Therefore, the combined controller is especially suitable for scenarios where a few periodic components contribute major proportion of unknown dynamics in amplitude. By introducing the adaptive compensation term, loads on the feedback controllers can be greatly relieved; hence good tracking performance can be achieved with moderate controller gains.

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The rest of this paper is organized as follows. In Section 1, an uncertain time-delay system and two assumptions are presented. Moreover, a new method to solve the equation, which guarantees the existence of a reference model, is proposed. Based on the assumptions presented in Section 1, the adaptive compensation term is designed and analyzed in Section 2. In Section 3, we demonstrate the feasibility of the proposed controller through numerical examples.

Notations. $\mathbf{R}^n$ is the Euclidean space of dimension n and $\mathbf{R}_+$ denotes the space of positive real numbers in $\mathbf{R}$. I_n is an identity matrix with dimension n. "0" denotes the zero matrix of appropriate dimensions. $\|\cdot\|$ is defined as the Euclidean norm or a matrix norm induced by the Euclidean norm. $\|\boldsymbol{x}\|_\infty = \sup_{t\in[t_0,+\infty)} \|\boldsymbol{x}(t)\|$. $\mathcal{C}^n_{PT}$ is the space of continuous and periodic functions with periodicity $T: \boldsymbol{x}(t) = \boldsymbol{x}(t-T)$, $\boldsymbol{x}(t)\in\mathbf{R}^n$.

1 Problem formulation and controller structure

1.1 Problem formulation

Consider the uncertain time-delay systems of the following form:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= [A+\Delta A(\upsilon,t)]\,\boldsymbol{x}(t) + A_d(\zeta,t)\boldsymbol{x}(t-\tau)+\\ &\quad\ [B+\Delta B(\nu,t)]\,\boldsymbol{u}(t)+\boldsymbol{\omega}(q,t)\\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t)\\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t),\ t\in[-\tau,0]\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^{n\times 1}$, $\boldsymbol{u}(t)\in\mathbf{R}^{m\times 1}$, and $\boldsymbol{y}(t)\in\mathbf{R}^{p\times 1}$ represent the state, input, and output vectors, respectively; $\tau\in\mathbf{R}_+$ is the unknown time delay; $A\in\mathbf{R}^{n\times n}$, $B\in\mathbf{R}^{n\times m}$, $C\in\mathbf{R}^{p\times n}$ are constant matrices; $\Delta A(\upsilon,t)$, $A_d(\zeta,t)\in\mathbf{R}^{n\times n}$ represent system uncertainties; $\Delta B(\nu,t)$ represents the input matrix uncertainty; and $\boldsymbol{\omega}(q,t)\in\mathbf{R}^{n\times 1}$ is an additive disturbance vector. The uncertain parameters $(\upsilon,\zeta,\nu,q)\in\Psi$ are Lebesgue measurable and take values in a known compact set Ω. It is assumed that the right-hand side of (1) is continuous and satisfies enough smoothness conditions to ensure the existence and uniqueness of the solution through every initial condition $\boldsymbol{\phi}(t)$.

A generalized assumption on system (1) is proposed in [2, 4]:

Assumption 1. For all $(\upsilon,\zeta,\nu,q)\in\Psi$, $\Delta A(\cdot)$, $A_d(\cdot)$, $\Delta B(\cdot)$, and $\boldsymbol{\omega}(\cdot)$ are continuous bounded matrix functions, and moreover, $\Delta B(\nu,t) = BE(\nu,t)$ and $E(\nu,t)^{\mathrm{T}}+E(\nu,t)+I_m>0$.

Our objective is to make $\boldsymbol{y}(t)$ in (1) track a reference trajectory $\boldsymbol{y}_m(t)$ which is generated by

$$\begin{aligned}\dot{\boldsymbol{x}}_m(t) &= A_m\boldsymbol{x}_m(t)\\ \boldsymbol{y}_m(t) &= C_m\boldsymbol{x}_m(t)\end{aligned} \tag{2}$$

where $\boldsymbol{x}_m(t)\in\mathbf{R}^{n_m\times 1}$ is bounded for all time, $A_m\in\mathbf{R}^{n_m\times n_m}$, and $C_m\in\mathbf{R}^{p\times n_m}$. The tracking error is defined as

$$\boldsymbol{e}(t) = \boldsymbol{y}(t)-\boldsymbol{y}_m(t) \tag{3}$$

As pointed out in [1−4], not all models in the form of (2) can be tracked by system (1). The requirement is that there exist $G\in\mathbf{R}^{n\times n_m}$ and $H\in\mathbf{R}^{m\times n_m}$ that satisfy the following equation:

$$\left\{\begin{aligned} &AG+BH = GA_m\\ &CG = C_m\end{aligned}\right. \tag{4}$$

By using direct product $\otimes$ of matrices and the "vec(·)" operation defined in [6], (4) is equivalent to

$$\Pi\boldsymbol{\chi} = R \tag{5}$$

where

$$\Pi = \begin{bmatrix} I_{n_m} \otimes A - A_m^{\mathrm{T}} \otimes I_n & I_{n_m} \otimes B \\ I_{n_m} \otimes C & 0 \end{bmatrix}$$

$$\boldsymbol{\chi} = \begin{bmatrix} \mathrm{vec}(G) \\ \mathrm{vec}(H) \end{bmatrix},\ R = \begin{bmatrix} 0 \\ \mathrm{vec}(C_m) \end{bmatrix}$$

The solutions to (4) exist iff $\mathrm{rank}\,[\Pi, R] = \mathrm{rank}\,\Pi$. One of the solutions is

$$\boldsymbol{\chi} = \Pi^{\dagger} R \tag{6}$$

where $\Pi^{\dagger}$ denotes the Moore-Penrose inverse of Π. We then can obtain the solution G and H from $\boldsymbol{\chi}$. If a solution cannot be found to satisfy (5), then a different reference model must be chosen.

1.2 Controller structure

The controller $\boldsymbol{u}(t)$ is designed with two parts:

$$\boldsymbol{u}(t) = \boldsymbol{u}_{ori}(t) + \boldsymbol{u}_f(t) \tag{7}$$

where $\boldsymbol{u}_{ori}(t)$ is an original controller proposed in other literature as [2], which is used to stabilize the closed-loop system; $\boldsymbol{u}_f(t)$ is the adaptive compensation term, which aims to compensate for the periodic unknown dynamics. We will focus on designing $\boldsymbol{u}_f(t)$ in this paper. The original controller $\boldsymbol{u}_{ori}(t)$ is represented as follows:

$$\boldsymbol{u}_{ori}(t) = H\boldsymbol{x}_m(t) + \boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t) \tag{8}$$

where $H \in \mathbf{R}^{m\times n_m}$ satisfies (4), $\boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t) \in \mathbf{R}^{m\times 1}$ is the feedback controller, and $\boldsymbol{x}_m(t)$ is the state of reference model (2).

Applying (7) to (1) yields the auxiliary system described by[1−2]

$$\begin{aligned}\dot{\boldsymbol{z}}(t) = {} & f(\boldsymbol{z}, \upsilon, \zeta, t) + \\ & [B + \Delta B(\nu, t)]\,[\boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t) + \boldsymbol{u}_f(t)] + \boldsymbol{\eta}(t)\end{aligned} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{z}(t) &= \boldsymbol{x}(t) - G\boldsymbol{x}_m(t) \\ f(\boldsymbol{z}, \upsilon, \zeta, t) &= [A + \Delta A(\upsilon, t)]\,\boldsymbol{z}(t) + A_d(\zeta, t)\boldsymbol{z}(t-\tau) \\ \boldsymbol{\eta}(t) &= \boldsymbol{\eta}(\upsilon, \zeta, \nu, q, t) = \\ & \Delta A(\upsilon, t)G\boldsymbol{x}_m(t) + A_d(\zeta, t)G\boldsymbol{x}_m(t-\tau) + \\ & \Delta B(\nu, t)H\boldsymbol{x}_m(t) + \boldsymbol{\omega}(q, t) \end{aligned}$$

Since $CG = C_m$ by (4), it follows that

$$C\boldsymbol{z}(t) = C\boldsymbol{x}(t) - CG\boldsymbol{x}_m(t) = \boldsymbol{e}(t)$$

consequently,

$$\|\boldsymbol{e}(t)\| \leq \|C\|\,\|\boldsymbol{z}(t)\|$$

In this case, $\boldsymbol{z}(t)$ can be considered as a new tracking error.

To make the problem tractable, we impose the following assumption to describe the stability of system (9) with $\boldsymbol{\eta}(t) \equiv \mathbf{0}$ and $\boldsymbol{u}_f(t) \equiv \mathbf{0}$.

Assumption 2. There exists a known continuously differentiable functional $V_1(t) \in \mathbf{R}_+ \cup \{0\}$ such that

$$\mu_1 \|\boldsymbol{z}(t)\|^2 \leq V_1(t) \leq \mu_2 \|\boldsymbol{z}(t)\|^2 + \mu_3 \int_{t-\tau}^{t} \|\boldsymbol{z}(s)\|^2\,\mathrm{d}s \tag{10}$$

and

$$\|\partial_z V_1(t)\| \leq b_{V_1}\|\boldsymbol{z}(t)\| \tag{11}$$

where $\partial_z V_1 = \frac{\partial V_1}{\partial z} \in \mathbf{R}^{n\times 1}$, $\mu_1, \mu_2, b_{V_1} \in \mathbf{R}_+$, and $\mu_3 \in \mathbf{R}_+ \cup \{0\}$. Moreover, the derivative of $V_1(t)$ along the trajectories of (9) with $\boldsymbol{\eta}(t) \equiv \mathbf{0}$ and $\boldsymbol{u}_f(t) \equiv \mathbf{0}$ satisfies

$$\dot{V}_1(t) \leq -\mu_4 \|\boldsymbol{z}(t)\|^2 \tag{12}$$

where $\mu_4 \in \mathbf{R}_+$.

Remark 1. Assumption 2 is imposed on the auxiliary system (9) without $\boldsymbol{u}_f(t)$ and any external disturbance, and makes us focus on designing the adaptive compensation. The control scheme proposed in [2] satisfies Assumption 2.

2 Adaptive compensation term design and analysis

The proposed adaptive compensation term $\boldsymbol{u}_f(t)$ is introduced to compensate for periodic components in unknown dynamics $\boldsymbol{\eta}(t)$. In Subsection 2.1, we first determine periods of major periodic components that exist in unknown dynamics $\boldsymbol{\eta}(t)$. Then, the adaptive compensation term is designed, and the stability of system (9) is analyzed in Subsections 2.2 and 2.3, respectively.

2.1 Major periods in unknown dynamics

In practice, systems such as industrial robots, magnetic disk, and CD drives, etc., operate repetitively over a fixed time interval and are subject to periodic uncertainties and disturbances[7−9]. In the above-mentioned systems, periodic components contribute a major proportion of the unknown dynamics $\boldsymbol{\eta}(t)$ in amplitude. According to this, the unknown dynamics $\boldsymbol{\eta}(t)$ in (9) can be decomposed into two parts:

$$\boldsymbol{\eta}(t) = B\boldsymbol{g}_p(t) + \boldsymbol{g}_r(t) \tag{13}$$

where $\boldsymbol{g}_p(t) = \boldsymbol{g}_p(\upsilon, \zeta, \nu, q, t)$ is the major periodic components of $\boldsymbol{\eta}(t)$, denoted by $\boldsymbol{g}_p(t) = -\sum_{i=1}^{N_p}\boldsymbol{g}_{T_i}(t)$, $\boldsymbol{g}_{T_i}(t) \in \mathcal{C}^m_{PT_i}$, and $\boldsymbol{g}_r(t) = \boldsymbol{g}_r(\upsilon, \zeta, \nu, q, t)$ is the remaining part of $\boldsymbol{\eta}(t)$, i.e., $\boldsymbol{g}_r(t) = \boldsymbol{\eta}(t) - B\boldsymbol{g}_p(t)$. The periods T_i, $i = 1, \cdots, N_p$ are the periods of major periodic components, which exist in unknown dynamics $\boldsymbol{\eta}(t)$. It should be noted that the concrete forms of $\boldsymbol{g}_p(t)$, $\boldsymbol{g}_r(t)$ are unknown, and this decomposition is only used for analysis.

In most cases, the periodicity of the disturbance and the states are related to the periodicity of the reference trajectory. Therefore, roughly speaking, T_i, $i = 1, \cdots, N_p$ can be determined based on analysis of the major periods of the reference trajectory $\boldsymbol{y}_m(t)$. Even if T_i, $i = 1, \cdots, N_p$ are unknown as a priori, we can also extract the major periods from the output data. First, we design a controller $\boldsymbol{u}(t) = \boldsymbol{u}_{ori}(t)$, then, (9) becomes

$$\begin{aligned}\dot{\boldsymbol{z}}(t) = {} & [A + \Delta A(\upsilon, t)]\,\boldsymbol{z}(t) + A_d(\zeta, t)\boldsymbol{z}(t-\tau) + \\ & [B + \Delta B(\nu, t)]\,\boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t) + \boldsymbol{\eta}(t)\end{aligned} \tag{14}$$

The above system is a perturbed time-varying system. If $\boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t)$ is chosen as in [2] and the conditions of Theorem 2 in [2] are satisfied, then the trivial solution of system (14) with $\boldsymbol{\eta}(t) \equiv \mathbf{0}$ is uniformly asymptotically stable. The solution $\boldsymbol{z}(t)$ of (14) with initial condition $\boldsymbol{z}_{t_0}$ can be represented[10] to be

$$\boldsymbol{z}(\boldsymbol{z}_{t_0}, t_0, t) = \Phi_{\upsilon,\zeta,\nu}(t, t_0)\boldsymbol{z}_{t_0} + \int_{t_0}^{t} \Phi_{\upsilon,\zeta,\nu}(t, s)X_0\boldsymbol{\eta}(s)\mathrm{d}s = \Phi_{\upsilon,\zeta,\nu}(t, t_0)\boldsymbol{z}_{t_0} - \sum_{i=1}^{N_p}\boldsymbol{z}_{g_{T_i}}(t) + \boldsymbol{z}_r(t)$$

where X_0 is the special matrix function given by $X_0(\theta) = 0$, $-\tau \leq \theta < 0$, $X_0(0) = I_n$, $\boldsymbol{z}_{g_{T_i}}(t) = \int_{t_0}^{t} \Phi_{\upsilon,\zeta,\nu}(t,s) X_0 B \boldsymbol{g}_{T_i}(s) \mathrm{d}s$, and $\boldsymbol{z}_r(t) = \int_{t_0}^{t} \Phi_{\upsilon,\zeta,\nu}(t,s) X_0 \boldsymbol{g}_r(s) \mathrm{d}s$. Since $\Delta A(\upsilon,t)$, $\Delta B(\nu,t)$, and $A_d(\zeta,t)$ are all bounded, there exist constants $K, \alpha \in \mathbf{R}_+$ such that[10]

$$\|\Phi_{\upsilon,\zeta,\nu}(t,s)\| \leq K\mathrm{e}^{-\alpha(t-s)}$$

for all $s \in \mathbf{R}_+ \cup \{0\}$, and consequently, the terms $\boldsymbol{z}_{g_{T_i}}(t)$ approach periodic functions with period T_i as $t \to +\infty$[11], $i = 1, \cdots, N_p$. If the periodic components $\boldsymbol{g}_{T_i}(t), i = 1, \cdots, N_p$ contribute a major proportion of $\boldsymbol{\eta}(t)$ in amplitude, accordingly, the solution $\boldsymbol{z}(t)$ in (14) will be consisted of more periodic components with periods T_i, $i = 1, \cdots, N_p$. We can extract these periods T_i, $i = 1, \cdots, N_p$ by computation of the autocorrelation function[12].

2.2 Adaptive compensation term design

According to the above discussion, in order to compensate for $\boldsymbol{g}_{T_i}(t)$, $i = 1, \cdots, N_p$, $\boldsymbol{u}_f(t)$ in (7) is designed as

$$\begin{aligned} &\boldsymbol{u}_f(t) = \sum_{i=1}^{N_p} \boldsymbol{u}_{f_i}(t) \\ &\boldsymbol{u}_{f_i}(t) = \mathrm{sat}_{\boldsymbol{\beta}}\left[\boldsymbol{u}_{f_i}(t - T_i)\right] - \bar{k}_i(t) B^{\mathrm{T}} \partial_z V_1(t) \\ &\boldsymbol{u}_{f_i}(t) = 0,\ t \in [t_0 - T_i, t_0),\ i = 1, \cdots, N_p \end{aligned} \tag{15}$$

where $\bar{k}_i(t) \in \mathbf{R}$ has the form as

$$\bar{k}_i(t) = \begin{cases} 0, & t \in [t_0 - T_i, t_0) \\ \lambda_i(t), & t \in [t_0, t_0 + T_i) \\ k_i, & t \in [t_0 + T_i, +\infty) \end{cases}$$

and is chosen to be a monotone and continuous function on $[-T_i, +\infty)$, $V_1(t)$ is the functional defined in Assumption 2, and $\mathrm{sat}_{\boldsymbol{\beta}}(\cdot) \in \mathbf{R}^{m \times 1}$ is a vector function whose elements are defined as follows[13]:

$$\mathrm{sat}_{\beta_i}(\xi_i) = \begin{cases} \xi_i, & |\xi_i| \leq \beta_i \\ \mathrm{sgn}(\xi_i)\beta_i, & |\xi_i| > \beta_i \end{cases}$$
$$\forall \xi_i \in \mathbf{R},\ i = 1, \cdots, m$$

where $\boldsymbol{\beta} = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_m \end{bmatrix}^{\mathrm{T}}$.

For $\boldsymbol{g}_{T_i}(t)$, $i = 1, \cdots, N_p$ in (13), the following equations are assumed to be satisfied

$$\mathrm{sat}_{\boldsymbol{\beta}}\left[\boldsymbol{g}_{T_i}(t)\right] = \boldsymbol{g}_{T_i}(t),\ i = 1, \cdots, N_p \tag{16}$$

If β_i is chosen large enough, $i = 1, \cdots, m$, then (16) can be satisfied. In the worst case, $\exists k$, $\mathrm{sat}_{\boldsymbol{\beta}}\left[\boldsymbol{g}_{T_k}(t)\right] \neq \boldsymbol{g}_{T_k}(t)$, i.e., (16) does not hold, $\boldsymbol{g}_{T_k}(t)$ can be written as

$$\boldsymbol{g}_{T_k}(t) = \bar{\boldsymbol{g}}_{T_k}(t) + \left[\boldsymbol{g}_{T_k}(t) - \bar{\boldsymbol{g}}_{T_k}(t)\right]$$

where $\mathrm{sat}_{\beta}\left[\bar{\boldsymbol{g}}_{T_k}(t)\right] = \bar{\boldsymbol{g}}_{T_k}(t)$ and $\bar{\boldsymbol{g}}_{T_k}(t) \in \mathcal{C}_{PT_k}^{m}$. Thus, $\boldsymbol{\eta}(t)$ in (13) can be still decomposed into two parts as follows:

$$\boldsymbol{\eta}(t) = B\bar{g}_p(t) + \left[\boldsymbol{g}_r(t) + B(\boldsymbol{g}_{T_k}(t) - \bar{\boldsymbol{g}}_{T_k}(t))\right]$$

where $\bar{\boldsymbol{g}}_p(t) = \sum_{i=1, i\neq k}^{N_p} \boldsymbol{g}_{T_k}(t) + \bar{\boldsymbol{g}}_{T_k}(t)$. Without loss of generality, we assume that (16) is always satisfied here.

Remark 2. Similar to [14], the reason of introducing $\bar{k}_i(t)$ is to ensure that $\boldsymbol{u}_{f_i}(t)$ is continuous on $[-T_i, +\infty)$.

2.3 Adaptive compensation term analysis

We will analyze the stability of system (9) with $\boldsymbol{u}_f(t)$ designed as (15) in this section. Before proceeding further with the development of this work, the following preliminary result is needed.

Lemma 1. Assume that there exists a continuously differentiable functional $V(t) \in \mathbf{R}_+ \cup \{0\}$ such that

$$\gamma_1 \|\boldsymbol{z}(t)\|^2 \leq V(t) \leq \gamma_2 \|\boldsymbol{z}(t)\|^2 + \gamma_3 \int_{t-\tau}^{t} \|\boldsymbol{z}(s)\|^2 \mathrm{d}s + c \tag{17}$$

where $\gamma_1, \gamma_2 \in \mathbf{R}_+$ and $\gamma_3, c \in \mathbf{R}_+ \cup \{0\}$. If the derivative of $V(t)$ along the trajectories of (9) satisfies

$$\dot{V}(t) \leq -\gamma_4 \|\boldsymbol{z}(t)\|^2 + \delta \tag{18}$$

where $\gamma_4 \in \mathbf{R}_+, \delta \in \mathbf{R}_+ \cup \{0\}$, then the solution $\boldsymbol{z}(t)$ of (9) with a bounded initial condition is uniformly ultimately bounded.

Proof. Please see Appendix. □

Theorem 1. Assume that (4) has a solution. Under Assumptions 1 and 2, controller (7) with and without adaptive compensation term $\boldsymbol{u}_f(t)$ both guarantee uniform ultimate boundedness of the tracking error.

Proof. From Assumption 1, $\boldsymbol{\eta}(t) = \boldsymbol{\eta}(\upsilon, \zeta, \nu, q, t)$ in (8) is continuous bounded, denoted by $b_\eta = \sup_{(\upsilon,\zeta,\nu,q)\in\Psi} \|\boldsymbol{\eta}(\upsilon,\zeta,\nu,q,t)\|_\infty$. By using Assumption 2, the derivative of $V_1(t)$ along the trajectories of (9) with $\boldsymbol{u}_f(t) \equiv \boldsymbol{0}$ satisfies

$$\begin{aligned} \dot{V}_1(t) \leq & -\mu_4 \|\boldsymbol{z}(t)\|^2 + \left[\partial_z V_1(t)\right]^{\mathrm{T}} \boldsymbol{\eta}(t) \leq \\ & -\mu_4 \|\boldsymbol{z}(t)\|^2 + b_{V_1} b_\eta \|\boldsymbol{z}(t)\| \end{aligned}$$

Note that

$$b_{V_1} b_\eta \|\boldsymbol{z}(t)\| \leq \frac{\mu_4}{2} \|\boldsymbol{z}(t)\|^2 + \frac{1}{2\mu_4} b_{V_1}^2 b_\eta^2$$

Hence, $\dot{V}_1(t)$ is further bounded as

$$\dot{V}_1(t) \leq -\frac{\mu_4}{2} \|\boldsymbol{z}(t)\|^2 + \frac{1}{2\mu_4} b_{V_1}^2 b_\eta^2$$

Moreover, $V_1(t)$ has the form as (17) with $c = 0$. Therefore, controller (7) without adaptive compensation term $\boldsymbol{u}_f(t)$ guarantees uniform ultimate boundedness of the tracking error by Lemma 1.

Next, we will prove that controller (7) with $\boldsymbol{u}_f(t)$ can also guarantee uniform ultimate boundedness of the tracking error.

By using (13), system (9) can be further written as

$$\begin{aligned} \dot{\boldsymbol{z}}(t) = & f(\boldsymbol{z}, \upsilon, \zeta, t) + \left[B + \Delta B(\nu,t)\right] \boldsymbol{u}_b(\boldsymbol{x}, \boldsymbol{x}_m, t) + \\ & \boldsymbol{g}_r(t) + B \sum_{i=1}^{N_p} \tilde{\boldsymbol{u}}_{f_i}(t) + \Delta B(\nu,t) \boldsymbol{u}_f(t) \end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{u}}_{f_i}(t) = \boldsymbol{u}_{f_i}(t) - \boldsymbol{g}_{T_i}(t)$.

Design a candidate Lyapunov-Krasovskii functional as follows:

$$V(t) = V_1(t) + V_2(t) \tag{20}$$

where $V_1(t)$ is defined in Assumption 2. By Assumption 2, taking the derivative of $V_1(t)$ along the solution of (19) yields

$$\begin{aligned}\dot{V}_1(t) \le & -\mu_4 \|\boldsymbol{z}(t)\|^2 + [\partial_z V_1(t)]^{\mathrm{T}} \boldsymbol{g}_r(t) + \\ & [\partial_z V_1(t)]^{\mathrm{T}} B\sum_{i=1}^{N_p}\tilde{\boldsymbol{u}}_{f_i}(t) + \\ & [\partial_z V_1(t)]^{\mathrm{T}} \Delta B(\nu, t)\boldsymbol{u}_f(t)\end{aligned} \tag{21}$$

In order to eliminate the term $[\partial_z V_1(t)]^{\mathrm{T}} B\sum_{i=1}^{N_p}\tilde{\boldsymbol{u}}_{f_i}(t)$ in (21), we define the nonnegative function $V_2(t) \in \mathbf{R}_+ \cup \{0\}$ as follows:

$$V_2(t) = \sum_{i=1}^{N_p} W_i(t) \tag{22}$$

where $W_i(t) = 1/(2k_i)\int_{t-T_i}^{t} \tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(s)^{\mathrm{T}}\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(s)\mathrm{d}s$ and $\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t) = \mathrm{sat}_\beta[\boldsymbol{g}_{T_i}(t)] - \mathrm{sat}_\beta[\boldsymbol{u}_{f_i}(t)]$, $i = 1, \cdots, N_p$.

Recalling (16) and the proof of Theorem 1 in [13], we have

$$\begin{aligned}&\dot{W}_i(t) = \\ &\frac{1}{2k_i}\left[\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t)^{\mathrm{T}}\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t) - \tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t-T_i)^{\mathrm{T}}\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t-T_i)\right] \le \\ & -[\partial_z V_1(t)]^{\mathrm{T}} B\tilde{u}_{f_i}(t) - \frac{1}{2}k_i[\partial_z V_1(t)]^{\mathrm{T}} BB^{\mathrm{T}}\partial_z V_1(t)\end{aligned} \tag{23}$$

when $t \in [t_0 + T_i, +\infty)$, $i = 1, \cdots, N_p$.

Then, combining (21) and (24), we obtain

$$\begin{aligned}\dot{V}(t) \le & -\mu_4 \|\boldsymbol{z}(t)\|^2 + [\partial_z V_1(t)]^{\mathrm{T}} \boldsymbol{g}_r(t) - \\ & \frac{1}{2}k_e[\partial_z V_1(t)]^{\mathrm{T}} BB^{\mathrm{T}}\partial_z V_1(t) + \\ & [\partial_z V_1(t)]^{\mathrm{T}} \Delta B(\nu, t)\boldsymbol{u}_f(t)\end{aligned}$$

when $t \in [t_0 + T_M, +\infty)$, where $T_M = \max_{i=1,\cdots,N_p} T_i$ and $k_e = \sum_{i=1}^{N_p} k_i$.

Substituting $\Delta B(\nu, t) = BE(\nu, t)$ (see Assumption 1) and (15) into the above inequality yields

$$\begin{aligned}\dot{V}(t) \le & -\mu_4 \|\boldsymbol{z}(t)\|^2 + [\partial_z V_1(t)]^{\mathrm{T}} \boldsymbol{g}_r(t) - \\ & \frac{1}{2}k_e[\partial_z V_1(t)]^{\mathrm{T}} BB^{\mathrm{T}}\partial_z V_1(t) + \\ & [\partial_z V_1(t)]^{\mathrm{T}} \Delta B(\nu, t)\sum_{i=1}^{N_p}\mathrm{sat}_{\boldsymbol{\beta}}[\boldsymbol{u}_{f_i}(t-T_i)] - \\ & k_e[\partial_z V_1(t)]^{\mathrm{T}} B\frac{E(\nu,t)^{\mathrm{T}} + E(\nu,t)}{2}B^{\mathrm{T}}\partial_z V_1(t)\end{aligned}$$

Note that $E(\nu,t)^{\mathrm{T}} + E(\nu,t) + I_m > 0$ by Assumption 1. Hence

$$\begin{aligned}\dot{V}(t) \le & -\mu_4 \|\boldsymbol{z}(t)\|^2 + [\partial_z V_1(t)]^{\mathrm{T}} \boldsymbol{g}_r(t) + \\ & [\partial_z V_1(t)]^{\mathrm{T}} \Delta B(\nu, t)\sum_{i=1}^{N_p}\mathrm{sat}_{\boldsymbol{\beta}}[\boldsymbol{u}_{f_i}(t-T_i)]\end{aligned}$$

Moreover, by (11) in Assumption 2, the above inequality becomes

$$\dot{V}(t) \le -\mu_4 \|\boldsymbol{z}(t)\|^2 + \rho\|\boldsymbol{z}(t)\| \tag{24}$$

when $t \in [t_0 + T_M, +\infty)$, where $\rho = b_{V_1}b_{g_r} + b_{V_1}b_{\Delta B}N_p\|\boldsymbol{\beta}\|$, $b_{\Delta B} = \sup_{\nu\in\Psi}\|\Delta B(\nu,t)\|_\infty$, $b_{g_r} = \sup_{(\upsilon,\zeta,\nu,q)\in\Psi}\|\boldsymbol{g}_r(\upsilon,\zeta,\nu,q,t)\|_\infty$. Since

$$\rho\|\boldsymbol{z}(t)\| \le \frac{\mu_4}{2}\|\boldsymbol{z}(t)\|^2 + \frac{1}{2\mu_4}\rho^2$$

inequality (24) becomes

$$\dot{V}(t) \le -\frac{\mu_4}{2}\|\boldsymbol{z}(t)\|^2 + \frac{1}{2\mu_4}\rho^2 \tag{25}$$

when $t \in [t_0 + T_M, +\infty)$.

In light of (10) in Assumption 2, since every $W_i(t)$ in (22) is bounded (noticing the form of $\tilde{\boldsymbol{u}}_{\mathrm{sat}}^i(t)$ in (22)), $V(t)$ has the form

$$\begin{aligned}&\mu_1\|\boldsymbol{z}(t)\|^2 \le V(t) \le \\ &\qquad \mu_2\|\boldsymbol{z}(t)\|^2 + \mu_3\int_{t-\tau}^{t}\|\boldsymbol{z}(s)\|^2\mathrm{d}s + c_1\end{aligned} \tag{26}$$

where $c_1 \in \mathbf{R}_+ \cup \{0\}$ is a constant.

Since $V(t)$ on $[t_0, t_0 + T_M]$ is bounded, which implies $\boldsymbol{z}(t)$ is bounded on $[t_0, t_0 + T_M]$, the initial condition at time $t = t_0 + T_M$ is bounded. Recalling (25) and (26), we can conclude that controller (7) with adaptive compensation term $\boldsymbol{u}_f(t)$ guarantees uniform ultimate boundedness of the tracking error by Lemma 1. □

Remark 3. If the original controllers (8) as in [1−2] have guaranteed that the tracking error is uniformly ultimately bounded, then these major periods from $\boldsymbol{z}(t)$ can be extracted (see Subsection 2.1). By using the major periods and combining the original controllers, combined controllers (7) can be designed. Since the adaptive compensation term in (7) does not affect the stability of the closed-loop system and merely plays a role in improving the tracking performance, hence it is flexible in practice to decide whether to include the adaptive terms depending on the tracking performance requirements and controller gain constraints.

3 Numerical examples

3.1 Example 1

Compared with the methods of solving (7) in [1, 3], the proposed method in this paper is easier to be implemented, and furthermore, it can handle some problems that cannot be solved by the methods in [1, 3]. When $A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} -1 & 1 \end{bmatrix}$, $A_m = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $\boldsymbol{C}_m = \begin{bmatrix} 1 & 0 \end{bmatrix}$, we can verify that $\mathrm{rank}\begin{bmatrix} A & \boldsymbol{B} \\ \boldsymbol{C} & 0 \end{bmatrix} = 2 < 3$. However, the methods of solving (4) in [1, 3] require $\begin{bmatrix} A & \boldsymbol{B} \\ \boldsymbol{C} & 0 \end{bmatrix}$ to be full row rank. Therefore, it implies that G and $\boldsymbol{H}$ cannot be solved by using the methods in [1, 3]. However, by (6), G and $\boldsymbol{H}$ can be solved as $G = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ and $\boldsymbol{H} = \begin{bmatrix} 2 & 0 \end{bmatrix}$.

3.2 Example 2

Consider a nominal system[2]

$$\begin{aligned}&\dot{\boldsymbol{x}}(t) = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 2 \\ -1 & -2 & 0 \end{bmatrix}}_{A}\boldsymbol{x}(t) + \underbrace{\begin{bmatrix} 0 \\ 0.1 \\ 1 \end{bmatrix}}_{\boldsymbol{B}}u(t) \\ &y(t) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\boldsymbol{C}}\boldsymbol{x}(t) \\ &\boldsymbol{x}(t) = 0, t \in [-\tau, 0], t_0 = 0\end{aligned} \tag{27}$$

subject to uncertainties:

$$\Delta A(v,t) = \boldsymbol{B}\left[\begin{array}{ccc} 0.15\sin(0.7t) & 0 & 0.2 \end{array}\right]$$
$$A_d(\zeta,t) = \boldsymbol{B}\left[\begin{array}{ccc} 0 & 0.1 & 0.15\sin(0.3t) \end{array}\right]$$
$$\boldsymbol{\omega}(q,t) = 0.5\sin(t+1)\boldsymbol{B}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^{3\times1}$, $u(t) \in \mathbf{R}$, and $y(t) \in \mathbf{R}$. The rest of uncertainties are zero. The time delay is $\tau = 1$. Thus, Assumption 1 is satisfied. The objective is to find a controller that can drive the output of the system (27) with uncertainties to follow the output of the reference model given by

$$\dot{\boldsymbol{x}}_m(t) = \left[\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right]\boldsymbol{x}_m(t)$$
$$y_m(t) = \left[\begin{array}{cc} 1 & 0 \end{array}\right]\boldsymbol{x}(t), \quad \boldsymbol{x}_m(0) = \left[\begin{array}{cc} 1 & 0 \end{array}\right]^{\mathrm{T}}$$

The original controller $u_{ori}(t)$ is chosen as in [2]

$$u_{ori}(t) = \boldsymbol{H}\boldsymbol{x}_m(t) + (\boldsymbol{K} - \gamma\boldsymbol{B}^{\mathrm{T}}P)\left[\boldsymbol{x}(t) - G\boldsymbol{x}_m(t)\right] \quad (28)$$

If $\gamma > 4.1$,

$$\boldsymbol{K} = \left[\begin{array}{ccc} -3.2649 & -7.0158 & -5.2984 \end{array}\right]$$
$$G = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ -0.5786 & -0.5711 \end{array}\right], \quad \boldsymbol{H} = \left[\begin{array}{cc} 1.5711 & 1.4214 \end{array}\right]$$
$$P = \left[\begin{array}{ccc} 3.5587 & 1.5197 & 0.1416 \\ 1.5197 & 3.2 & 0.3965 \\ 0.1416 & 0.3965 & 0.3176 \end{array}\right]$$

are chosen as in Example 1 of [2]; then, Assumption 2 is satisfied with $\partial_z V_1(t) = P\boldsymbol{z}(t)$.

We first determine periods of major periodic components that exist in unknown dynamics. System (1) is driven by controller $u_{ori}(t)$ in (28) first. The new tracking error $\boldsymbol{z}(t)$ is uniformly ultimate bounded. Take $z_1(t)$ for example, where $z_1(t)$ represents the first variable of $\boldsymbol{z}(t)$, we use the autocorrelation function to determine major periods of $z_1(t)$. The curve of $z_1(t)$ on time interval $[5\times2\pi, 25\times2\pi]$ and its raw autocorrelation are shown in Fig. 1 (see function "xcorr" in Signal Processing Toolbox of Matlab Help).

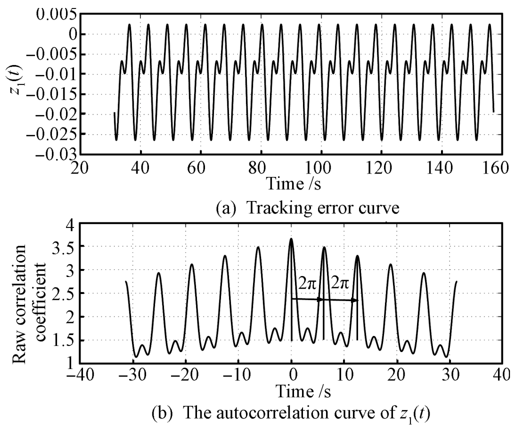

Fig. 1 Detect major periods by the autocorrelation function

As shown in Fig. 1, a major period of $\boldsymbol{z}(t)$ is 2π. Therefore, $T_1 = 2\pi$ is chosen as the major known period in this example. The combined controller and the adaptive compensation term are then designed as follows:

$$u(t) = u_{ori}(t) + u_{f_1}(t)$$
$$u_{f_1}(t) = \mathrm{sat}_\beta\left[u_{f_1}(t-T_1)\right] - \bar{k}_1(t)\boldsymbol{B}^{\mathrm{T}}P\boldsymbol{z}(t), \quad t \in [0,+\infty)$$
$$u_{f_1}(t) = 0,\ t\in[-T_1,0),\ T_1 = 2\pi,\ \beta = 1 \quad (29)$$

where $u_{ori}(t)$ is chosen as (28) and

$$\bar{k}_1(t) = \begin{cases} 0, & t\in[-2\pi,0) \\ k_1\dfrac{t}{2\pi}, & t\in[0,2\pi) \\ k_1, & t\in[2\pi,+\infty) \end{cases}$$

The comparison of tracking performance between controllers (28) and (29) is depicted in Fig. 2, where $\|e_i\|_T = \sup_{s\in[0,T_1]}\|e(iT_1+s)\|$.

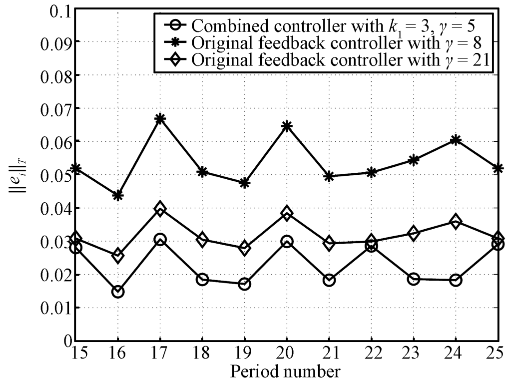

Fig. 2 Change of maximum Euclidean norm $\|e_i\|_T$ of error with the i-th period

As shown in Fig. 2, the tracking performance can be improved by increasing the control gain γ of original controller (28). However, the tracking performance of the combined controller (29) with $k_1 = 3$ and $\gamma = 5$ is best. It should be noted that the control gain γ of the combined controller (29) is smaller than that of original controller (28). Therefore, the combined controller is especially suitable for scenarios where a few periodic components contribute the major proportion of unknown dynamics in amplitude. By introducing the adaptive compensation term, loads on the feedback controllers can be greatly relieved; hence, good tracking performance can be achieved with moderate controller gains.

4 Conclusions

Various high-gain control schemes have been proposed for a class of uncertain dynamic delay systems to track dynamic outputs of a nondelay reference model in this paper. However, the drawbacks of the high-gain feedback solutions are related to the fact that they may saturate the actuators, excite high-frequency modes, etc. In order to overcome these drawbacks and relieve loads on the feedback controllers, an adaptive compensation term is introduced into the original controller to compensate for periodic components in the unknown dynamics. The main contributions of this paper are: 1) The introduction of the adaptive compensation term can help to achieve good performance with a moderate controller gain; 2) It is proven that adding or removing the adaptive compensation term does not affect the stability of closed-loop systems; 3) A scheme is given to determine the periods of major periodic components, which exist in unknown dynamics; 4) A new method is proposed to solve (4), which guarantees the existence of the reference model; and furthermore it can handle some problems that cannot be solved by the methods proposed in [1, 3].

Appendix The proof of Lemma 1

By (18), we can obtain

$$\|\boldsymbol{z}(t)\|^2 \leq \frac{\delta - \dot{V}(t)}{\gamma_4} \tag{A1}$$

Substituting (A1) into (17) yields

$$V(t) \leq \gamma_2 \frac{\delta - \dot{V}(t)}{\gamma_4} + \gamma_3 \int_{t-\tau}^{t} \frac{\delta - \dot{V}(s)}{\gamma_4} \mathrm{d}s + c$$

Note that

$$\int_{t-\tau}^{t} \dot{V}(s)\mathrm{d}s = V(t) - V(t-\tau)$$

Thus,

$$\dot{V}(t) \leq -\frac{\gamma_3 + \gamma_4}{\gamma_2} V(t) + \frac{\gamma_3}{\gamma_2} V(t-\tau) + \delta_b$$

where $\delta_b = (1 + \gamma_3\tau/\gamma_2)\delta + c\gamma_4/\gamma_2$.

Define a $U(t)$ such as

$$\dot{U}(t) = -\frac{\gamma_4 + \gamma_3}{\gamma_2} U(t) + \frac{\gamma_3}{\gamma_2} U(t-\tau) + \delta_b \tag{A2}$$

where its initial value satisfies $U_{t_0} = V_{t_0} = V(t_0 + s)$, $s \in [-\tau, 0]$. By the comparison lemma[15], $V(t) \leq U(t)$ holds. Since $(\gamma_4 + \gamma_3)/\gamma_2 > \gamma_3/\gamma_2$, the trivial solution of (A2) with $\delta_b = 0$ is exponentially stable[16]. $U(t)$ can be represented by[10]

$$U(t) = T(t, t_0)U_{t_0} + \delta_b \int_{t_0}^{t} T(t, s)X_0 \mathrm{d}s$$

where $\|T(t,s)\| \leq K'\mathrm{e}^{-\alpha'(t-s)}$, $K', \alpha' \in \mathbf{R}_+$, consequently,

$$\gamma_1 \|\boldsymbol{z}(t)\|^2 \leq V(t) \leq U(t) \leq K'\mathrm{e}^{-\alpha'(t-t_0)} \|V_{t_0}\| + \frac{K'}{\alpha'}\delta_b,\ t \geq t_0$$

by (17). Since $K'\delta_b/\alpha'$ is independent of the initial value, $\boldsymbol{z}(t)$ is uniformly ultimately bounded. □

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