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FILTERED REPETITIVE CONTROL OF ROBOT MANIPULATORS

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Abstract. *In this paper, a filtered repetitive controller (FRC) is proposed for robot manipulator tracking. Error dynamics are obtained first, leaving an unknown periodic signal to be compensated. A new model to describe the periodic signal is proposed then. By this model, an FRC is designed to compensate for the unknown periodic signal. The resulting closed-loop error dynamics are analyzed with the help of a Lyapunov-Krasovskii functional. In order to compare stability of the closed-loop error dynamics with the FRC and corresponding repetitive controller, the convergence rate is proposed to measure their stability. It is shown that the resulting closed-loop error dynamics with the FRC is more stable. In comparison with existing repetitive controllers, the proposed controller provides the flexibility to choose parameters to achieve a tradeoff between tracking performance and stability. Numerical simulations demonstrate the effectiveness of the proposed controller.*
Keywords: Repetitive control (RC), Robot manipulator, Disturbance

1. Introduction.

Repetitive control (RC) is an internal-model-based control approach in which the infinite-dimensional internal model $\frac{1}{1-e^{-sT}}$ gives rise to an infinite number of poles on the imaginary axis. It was proved in [1] that, for a class of general linear plants, exponential stability of RC systems could be achieved only when the plant is proper but not strictly proper. Moreover, the internal model $\frac{1}{1-e^{-sT}}$ may destabilize the system. To enhance stability, a suitable filter is introduced as shown in Figure 1, forming a *filtered repetitive controller*[1] (FRC or *filtered repetitive control*, also designated FRC) in which the loop gain is reduced at high frequencies. Stability results only at some sacrifice of high frequency performance. With appropriate design, however, an FRC can often achieve an acceptable tradeoff between tracking performance and stability, a tradeoff which broadens the application of RC in practice [1-7].

Over the years, RC of robot manipulators has received considerable attention [8-12]. The works of these literatures require the disturbances to be periodic. It is well known that, besides a periodic disturbance, robot manipulator tracking is often subject to a persistent nonperiodic disturbance as well. However, to the best knowledge of the authors, few works have investigated such a case. Theoretically speaking, a persistent nonperiodic disturbance may lead to RC systems instability, because non-exponential stability of RC systems implies that input-to-state stability may not be satisfied. Inspired by the fact that an FRC system is more stable than the corresponding RC system, we attempt to apply an FRC to robot manipulator tracking in the presence of both types of disturbance.

It is not trivial to design an FRC for robot manipulator tracking. The theories of FRC proposed in [1-4] are derived in frequency domain and can be applied only with difficulty, if at all, to nonlinear systems. For this reason, tracking performance needs to

[1]In this paper, we have replaced the term "modified" in [1] with the more descriptive term "filtered".

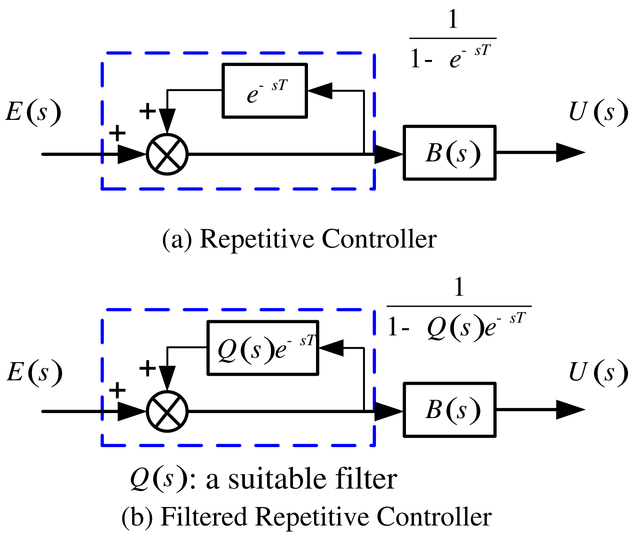

FIGURE 1. A suitable filter $Q(s)$ is introduced into a repetitive controller to form an FRC

be analyzed in the time domain. The major contributions of this paper are: (i) a method to design and analyze an FRC for robot manipulator tracking; (ii) the ability of our FRC to cope with persistent nonperiodic disturbances, namely the tracking error can be made uniformly ultimately bounded; and (iii) a tradeoff achieved by tuning filter parameters between tracking performance and stability.

We use the following notation. $\mathbb{R}^n$ is Euclidean space of dimension n and $\mathbb{R}_+$ denotes the space of nonnegative reals in $\mathbb{R}$. $\|\cdot\|$ denotes the Euclidean vector norm or induced matrix norm. $\|x\|_\infty \triangleq \sup\limits_{t\in[0,\infty)} \|x(t)\|$ and $\|x\|_{[a,b]} \triangleq \sup\limits_{t\in[a,b]} \|x(t)\|$. $\mathcal{C}_{PT}^n([0,\infty);\mathbb{R}^m)$ is the space of nth-order continuously differentiable functions $g:[0,\infty)\to\mathbb{R}^m$ which are T-periodic, i.e., $g(t+T)=g(t)$. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote the minimum and maximum eigenvalues respectively of a positive semidefinite matrix X. I_n is the identity matrix with dimension n.

2. **Problem Formulation.**

The dynamics of an m-degree-of-freedom manipulator are described by the following differential equation

$$D(q(t))\ddot{q}(t)+C(q(t),\dot{q}(t))\dot{q}(t)+G(q(t))=u(t)+w(t)+\varepsilon(t) \tag{1}$$

where $q(t)\in\mathbb{R}^m$ denotes the vector of generalized displacements in robot co-ordinates, $u(t)$ denotes the vector of generalized control input forces in robot coordinates; $D(q(t))\in\mathbb{R}^{m\times m}$ is the manipulator inertial matrix, $C(q(t),\dot{q}(t))\in\mathbb{R}^{m\times m}$ is the vector of centripetal and Coriolis torques and $G(q(t))\in\mathbb{R}^m$ is the vector of gravitational torques; in this paper, $w\in\mathcal{C}_{PT}^1([0,\infty);\mathbb{R}^m)$ and $\varepsilon\in\mathcal{L}_\infty$ are the T-periodic and persistent nonperiodic disturbances, respectively. It is assumed that only $q(t)$ and $\dot{q}(t)$ are available from measurements.

Define the filtered tracking error as

$$e(t)=\dot{\tilde{q}}(t)+\mu\tilde{q}(t) \tag{2}$$

where μ is a positive real number, $\tilde{q}(t)=q_d(t)-q(t)$ and $q_d(t)$ is a desired trajectory.

We will need the following assumptions, which are common to robot manipulators [8-17].

(A1) The inertial matrix $D(q(t))$ is symmetric, uniformly positive definite and bounded, i.e.,

$$0 < \lambda_m I_m \leq D(q(t)) \leq \lambda_M I_m, \quad \forall q(t) \in \mathbb{R}^m \tag{3}$$

where λ_m and λ_M are positive real numbers.

(A2) The matrix $\dot{D}(q(t)) - 2C(q(t), \dot{q}(t))$ is skew-symmetric, hence,

$$x^T \left[\dot{D}(q(t)) - 2C(q(t), \dot{q}(t))\right] x = 0, \quad \forall x \in \mathbb{R}^m.$$

(A3) The linear-in-the-parameters property [11] is written as

$$D(q(t))\ddot{q}_e(t) + C(q(t), \dot{q}(t))\dot{q}_e(t) + G(q(t)) = \Psi(q, \dot{q}, \dot{q}_e, \ddot{q}_e, t)\, p \tag{4}$$

where $\dot{q}_e(t) = \dot{q}_d(t) + \mu\tilde{q}(t)$, $\ddot{q}_e(t) = \ddot{q}_d(t) + \mu\dot{\tilde{q}}(t)$, $p \in \mathbb{R}^l$ is the vector of unknown constant parameters, and $\Psi(q, \dot{q}, \dot{q}_e, \ddot{q}_e, t) \in \mathbb{R}^{m \times l}$ is a known matrix, denoted by $\Psi(t)$ for brevity.

For a given desired trajectory $q_d \in \mathcal{C}^2_{PT}([0, \infty); \mathbb{R}^m)$, our objective is to design a controller with the following two properties: (i) with certain parameters, $\lim\limits_{t \to \infty} e(t) = 0$ when $\varepsilon(t) \equiv 0$; (ii) with another set of appropriate parameters, for any value of $e(0)$, $e(t)$ is uniformly ultimately bounded (for the definition see [18]) when $\varepsilon \in \mathcal{L}_\infty$.

Remark 2.1. *From (2), we know that both $\tilde{q}(t)$ and $\dot{\tilde{q}}(t)$ can be viewed as outputs of a stable system with $e(t)$ as input, which means that $\tilde{q}(t)$ and $\dot{\tilde{q}}(t)$ are bounded or approach zero if $e(t)$ is bounded or approaches zero. Assumptions (A1) and (A2) are very common to a robot manipulator; and (A3) illustrates separation of the unknown parameters and the known functions, which is often used in literature on adaptive control [11,15-17].*

3. Controller Design.

3.1. Controller structure.

Design $u(t)$ as follows:

$$u(t) = Pe(t) + \Psi(t)\hat{p}(t) + \hat{w}(t) \tag{5}$$

where $P \in \mathbb{R}^{m \times m}$ is a positive definite matrix, $\hat{p}(t) \in \mathbb{R}^l$ is the estimate of p in (A3), and $\hat{w}(t) \in \mathbb{R}^m$ is the estimate of $w(t)$. By employing (4) and (5), the filtered error dynamics can be obtained as follows:

$$D(q(t))\dot{e}(t) + C(q(t), \dot{q}(t))e(t) = -Pe(t) + R(t)[v(t) - \hat{v}(t)] + \varepsilon(t) \tag{6}$$

where

$$R(t) = \begin{bmatrix} \Psi(t) & I_m \end{bmatrix}, \quad v(t) = \begin{bmatrix} p^T & w^T(t) \end{bmatrix}^T, \quad \hat{v}(t) = \begin{bmatrix} \hat{p}^T(t) & \hat{w}^T(t) \end{bmatrix}^T.$$

Then, $u(t)$ in (5) is rewritten as

$$u(t) = Pe(t) + R(t)\hat{v}(t). \tag{7}$$

Here, the unknown signal $v \in \mathcal{C}^1_{PT}\left([0, \infty); \mathbb{R}^{m+l}\right)$ contains the periodic disturbance $w \in \mathcal{C}^1_{PT}([0, \infty); \mathbb{R}^m)$ and the unknown parameter $p \in \mathbb{R}^l$, and $\hat{v}(t) \in \mathbb{R}^m$ is an estimate of $v(t)$ which is provided by the designed FRC.

3.2. **A new model of periodic signals.** In this section, a new model to describe the periodic signal $v(t)$ is proposed. In order to show the difference, the usual model of periodic signals is given first. Any $v \in \mathcal{C}_{PT}^{1}\left([0,\infty);\mathbb{R}^{m+l}\right)$ can be generated by the model [1]:

$$\begin{aligned} x(t) &= x(t-T) \\ v(t) &= x(t) \\ x(\theta) &= v(T+\theta), \quad \theta \in [-T,0] \end{aligned} \tag{8}$$

where $x(t) \in \mathbb{R}^{m+l}$ is the state. By the internal model principle [19], it is expected that asymptotic rejection of a periodic disturbance can be achieved by incorporating the model (8), i.e., $\frac{1}{1-e^{-sT}} I_{m+l}$ (the transfer function of (8)), into the closed-loop system. This is also the basic idea of RC [1].

A new model to describe $v \in \mathcal{C}_{PT}^{1}\left([0,\infty);\mathbb{R}^{m+l}\right)$, which will help to design the FRC for the nonlinear system (6), is given in Lemma 3.1.

Lemma 3.1. *For any given $v \in \mathcal{C}_{PT}^{1}\left([0,\infty);\mathbb{R}^{m+l}\right)$ and $\epsilon \in \mathbb{R}_{+}$, there exists a function $\sigma \in \mathcal{C}_{PT}^{0}\left([0,\infty);\mathbb{R}^{m+l}\right)$ such that*

$$\epsilon \dot{v}(t) = -v(t) + (1-\epsilon\alpha) v(t-T) + \sigma(t) \tag{9}$$

with a bounded initial condition. Moreover, the function σ satisfies

$$\|\sigma(t)\| \leq \epsilon \|\dot{v} + \alpha v\|_{[0,T]} \tag{10}$$

for all $t \in \mathbb{R}_{+}$.

Proof: See Appendix A.1.

Remark 3.1. *Similarly, we can also obtain a new model of periodic signals in discrete form. The usual model of periodic signals with a period NT_p can be represented in discrete form as $v(k) = v(k-N)$, where T_p is the sampling period. Let $v(t) := v(k)$, $v(t-T) := v(k-N)$, $\dot{v}(t) := \frac{v(k)-v(k-1)}{T_p}$. Then, system (9) becomes*

$$v(k) = \frac{\epsilon}{\epsilon + T_p} v(k-1) + \frac{(1-\epsilon\alpha) T_p}{\epsilon + T_p} v(k-N) + \sigma(k) \tag{11}$$

where $\sigma(k) = \frac{\epsilon}{\epsilon+T_p}\left[(1+\alpha T_p) v(k) - v(k-1)\right]$. The proof is similar to that of Lemma 3.1. In particular, if $\epsilon = 0$, then the system (11) reduces to $v(k) = v(k-N)$, where $\sigma(k) \equiv 0$. The new model of periodic signals in discrete form can be applied to design FRCs for discrete-time systems.

3.3. **FRC design.** In this section, an FRC is designed with the help of the new model (9). The closed-loop error dynamics are then analyzed with the help of a Lyapunov-Krasovskii functional.

By model (9), the estimate $\hat{v}(t)$ is given by

$$\begin{aligned} &\epsilon \dot{\hat{v}}(t) = -\hat{v}(t) + (1-\epsilon\alpha) \hat{v}(t-T) + h(t, e(t)) \\ &\hat{v}(\theta) = 0, \quad \theta \in [-T,0] \end{aligned} \tag{12}$$

where $h : \mathbb{R}_{+} \times \mathbb{R}^{m} \to \mathbb{R}^{m+l}$. The Laplace transform of (12) is

$$\hat{v}(s) = \frac{1}{1 - Q(s) e^{-sT}} I_{m+l} \cdot \frac{1}{1-\epsilon\alpha} Q(s) h_e(s) \tag{13}$$

where $Q(s) = \frac{1-\epsilon\alpha}{\epsilon s+1}$ is a filter; $\hat{v}(s)$ and $h_e(s)$ are the Laplace transforms of $\hat{v}(t)$ and $h(t, e(t))$, respectively. Therefore, as shown in Figure 1, (12) can be viewed as an FRC. In particular, if $\epsilon = 0$, then (12) reduces to

$$\begin{aligned} \hat{v}(t) &= \hat{v}(t-T) + h(t, e(t)) \\ \hat{v}(\theta) &= 0, \quad \theta \in [-T, 0] \end{aligned} \tag{14}$$

which is the corresponding repetitive controller. Compared with the repetitive controller (14), a major advantage of the proposed FRC (12) is that the choice of the parameter ϵ provides the flexibility to satisfy different performance requirements.

Subtracting (12) from (9) yields

$$\epsilon\dot{\tilde{v}}(t) = -\tilde{v}(t) + (1-\epsilon\alpha)\tilde{v}(t-T) - h(t, e(t)) + \sigma(t) \tag{15}$$

where $\tilde{v} \triangleq v - \hat{v}$. In (15), the initial value of $\tilde{v}$ is bounded. We do not concern ourselves with the concrete initial value as the following results hold globally.

3.4. **Stability analysis.** The closed-loop error dynamics forming by (6) and (15) will be analyzed with the help of a Lyapunov-Krasovskii functional, with the results stated in Theorem 3.1. Denote $z \triangleq \left[\begin{array}{cc} \tilde{v}^T & e^T \end{array}\right]$ and $\bar{\sigma} = \|\sigma\|_\infty + \|\varepsilon\|_\infty$ here.

Theorem 3.1. *Suppose (i) (A1) – (A3) hold; (ii) the function C: $\mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is bounded when $q(t)$ and $\dot{q}(t)$ are bounded on $\mathbb{R}_+$, the function R: $\mathbb{R}_+ \times \mathbb{R}^m \to \mathbb{R}^{m\times(m+l)}$ is bounded when $e(t)$ is bounded on $\mathbb{R}_+$; (iii) in controller (7), the estimate $\hat{v}(t)$ is designed as (12) with*

$$h(t, e(t)) = R^T(t)\, e(t), \quad 0 \le \epsilon\alpha < 1, \quad \alpha > 0, \quad \epsilon > 0. \tag{16}$$

We claim that (i) if $\varepsilon(t) \equiv 0$ and $\epsilon = 0$, then $\lim_{t\to\infty} e(t) = 0$ for any bounded initial conditions; (ii) if $\varepsilon \in \mathcal{L}_\infty$ and $\epsilon > 0$, there exists a positive real number k such that

$$\|z(t)\| \le \sqrt{\frac{b}{\gamma_1\gamma_3}(\gamma_2 + T)} + \sqrt{\frac{k}{\gamma_1}} e^{-\frac{\eta}{2}t}, \tag{17}$$

where the convergence rate η is the unique solution to the equation

$$-\eta = -\frac{\gamma_3 + 1}{\gamma_2} + \frac{1}{\gamma_2} e^{\eta T} \tag{18}$$

and $\gamma_1 = \min(\lambda_m, \epsilon)$, $\gamma_2 = \max(\lambda_M, \epsilon)$, $\gamma_3 = \min[\lambda_{\min}(P), \epsilon\alpha(2-\epsilon\alpha)/2]$, $b = \frac{\bar{\sigma}^2}{\min(\lambda_{\min}(P), \epsilon\alpha(2-\epsilon\alpha)/2)}$.

Proof: See Appendix A.2.

Remark 3.2. *When $\varepsilon \in \mathcal{L}_\infty$, if $\epsilon = 0$ is chosen, i.e., the repetitive controller is chosen, then, stability of these closed-loop systems is difficult to establish (refer to (24) in Appendix). However, the FRC (12) with $\epsilon > 0$ can cope with this case and the tracking error is uniformly ultimately bounded (refer to (17)).*

Remark 3.3. *Obviously, the time-domain analysis proposed in this paper can also apply to linear systems. Compared with the frequency-domain analysis, the proposed analysis can give some indexes of the tracking error in time domain, such as the ultimate bound (refer to (17)).*

Remark 3.4. *The filtered error dynamics (6) can be written to be a kind of general error dynamics as*

$$\dot{e}(t) = f(t, e(t)) + b(t, e(t))[v(t) - \hat{v}(t)] + \varepsilon_a(t)$$

with

$$f(t,e(t)) = -D^{-1}(q(t))\,C(q(t),\quad \dot{q}(t))\,e(t) - D^{-1}(q(t))\,Pe(t),$$
$$b(t,e(t)) = D^{-1}(q(t))\,R(t),\quad \varepsilon_a(t) = D^{-1}(q(t))\,\varepsilon(t).$$

By using appropriate assumptions and following the similar idea, an FRC can be designed for the kind of general error dynamics as well. This broadens the application of the design idea here.

In the remainder of this section, we will propose an index, namely convergence rate, to measure the stability of the closed-loop system under consideration. To begin with, we show that the index is reasonable. Consider a simple linear system $\dot{x}(t) = Ax(t)$ first. There exists a positive real number k' such that $\|x(t)\| \leq k'e^{-\eta' t}$, where $\eta' = -\max$ $\mathrm{Re}(\lambda(A))$ is the convergence rate. If A is marginally stable, then $\eta' = 0$. On the other hand, if A is stable, then $\eta' > 0$. It is well known that the latter is more stable than the former, i.e., a larger convergence rate implies a better stability. Linear RC systems are in fact neutral type time-delay systems in a critical case where asymptotic stability is not equivalent to exponential stability [20, 21]. This implies that there is no positive real number η' such that $\|x(t)\| \leq k'e^{-\eta' t}$, even if linear RC systems are asymptotically stable. Whereas, linear FRC systems are in fact retarded type systems where asymptotic stability is equivalent to exponential stability. This implies that there is a positive η' such that $\|x(t)\| \leq k'e^{-\eta' t}$, if linear FRC systems are asymptotically stable. It is well known that an FRC system is more stable than the corresponding RC system. This confirms again that a larger convergence rate implies the better stability. Therefore, we use the convergence rate η in (17) to measure stability of the closed-loop system under consideration. Similarly, the larger convergence rate η implies better stability. In the following, we show in Theorem 3.2 that stability of the resulting closed-loop error dynamics is improved as ϵ increases, namely $\partial\eta(\epsilon)/\partial\epsilon > 0$.

Theorem 3.2. *Suppose (i) the conditions of Theorem 3.1 hold, (ii)* $\lambda_M > \epsilon$ *and* $\epsilon\alpha(2 - \epsilon\alpha)/2 < \lambda_{\min}(P)$. *Then,* $\partial\eta(\epsilon)/\partial\epsilon > 0$.

Proof: See Appendix A.3.

Remark 3.5. *The parameter* α *is often set to be a small positive number, hence, the condition (ii) of Theorem 3.2 is satisfied. According to the proposed stability index, stability of the resulting closed-loop error dynamics is improved as* ϵ *increases.*

4. **Numerical Simulations.** The robot used for computer simulation is a three-link manipulator [15, pp.62-63]. According to the dynamic equations, we obtain the following expression:

$$p = \begin{bmatrix} I_1 & l_2 & l_3 \end{bmatrix}^T$$
$$\Psi(t) = \begin{bmatrix} \ddot{q}_{e,1}(t) & 0 & q_3^2(t)\,\ddot{q}_{e,1}(t) + q_3(t)\,\dot{q}_3(t)\,\dot{q}_{e,1}(t) + q_3(t)\,\dot{q}_1(t)\,\dot{q}_{e,3}(t) \\ 0 & \ddot{q}_{e,2}(t) + g & \ddot{q}_{e,2}(t) + g \\ 0 & 0 & \ddot{q}_{e,3}(t) - q_3(t)\,\dot{q}_1(t)\,\dot{q}_{e,1}(t) \end{bmatrix}$$

where $I_1 = 0.8$ kg/m^2, $l_2 = 2$ m, $l_3 = 1$ m, $g = 9.8$ m/s^2 and x_j denote the jth element in the vector x, $x = \{q, \dot{q}, \dot{q}_e, \ddot{q}_e\}$. The parameters I_1, l_2, l_3 are assumed unknown for the controller design. The initial condition and the tracking task of the robot manipulator in the numerical simulation are the same as that in [11]. We assume that the periodic disturbance is

$$w(t) = \begin{bmatrix} 0.5\left(1 - \cos\left(\frac{2\pi t}{3}\right)\right) & 0.5\sin\left(\frac{2\pi}{3}t\right) & 0.5\left(\cos\left(\frac{2\pi t}{3}\right) - 1\right) \end{bmatrix}^T$$

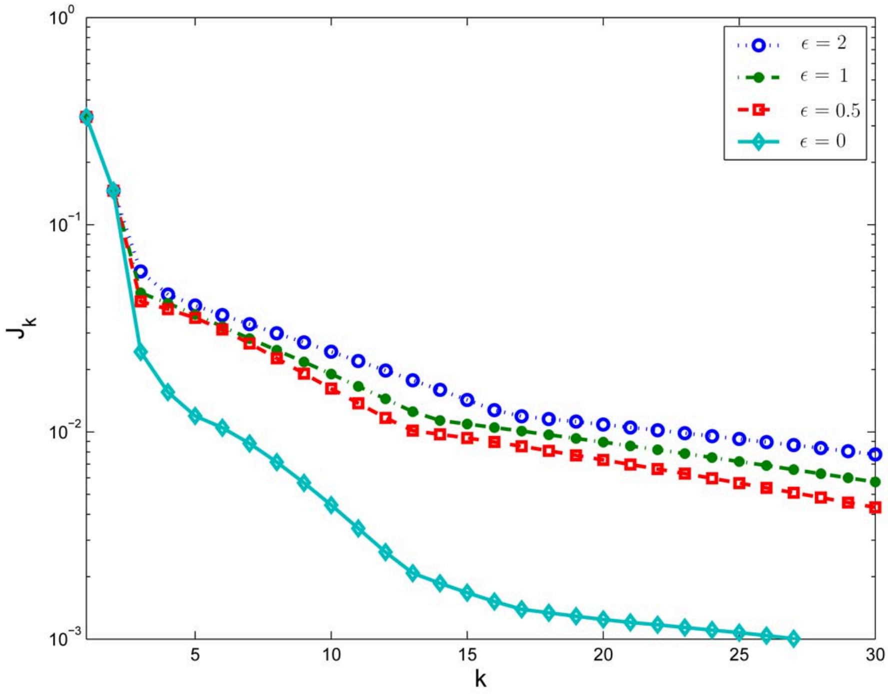

FIGURE 2. Tracking performance with different ϵ

and the nonperiodic disturbance $\varepsilon(t)$ is uniformly distributed random noise in $[-0.05, 0.05]$, which implies $\varepsilon \in \mathcal{L}_\infty$.

In (7), choose $\mu = 1$, $P = 10I_3$ and design $\hat{v}(t)$ as (12) with $h(t, e) = R^T(t)\, e(t)$, $\epsilon = 2, 1, 0.5, 0$ and $\alpha = 0.01$. By Theorem 3.1, we have (i) without ε, if $\epsilon = 0$ then $\lim_{t\to\infty} \tilde{q}(t) = 0$ for any bounded initial conditions; (ii) with ε, if $\epsilon = 2, 1, 0.5$, then $\tilde{q}(t)$ is uniformly ultimately bounded.

For tracking performance[2] comparison, we introduce the performance index as $J_k = \sup_{t\in[(k-1)T, kT]} \|\tilde{q}(t)\|$, where $k = 1, 2, \cdots$. As seen in Figure 2, when choose $\epsilon = 0$, the performance index J_k approaches zero as k increases. This implies that $\tilde{q}(t)$ approaches zero as $t \to \infty$, which is consistent with the conclusion (i) in Theorem 3.1. As seen in Figure 2, when choose $\epsilon = 2,\ 1,\ 0.5$ and the performance index J_k is bounded, which is consistent with the conclusion (ii) in Theorem 3.1.

From Figure 2, tracking performance decreases as the parameter ϵ increases, however, stability of the closed-loop system dynamics increases (see Theorem 3.2), vice versa. Therefore, a tradeoff between tracking performance and stability can be achieved by tuning the parameter ϵ.

5. **Conclusions.** In this paper, an FRC is proposed to compensate for a periodic disturbances in robot manipulator tracking. In the presence of both a periodic disturbance and a persistent nonperiodic disturbance, the FRC with $\epsilon > 0$ causes the tracking error to be uniformly ultimately bounded. The convergence rate of the resulting closed-loop system is proposed to measure its stability, and it is shown that the resulting closed-loop error dynamics with an FRC is more stable than that with the corresponding repetitive controller. Numerical simulations demonstrate that a tradeoff between tracking performance and stability can be achieved by tuning the parameter ϵ.

[2]In this paper, tracking performance means tracking accuracy.

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Appendix.

A.1. Proof of Lemma 3.1. For any given $v \in \mathcal{C}_{PT}^{1}\left([0,\infty);\mathbb{R}^{m+l}\right)$, there exists a solution x^* of (8) such that

$$\begin{aligned} x^*(t) &= x^*(t-T) \\ v(t) &= x^*(t) \end{aligned}$$

for all $t \in \mathbb{R}_+$, where $x^*(\theta) = v(\theta + T), \theta \in [-T, 0]$. Since the derivative of $v(t)$ exists and $v(t) = x^*(t)$ for all $t \in \mathbb{R}_+$, the equation above can be rewritten as follows:

$$\begin{aligned} \alpha\dot{x}^*(t) &= -x^*(t) + (1-\epsilon\alpha)x^*(t-T) + \sigma^*(t) \\ v(t) &= x^*(t) \end{aligned} \tag{19}$$

where $\sigma^*(t) = \alpha\dot{x}^*(t) + x^*(t) - (1-\epsilon\alpha)x^*(t-T)$. Note that $v(t) = x^*(t) = x^*(t-T)$ for all $t \in \mathbb{R}_+$, we have

$$\sigma^*(t) = \epsilon\left[\dot{v}(t) + \alpha v(t)\right] \tag{20}$$

where $\sigma^* \in \mathcal{C}_{PT}^{0}\left([0,\infty);\mathbb{R}^{m+l}\right)$. Therefore, for any given $v \in \mathcal{C}_{PT}^{1}\left([0,\infty);\mathbb{R}^{m+l}\right)$, there exists such a function $\sigma^* \in \mathcal{C}_{PT}^{0}\left([0,\infty);\mathbb{R}^{m+l}\right)$ that (19) holds. This implies that (9) holds with a bounded initial condition. From (20), inequality (10) can be verified easily.

A.2. Proof of Theorem 3.1. For the closed-loop error dynamics forming by (6) and (15), choose a nonnegative function $V(t)$ to be

$$V(t) = e^T(t)D(q(t))e(t) + \alpha\tilde{v}^T(t)\tilde{v}(t) + \int_{t-T}^{t}\tilde{v}^T(s)\tilde{v}(s)\,ds \tag{21}$$

where $\epsilon \in \mathbb{R}_+$ and $D(q(t))$ is symmetric uniformly positive definite and bounded by (A1). Taking the time derivative $V(t)$ along the closed-loop error dynamics yields

$$\begin{aligned} \dot{V}(t) = {} & e^T(t)\dot{D}(q(t))e(t) \\ & +2e^T(t)\left[-C(q(t),\dot{q}(t))e(t) - Pe(t) + R(t)\tilde{v}(t) + \varepsilon(t)\right] \\ & +\rho_1\tilde{v}^T(t)\tilde{v}(t) + d(t,\tilde{v}) - 2\tilde{v}^T(t)h(t,e(t)) + 2\tilde{v}^T(t)\sigma(t) \end{aligned} \tag{22}$$

where $\rho_1 = \epsilon\alpha(2-\epsilon\alpha)$ and

$$d(t,\tilde{v}) = -(1-\epsilon\alpha)^2\tilde{v}^T(t)\tilde{v}(t) + 2(1-\epsilon\alpha)\tilde{v}^T(t)\tilde{v}(t-T) - \tilde{v}^T(t-T)\tilde{v}(t-T).$$

By employing (A2), $\dot{V}(t)$ becomes

$$\begin{aligned} \dot{V}(t) = {} & -2e^T(t)Pe(t) + 2e^T(t)R(t)\tilde{v}(t) + 2e^T(t)\varepsilon(t) \\ & -\rho_1\tilde{v}^T(t)\tilde{v}(t) + d(t,\tilde{v}) - 2\tilde{v}^T(t)h(t,e(t)) + 2\tilde{v}^T(t)\sigma(t). \end{aligned}$$

Using (16) and the fact $d(t,\tilde{v}) \le 0$, we obtain

$$\dot{V}(t) \le -2e^T(t)Pe(t) - \rho_1\|\tilde{v}(t)\|^2 + 2\|e(t)\|\|\varepsilon(t)\| + 2\|\tilde{v}(t)\|\|\sigma(t)\|. \tag{23}$$

Based on the results above, the conclusions (i) and (ii) can be proved next.

(i) If $\epsilon = 0$, then, $\sigma(t) \equiv 0$ by the inequality (10) in Lemma 3.1 and $\rho_1 = 0$. Thus, (23) becomes

$$\dot{V}(t) \le -2\lambda_{\min}(P)\|e(t)\|^2 + 2\|e(t)\|\|\varepsilon(t)\|. \tag{24}$$

Since $\varepsilon(t) \equiv 0$, we obtain

$$\dot{V}(t) \le -2\lambda_{\min}(P)\|e(t)\|^2$$

From the inequality above, we obtain

$$V(t) \le V(0)$$

also

$$\int_0^t \|e(s)\|^2 ds \le \frac{1}{2\eta} V(0).$$

Therefore, $e \in \mathcal{L}_\infty \cap \mathcal{L}_2$. Next, we will prove $e(t)$ is uniformly continuous. If this is true, then $\lim_{t\to\infty} e(t) = 0$ by Barbalat's Lemma [22].

Since $e \in \mathcal{L}_\infty$, we have q, $\dot{q} \in \mathcal{L}_\infty$ by the definition of e. Therefore, by using condition (i), there exist b_F, $b_B \in \mathbb{R}_+$ such that

$$\begin{aligned} \left\| -D^{-1}(q(t))\left[P + C(q(t), \dot{q}(t))\right] e(t) \right\|_\infty &\le b_F \\ \left\| D^{-1}(q(t)) R(t) \right\|_\infty &\le b_B \end{aligned}$$

for all $t \in \mathbb{R}_+$. From (6), we have

$$\dot{e}(t) = -D^{-1}(q(t))\left[P + C(q(t), \dot{q}(t))\right] e(t) + D^{-1}(q(t)) R(t) \tilde{v}(t).$$

Let t_1 and t_2 be any real numbers such that $0 < t_2 - t_1 \le h_0$ and we have

$$\begin{aligned} \|e(t_2) - e(t_1)\| &= \left\| \int_{t_1}^{t_2} \dot{e}(s)\, ds \right\| \\ &= \left\| \int_{t_1}^{t_2} \left\{ -D^{-1}(q(s))\left[P + C(q(s), \dot{q}(s))\right] e(s) \right. \right. \\ &\qquad \left. \left. + D^{-1}(q(s)) R(s) \tilde{v}(s) \right\} ds \right\| \\ &\le b_F (t_2 - t_1) + b_B \int_{t_1}^{t_2} \|\tilde{v}(s)\|\, ds. \end{aligned} \tag{25}$$

Note that $V(t)$ is bounded for all $t \in \mathbb{R}_+$ with respect to the bound $V(0)$, hence,

$$\int_{t-T}^{t} \|\tilde{v}(s)\|^2 ds \le V(0)$$

for all $t \in \mathbb{R}_+$. Thus,

$$\int_{t_1}^{t_2} \|\tilde{v}(s)\|^2 ds \le NV(0) \tag{26}$$

where $N = \lfloor (t_2 - t_1)/T \rfloor + 1$ and $\lfloor (t_2 - t_1)/T \rfloor$ represents the floor integer of $(t_2 - t_1)/T$. Based on the Cauchy-Schwarz inequality, we have

$$\int_{t_1}^{t_2} \|\tilde{v}(s)\|\, ds \le \left(\int_{t_1}^{t_2} 1^2 ds \right)^{\frac{1}{2}} \left[\int_{t_1}^{t_2} \|\tilde{v}(s)\|^2 ds \right]^{\frac{1}{2}}.$$

Consequently, $\|e(t_2) - e(t_1)\|$ in (25) is further bounded by

$$\|e(t_2) - e(t_1)\| \le b_F h + h_1 \sqrt{h_0}$$

where (26) is utilized and $h_1 = \sqrt{NV(0)}\, b_B$. Therefore, for any $\varepsilon' > 0$ there exists

$$h_0 = \left[\frac{-h_1 + \sqrt{h_1^2 + b_F \varepsilon'}}{2b_F} \right]^2 > 0$$

such that $\|e(t_2) - e(t_1)\| < \varepsilon'$ for any $0 < t_2 - t_1 < h_0$. This implies that $e(t)$ is uniformly continuous.

(ii) Before proving conclusion (ii) of *Theorem 3.1*, the following result is needed.

Lemma A.1. ([23]) *Let $g(t)$ be a continuous function with $g(t) \geq 0$ for all $t \geq t_0 - r_0$ and $k_0 > \sup_{s\in[-r_0,0]} g(t_0+s)$. Let $\dot{g}(t) \leq -\alpha_1 g(t) + \alpha_2 \sup_{s\in[-r_0,0]} g(t+s) + \delta$ for $t \geq t_0$ where r_0, α_1, α_2, $\delta \in \mathbb{R}_+$. If $\alpha_1 > \alpha_2$, then $g(t) \leq g_0 + k_0 e^{-\eta_0(t-t_0)}$, for $t \geq t_0$ where $g_0 = \delta/(\alpha_1 - \alpha_2)$ and η_0 is the unique solution to the equation $-\eta_0 = -\alpha_1 + \alpha_2 e^{\eta_0 r_0}$.*

With the help of Lemma A.1, we have the following proof. From the definition of $V(t)$ in (21), we obtain

$$\gamma_1 \|z(t)\|^2 \leq V(t) \leq \gamma_2 \|z(t)\|^2 + \int_{t-T}^{t} \|z(s)\|^2 ds \tag{27}$$

where $\gamma_1 = \min(\lambda_m, \epsilon)$ and $\gamma_2 = \max(\lambda_M, \epsilon)$. If $\epsilon > 0$, then, $\sigma \in \mathcal{L}_\infty$ by (10) in Lemma 3.1. The inequality (23) becomes

$$\dot{V}(t) \leq -2\min[\lambda_{\min}(P), \rho_1/2] \|z(t)\|^2 + 2\bar{\sigma}\|z(t)\| \tag{28}$$

where $\bar{\sigma} = \|\sigma\|_\infty + \|\varepsilon\|_\infty$. Since $\rho_1 > 0$ by (16), the following inequality holds

$$-\min[\lambda_{\min}(P), \rho_1/2] \|z(t)\|^2 + 2\bar{\sigma}\|z(t)\| - \frac{\bar{\sigma}^2}{\min[\lambda_{\min}(P), \rho_1/2]} \leq 0.$$

Therefore, (28) becomes

$$\dot{V}(t) \leq -\gamma_3 \|z(t)\|^2 + b \tag{29}$$

where $\gamma_3 = \min[\lambda_{\min}(P), \rho_1/2]$ and $b = \frac{\bar{\sigma}^2}{\min[\lambda_{\min}(P), \rho_1/2]}$. From (29), we can obtain

$$\|z(t)\|^2 \leq \frac{b - \dot{V}(t)}{\gamma_3}. \tag{30}$$

Substituting (30) into the right-hand side of (27) yields

$$V(t) \leq \gamma_2 \frac{b - \dot{V}(t)}{\gamma_3} + \int_{t-T}^{t} \frac{b - \dot{V}(s)}{\gamma_3} ds.$$

Note that $\int_{t-T}^{t} \dot{V}(s)\, ds = V(t) - V(t-T)$, thus,

$$\dot{V}(t) \leq -\frac{\gamma_3 + 1}{\gamma_2} V(t) + \frac{1}{\gamma_2} V(t-T) + (1 + \frac{T}{\gamma_2}) b$$

By Lemma A.1, we have $V(t) \leq \frac{b}{\gamma_3}(\gamma_2 + T) + k e^{-\eta t}$, where $k > \sup_{s\in[-T,0]} \|V(s)\|^2$ and η is the unique solution to the Equation (18). Since $\gamma_1 \|z(t)\|^2 \leq V(t)$, it follows that $\|z(t)\| \leq \sqrt{\frac{b}{\gamma_1\gamma_3}(\gamma_2 + T)} + \sqrt{\frac{k}{\gamma_1}} e^{-\frac{\eta}{2}t}$.

A.3. Proof of Theorem 3.2. By the condition (ii), we obtain $\gamma_2 = \lambda_M$ and $\gamma_3 = \epsilon\alpha(2 - \epsilon\alpha)/2$. We now investigate the relationship between ϵ and η. From (18), we have

$$\gamma_3 = \gamma_2 \eta - 1 + e^{\eta T}.$$

The effect of changes in η on γ_3 can be evaluated $\frac{\partial \gamma_3}{\partial \eta} = \gamma_2 + T e^{\eta T}$. Thus, the effect of changes in ϵ on η can be evaluated:

$$\frac{\partial \eta}{\partial \epsilon} = \frac{\partial \eta}{\partial \gamma_3} \frac{\partial \gamma_3}{\partial \epsilon} = \frac{\alpha(1 - \epsilon\alpha)}{\gamma_2 + T e^{\eta T}}.$$

Note that $0 \leq \epsilon\alpha < 1, \alpha > 0, \epsilon > 0$ by (16), we have $\frac{\partial \eta}{\partial \epsilon} > 0$.

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