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Brief Paper

Attitude control of a quadrotor aircraft subject to a class of time-varying disturbances

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Abstract: Here, the attitude control of a quadrotor aircraft subject to a class of disturbances is studied. Unlike disturbances mentioned in most of the existing literature, the disturbance considered here is time varying and non-vanished. An extended observer is designed to estimate the disturbance by treating it as a new unknown state. Based on the estimation, a feedback controller with a sliding mode term is designed to stabilise the attitude of the quadrotor. Furthermore, to avoid the discontinuity of the control law caused by the sliding mode term, a modified sliding mode term is designed. The resulting continuous feedback controller makes the attitude error uniformly ultimate bounded. Theoretical results are confirmed by numerical simulations.

Nomenclature

$(q_0\ q)$	$q_0 \in \mathbb{R}$, $q \in \mathbb{R}^3$ are the scalar part and vector part of the unit quaternion, respectively
w	angular velocity of the airframe in the body fixed frame
J	inertial matrix of a quadrotor aircraft
τ	control torque
C_T	lift coefficient of a rotor
C_M	moment coefficient of a rotor
T	thrust force of a rotor
ρ	air density
r	rotor radius
A	rotor-disc area
l	distance from the epicentre of a quadrotor to the rotor axes
F_i	ith rotor thrust
F	total rotor thrust
δ	sliding mode surface

1 Introduction

A quadrotor aircraft can take off and land in limited spaces and hover over a target easily [1]. In comparison with traditional helicopters, the mechanical construction is simpler, and it is notably more manoeuverable because of the torque provided by asymmetric lifts. Also, it is easier to manufacture because of the brief rotor system. For these reasons, the quadrotor aircraft and its control scheme have received extensive attention in recent years.

1.1 *Motivation*

Usually there exist various uncertainties in quadrotor aircraft control, such as inaccurate measurements, ground effects and the bias between the geometric centre and its centre of gravity etc. These factors will cause disturbance torques that may further lead to a remarkable undesired movement of the aircraft. For this reason, more attention should be paid to the attitude control of a quadrotor aircraft in order to achieve desirable position control in the presence of disturbance torques. Generally, these disturbance torques consist of constant components and vanishing components [2–6]. To attenuate this kind of disturbance torques, various control laws with integrators are employed. In fact, disturbances generated by gust, ground effect and alteration of the engine torques depend on the state of quadrotor aircraft and time. These time-varying disturbances should be considered in order to achieve better performance in position control, such as deck landing and missions in a hostile environment. In [7], an approach was proposed to handle a time-varying disturbance torque which is a combination of constant and sinusoidal functions. Unlike disturbance torques mentioned in [2–7], the disturbance torque considered here is time varying with bounded amplitude and bounded derivative, and can take disturbances mentioned in [2–7] as a special case. This is the first motivation of the study. In [8], a control law including a proportional derivative (PD) term and a sliding mode term was proposed to stabilise the attitude in quadrotor hovering experiments. In these experiments, the time-varying disturbance was not considered directly. This leads to undesired movements. Although the undesired movements caused by the time-varying disturbance were reduced by the position feedback, the effect of this kind would be amplified in outdoor experiments because the position and velocity cannot be measured accurately and rapidly. As a result, new controllers are needed to improve attitude control of the quadrotor and in turn, improve the position control as well. This is the second motivation.

1.2 Methodology

In this paper, two attitude controllers are developed for a quadrotor aircraft subject to a class of time-varying disturbances. An extended observer is designed to estimate the disturbance which is taken as a new unknown state in the observer. Based on the observer, a feedback controller with a sliding mode term (the first controller) is designed to stabilise the attitude of the quadrotor. Furthermore, to avoid discontinuous control caused by the sliding mode term, a modified sliding mode term is designed. The resulting continuous feedback controller (the second controller) can make the attitude error uniformly ultimate bounded. These results are confirmed by numerical simulations.

1.3 Organisation

The remainder of this paper is organised as follows. In Section 2, the attitude mathematical model of a quadrotor aircraft is introduced, and the model of the disturbance is formulated. In Section 3, an extended observer is designed to estimate the disturbance. Two feedback controllers based on the extended observer are designed in Section 4. In Section 5, numerical simulations are presented.

We use the following notation. $\mathbb{R}^n$ is Euclidean space of dimension n. $\|\cdot\|$ denotes the Euclidean vector norm or induced matrix norm.

2 Problem statement

The quadrotor aircraft, shown in Fig. 1, has four rotors to generate the propeller forces F_i, $i = 1, \ldots, 4$. The up (down) motion is achieved by increasing (decreasing) the rotor speeds altogether with the same quantity. The two pairs of rotors (front, end) and (left, right) turn in opposite directions in order to balance the moments and produce yaw motion as needed. On the other hand, forward (backward) motion is achieved by pitching in the desired direction by increasing the end (front) rotor thrust and decreasing the front (end) rotor thrust to maintain the total thrust. Finally, a sideways motion is achieved by rolling in the desired direction by increasing the left (right) rotor thrust and decreasing the right (left) rotor thrust to maintain the total thrust.

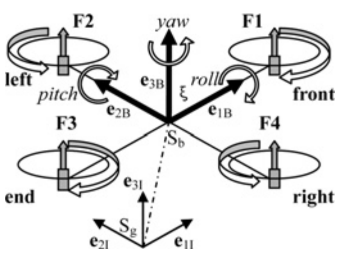

Fig. 1 *Sketch map of quadrotor aircraft*

The unit quaternion is a vector denoted by $(q_0\ q)$, where $q_0^2 + \|q\|^2 = 1$. In this paper, the unit quaternion, which is free of singularity, is used to represent the attitude kinematics of a quadrotor aircraft as follows [9]

$$\dot{q} = \frac{1}{2}(q \times w + q_0 w) \tag{1}$$

$$\dot{q}_0 = -\frac{1}{2} w^{\mathrm{T}} q \tag{2}$$

where $w \in \mathbb{R}^3$. The dynamic equation of attitude motion is

$$\dot{w} = -J^{-1} w \times Jw + J^{-1}\tau + d \tag{3}$$

where $J \in \mathbb{R}^{3\times 3}$ is known, $\tau \in \mathbb{R}^3$ and $d \in \mathbb{R}^3$ is the unknown disturbance vector.

Our major goal is to design τ to make q uniformly ultimate bounded with an ultimate bound $b_q > 0$.

We impose the following assumptions on the systems (1)–(3).

Assumption 1: The disturbance d satisfies

$$\dot{d} = \sigma(t) \tag{4}$$

where $d(0) = d_0$ is unknown, $\sigma(t)$ is an unknown function vector which satisfies $\sup_{t\in[0,\infty]} \|\sigma(t)\|\ ; \leq b_\sigma$, and b_σ is a positive constant.

Assumption 2: q and w are available from measurements.

Remark 1: Thrust force T of a rotor, the resultant of the vertical forces acting on all the blade elements, can be written as

$$T = C_T \rho A (wr)^2$$

Rolling moment M of a rotor, the integration over the entire rotor of the lift of each section acting at a given radius, can be written as

$$M = C_M \rho A (wr)^2 r$$

Therefore, rolling moment M can be written as

$$M = cT$$

where $c = C_M r / C_T$. The force F and torque τ produced by the propeller system of a quadrotor aircraft (four rotors) are [10, 11]

$$F = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix}, \quad \tau = \begin{bmatrix} l(F_2 - F_4) \\ l(F_3 - F_1) \\ c\sum_{i=1}^{4} (-1)^{i+1} F_i \end{bmatrix} \tag{5}$$

Readers can refer to [12] for the actuator dynamics. Given F and τ, the rotor thrusts F_i, $i = 1, \ldots, 4$ can be obtained by solving (5). Since F is determined by the attitude and the gravity of the quadrotor aircraft, and the attitude dynamics is faster than the position dynamics, we focus on designing τ here for simplicity.

Remark 2: The disturbance d can describe uncertainties such as inaccurate torques and lifts of the rotors, the ground effects and the bias between the geometric centre and its centre of gravity etc.

3 Design of extended observer

To deal with time-varying disturbances, the usual method is to employ a high-gain feedback controller to attenuate the disturbance. It is well known that the drawbacks of the high-gain feedback solutions are related to the fact that they may saturate the joint actuators, excite high-frequency modes etc. To avoid these drawbacks, a natural way is to design an observer to estimate the disturbance, then use the estimate to compensate for the disturbance.

Define

$$\tilde{w} = \hat{w} - w$$

$$\tilde{d} = \hat{d} - d$$

where $\hat{w}$ is the estimate of the angular velocity w and $\hat{d}$ is the estimate of the disturbance d. Although the angular velocity w is known from Assumption 2, the reason for introducing $\hat{w}$ is to construct an extended observer to obtain $\hat{d}$. Before introducing the proposed observer, we need

Lemma 1 [13, p. 475]: If A_{11} and A_{22} are square matrices, then

$$\det\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \det(A_{11})\det(A_{22} - A_{21}A_{11}^{-1}A_{12})$$

when A_{11}^{-1} exists, and

$$\det\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \det(A_{22})\det(A_{11} - A_{12}A_{22}^{-1}A_{21})$$

when A_{22}^{-1} exists.

Theorem 1: Under Assumptions 1–2, for system (3), if the designed extended observer is

$$\dot{\hat{w}} = -J^{-1}w \times Jw + J^{-1}\tau + \hat{d} - \frac{k_1}{\varepsilon}\tilde{w} \tag{6}$$

$$\dot{\hat{d}} = -\frac{k_2}{\varepsilon^2}\tilde{w}, \quad \hat{w}(0) = 0, \quad \hat{d}(0) = 0 \tag{7}$$

where k_1, k_2 and ε are all positive constants. Then we obtain

$$\begin{aligned} &\lim_{\varepsilon\to 0}\lim_{t\to\infty} \|\tilde{w}(t)\| = 0 \\ &\lim_{\varepsilon\to 0}\lim_{t\to\infty} \|\tilde{d}(t)\| = 0 \end{aligned} \tag{8}$$

Proof: Considering the difference between (6) and (3) and the difference between (7) and (4), we obtain

$$\dot{\zeta} = A_\varepsilon \zeta + L\sigma \tag{9}$$

where

$$\zeta = \begin{bmatrix} \tilde{w} \\ \tilde{d} \end{bmatrix}, \ A_\varepsilon = \begin{bmatrix} -\dfrac{k_1}{\varepsilon}I_3 & I_3 \\ -\dfrac{k_2}{\varepsilon^2}I_3 & 0_{3\times 3} \end{bmatrix}, \ L = \begin{bmatrix} 0 \\ -I_3 \end{bmatrix}$$

In the presence of σ, the transfer function from σ to ζ is

$$\begin{aligned} G_\varepsilon(s) &= (sI_6 - A_\varepsilon)^{-1}L \\ &= \varepsilon(s\varepsilon I_6 - \varepsilon A_\varepsilon)^{-1}L \\ &= \varepsilon\frac{\mathrm{adj}(s\varepsilon I_6 - \varepsilon A_\varepsilon)L}{\det(s\varepsilon I_6 - \varepsilon A_\varepsilon)} \end{aligned}$$

Since

$$\begin{aligned} \det(s\varepsilon I_6 - \varepsilon A_\varepsilon) &= \det\left(\begin{bmatrix} s\varepsilon I_3 + k_1 I_3 & -\varepsilon I_3 \\ \dfrac{k_2}{\varepsilon}I_3 & s\varepsilon I_3 \end{bmatrix}\right) \\ &= \det(s\varepsilon I_3 + k_1 I_3)\det[s\varepsilon I_3 + k_2(s\varepsilon I_3 + k_1 I_3)^{-1}] \end{aligned}$$

by Lemma 1 and

$$\begin{aligned} \mathrm{adj}(s\varepsilon I_6 - \varepsilon A_\varepsilon)L &= \begin{bmatrix} * & \varepsilon I_3 \\ * & (s\varepsilon I_3 + k_1 I_3)^{-1} \end{bmatrix} L \\ &= \begin{bmatrix} \varepsilon I_3 \\ (s\varepsilon I_3 + k_1 I_3)^{-1} \end{bmatrix} \end{aligned}$$

where I_3 is the identity matrix of dimension 3.
We have

$$\lim_{\varepsilon\to 0}\det(s\varepsilon I_6 - \varepsilon A_\varepsilon) = k_2^3$$

$$\lim_{\varepsilon\to 0}[\mathrm{adj}(s\varepsilon I_6 - \varepsilon A_\varepsilon)L] = \begin{bmatrix} 0_3 \\ k_1^{-1}I_3 \end{bmatrix}$$

Therefore $\lim_{\varepsilon\to 0} G_\varepsilon(s) = 0$. Note that as σ is an unknown and bounded function vector, we have

$$\lim_{\varepsilon\to 0}\lim_{t\to\infty}\zeta(t) = 0$$

That is to say, when ε is sufficiently small, the observer error will also be sufficiently small. This completes the proof. □

Remark 3: The external disturbance is applied to the system after a kind of proportional integral (PI) action in (6), that is, $\dot{\hat{w}} = -J^{-1}w \times Jw + J^{-1}\tau - (k_2/\varepsilon^2)\int_0^t \tilde{w}(s)ds - (k_1/\varepsilon)\tilde{w}$. By applying the high gain k_2/ε^2, the observer can estimate the disturbance quickly.

Remark 4: The mathematical proof shows that observer error $\tilde{d}$ vanishes as $\varepsilon \to 0$ and $t \to \infty$, but measurement noise and unmodelled high-frequency sensor dynamics will put a practical limit on how small ε could be. This implies that observer error $\tilde{d}$ can only be uniformly ultimate bounded in practice. According to (8), for a given disturbance d, there exists a time t_ε such that $\sup_{t\geq t_\varepsilon}\|\tilde{d}\| \leq b_{\tilde{d}}(t_\varepsilon, \varepsilon)$.

Remark 5: The observer is designed based on Assumption 1, where the derivative of disturbance is bounded. Another type

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of disturbance model may upset the observer, that is when the second-order derivative of disturbance is bounded. Similar to the design idea, we can also design another observer applicable to the disturbance whose second-order derivative is bounded.

4 Controller design

Based on systems (1)–(3), two attitude controllers by using the extended observer mentioned in Section 3 are developed to stabilise the attitude for the quadrotor aircraft. First, a feedback controller with a sliding mode term (the first controller) is designed to stabilise the attitude of the quadrotor. Furthermore, to avoid the discontinuous control law caused by the sliding mode term, a modified sliding mode term is further designed. The resulting continuous feedback controller (the second controller) can make the attitude error uniformly ultimate bounded.

Let $\delta = \mu q + w$ and $\mu > 0$, and select a Lyapunov function as $V_\delta = \delta^{\mathrm{T}}\delta$. The reason for choosing $\delta = \mu q + w$ is to make $\delta \equiv 0$ (in the proof of Theorem 2) or δ sufficiently small (in the proof of Theorem 3). Then by $w = -\mu q + \delta$, system (1)–(2) therefore becomes a stable autonomous system (in the proof of Theorem 2) or a stable autonomous system with a sufficiently small external signal (in the proof of Theorem 3).

Taking the derivative of V_δ, we have

$$\begin{aligned}\dot{V}_\delta &= 2\delta^{\mathrm{T}}\dot{\delta}\\ &= 2\delta^{\mathrm{T}}(\mu\dot{q} + \dot{w})\\ &= 2\delta^{\mathrm{T}}\left[\frac{1}{2}\mu(q\times w + q_0 w) - J^{-1}w\times Jw + J^{-1}\tau + d\right]\end{aligned}$$

Design τ to be

$$\tau = Jv - J\hat{d} + w\times Jw - \frac{1}{2}\mu J(q\times w + q_0 w) \tag{10}$$

then

$$\dot{V}_\delta = 2\delta^{\mathrm{T}}(v - \tilde{d}) \tag{11}$$

The remaining work is to design v. First, we design a feedback controller with v in the form of a sliding mode term and give the following theorem.

Theorem 2: Under Assumptions 1–2, design the controller as (10), where $\hat{d}$ is estimated by the extended observer (6)–(7) and

$$v = \begin{cases} -\eta\dfrac{\delta}{||\delta||} & ||\delta|| \neq 0\\ 0 & ||\delta|| = 0\end{cases} \tag{12}$$

where $\eta > 2b_{\tilde{d}}(t_\varepsilon, \varepsilon)$ and $b_{\tilde{d}}(t_\varepsilon, \varepsilon)$ is defined in Remark 4. Then $\lim_{t\to\infty} q(t) = 0$.

Proof: If v is designed as (12), then (11) becomes

$$\dot{V}_\delta = 2\delta^{\mathrm{T}}\left(-\eta\frac{\delta}{||\delta||} - \tilde{d}\right)$$

where $||\delta|| \neq 0$. Otherwise, $\dot{V}_\delta = 0$ and then $\delta \equiv 0$. From Remark 4, we have

$$\dot{V}_\delta \le 2[-\eta + b_{\tilde{d}}(t_\varepsilon, \varepsilon)]||\delta||$$

when $t \ge t_\varepsilon$. Since $\eta > 2b_{\tilde{d}}(t_\varepsilon, \varepsilon)$, we obtain

$$\dot{V}_\delta \le -\eta||\delta|| \tag{13}$$

when $t \ge t_\varepsilon$.

Based on the result above, we have $\delta = 0$ within a finite time t_f [14, p. 553]. Then if $t \ge t_\varepsilon + t_f$, we have $\delta(t) \equiv 0$. Therefore the attitude kinematics (1) and (2) will become

$$\dot{q} = -\frac{1}{2}\mu q_0 q \tag{14}$$

$$\dot{q}_0 = \frac{1}{2}\mu q^{\mathrm{T}} q \tag{15}$$

when $t \ge t_\varepsilon + t_f$. Select a Lyapunov function as

$$V = q^{\mathrm{T}}q + (1 - q_0)^2$$

then we have

$$\begin{aligned}\dot{V} &= 2q^{\mathrm{T}}\dot{q} - 2(1-q_0)\dot{q}_0\\ &= -\mu q_0 q^{\mathrm{T}}q - \mu(1-q_0)q^{\mathrm{T}}q\\ &= -\mu q^{\mathrm{T}}q\end{aligned}$$

Similar to [15, p. 77], we have $\lim_{t\to\infty} q(t) = 0$. □

From Theorem 2, for any given $b_q > 0$, the controller τ makes q uniformly ultimate bounded with the ultimate bound b_q. The control law given by (10) with v designed in (12) is a discontinuous function. Practically, the implementation of such a discontinuous controller is characterised by the phenomenon of chattering. This will not only result in low control accuracy, high heat losses in electrical power circuits [16, 17], but also excite high-frequency modes which may lead systems to instability in practice [14, p. 555]. Moreover, we should consider the relative degree condition that has to be fulfilled when designing sliding mode controllers. To avoid these problems, we design a feedback controller with a modified sliding mode term and give the following theorem.

Theorem 3: Under Assumptions 1–2, design the controller as (10), where $\hat{d}$ is estimated by the extended observer (6), (7) and

$$v = \begin{cases} -\eta\dfrac{\delta}{||\delta||} & ||\delta|| \ge \dfrac{\varepsilon}{\eta}\\ -\eta^2\dfrac{\delta}{\varepsilon} & ||\delta|| < \dfrac{\varepsilon}{\eta}\end{cases} \tag{16}$$

where $\varepsilon > 0$, $\eta > 2b_{\tilde{d}}(t_\varepsilon, \varepsilon)$, $\mu\eta - 1 - \varepsilon > 0$ and $b_{\tilde{d}}(t_\varepsilon, \varepsilon)$ is defined in Remark 4. Then q is uniformly ultimate bounded with an ultimate bound $\sqrt{2}\varepsilon/(\mu\eta - 1 - \varepsilon)$.

Proof: Note that $\delta = \mu q + w$, then we have

$$w = -\mu q + \delta$$

Consequently, systems (1) and (2) become

$$\dot{q} = \frac{1}{2}[q \times (-\mu q + \delta) + q_0(-\mu q + \delta)] \tag{17}$$

$$\dot{q}_0 = -\frac{1}{2}(-\mu q + \delta)^{\mathrm{T}} q \tag{18}$$

respectively. We select a Lyapunov function as

$$V = q^{\mathrm{T}} q + (1 - q_0)^2$$

The derivative of V along systems (17) and (18) is

$$\begin{aligned}\dot{V} &= 2q^{\mathrm{T}}\dot{q} - 2(1 - q_0)\dot{q}_0 \\ &= q^{\mathrm{T}}[q \times \delta + q_0(-\mu q + \delta)] + (1-q_0)(-\mu q + \delta)^{\mathrm{T}} q \\ &= -\mu q^{\mathrm{T}} q + \delta^{\mathrm{T}} q \end{aligned} \tag{19}$$

Noticing (13) in the proof of Theorem 2, we have $\dot{V}_\delta \le -\eta||\delta||$ when $||\delta|| \ge (\varepsilon/\eta)$. Therefore the trajectory δ will arrive $B_{\varepsilon/\eta} = \{||\delta|| \le (\varepsilon/\eta)\}$ within a finite time t'_f. Moreover, $\dot{V}_\delta \le 0$ when $||\delta|| = (\varepsilon/\eta)$, hence the trajectory δ will stay in $B_{\varepsilon/\eta}$ when $t > t_\varepsilon + t'_f$. This implies that $||\delta|| \le (\varepsilon/\eta)$ when $t > t_\varepsilon + t'_f$. Consequently, when $t > t_\varepsilon + t'_f$ $\dot{V}$ in (19) is bounded as

$$\dot{V} \le -\left(\mu - \frac{\varepsilon}{\eta}\right)||q||^2 + \frac{\varepsilon}{\eta}||q|| \tag{20}$$

Since

$$\begin{aligned} q^{\mathrm{T}} q + (1 - q_0)^2 &= 2 - 2q_0 \\ &\le 2 - 2q_0^2 \\ &= 2||q||^2 \end{aligned}$$

we can obtain

$$||q||^2 \le V \le 2||q||^2$$

Moreover, from (20), it is easy to verify that if $||q|| \ge (\varepsilon/(\mu\eta - 1 - \varepsilon))$, then

$$\dot{V} \le -\frac{1}{\eta}||q||^2$$

Therefore, from Theorem 4.18 in [14, p. 172], there exists a $T > 0$ such that

$$||q|| \le \frac{\sqrt{2}\varepsilon}{\mu\eta - 1 - \varepsilon}, \qquad t \ge t_\varepsilon + t'_f + T$$

That is to say q is uniformly ultimate bounded with an ultimate bound $\sqrt{2}\varepsilon/(\mu\eta - 1 - \varepsilon)$. □

Remark 6: It should be noted that the resulting control is continuous when (16) is adopted. For a given b_q, if ε is chosen to be

$$\varepsilon \le \frac{(\mu\eta - 1)b_q}{\sqrt{2} + b_q}$$

then q is uniformly ultimate bounded with an ultimate bound b_q. Since $w = -\mu q + \delta$ and δ is bounded, w is bounded. By noticing (7) and (9), $\hat{d}$ is bounded and continuous. Then τ is bounded by recalling the form of (10) and (16). Consequently, w is continuous by (3). Furthermore, the term v in (16) is continuous. We conclude that the control law τ is continuous when (16) is adopted.

5 Numerical simulations

The simulation parameters are chosen as follows: the inertial matrix J of a quadrotor aircraft is as in [8] that

$$J = \mathrm{diag}(0.16,\ 0.16,\ 0.32)\,\mathrm{kg\,m^2}$$

The initial condition of (1)–(3) is $q_0(0) = 0.707$, $q(0) = [-0.4 \quad -0.3 \quad 0.5]^{\mathrm{T}}$ and $w(0) = [0 \quad 0 \quad 0]^{\mathrm{T}}$ rad/s. The disturbance d is assumed to be

$$d = \begin{bmatrix} \sin(||q||) + 1 + 0.2\sin t \\ \sin(0.5||q||) + 0.5 + 0.2\cos 2t \\ 0.5 \end{bmatrix} \mathrm{N\,m}$$

where d depends on both the state of quadrotor aircraft and time and is non-vanishing.

Case 1: The extended observer is designed as (6), (7) with $k_1 = 1$, $k_2 = 1$ and $\varepsilon = 0.1$. The controller is designed as (10) with v designed in (12), where $\mu = 1$ and $\eta = 2$. Fig. 2 shows the estimate of the disturbance d by the extended observer. As shown in Fig. 2, the extended observer can estimate the time-varying disturbance d extremely well within 1 s. From the first second to the 10th second, the observer error is very small. This is consistent with Theorem 1. Fig. 3 shows the evolution of the quaternion in (1) and (2) driven by the controller mentioned in *Case 1*, where q_0 and q are the scalar part and vector part of the quaternion, respectively. As shown in Fig. 3, the vector part of the quaternion q approaches zero. This is consistent with Theorem 2. Fig. 4 shows the control input in Case 1, where the phenomenon of chattering occurs in about 2 s. In order to overcome the phenomenon of chattering, we design another feedback controller in Case 2.

Case 2: The extended observer is designed as (6), (7) with $k_1 = 1, k_2 = 1$ and $\varepsilon = 0.1$. The controller is designed as (10) with v designed in (12), where $\mu = 1$ and $\eta = 2$, $\varepsilon = 0.1$.

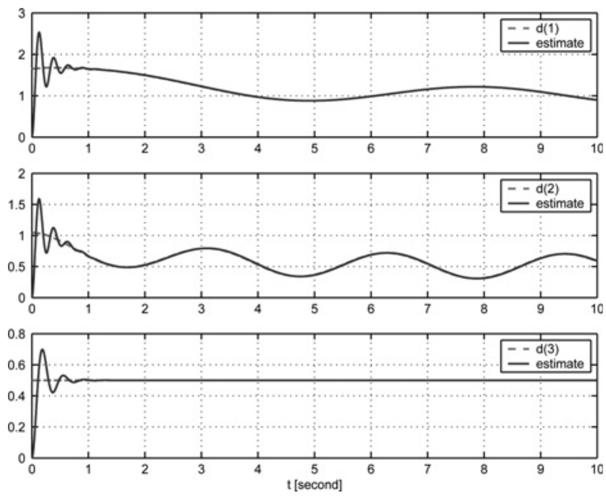

Fig. 2 *Estimate of the disturbance by the extended observer*

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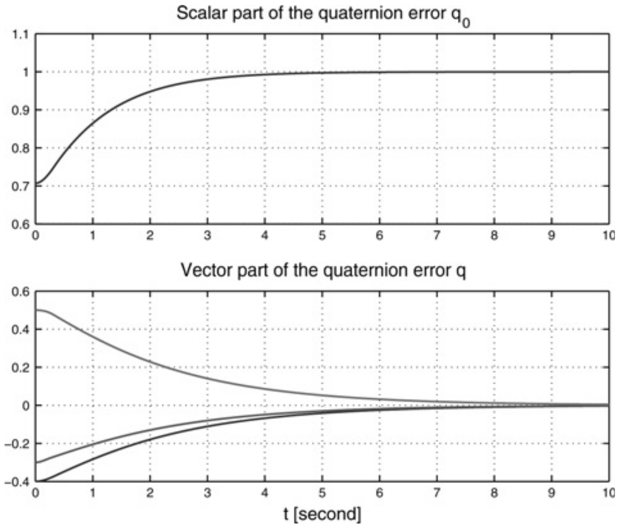

Fig. 3 *Quaternion driven by the controller in Case 1*

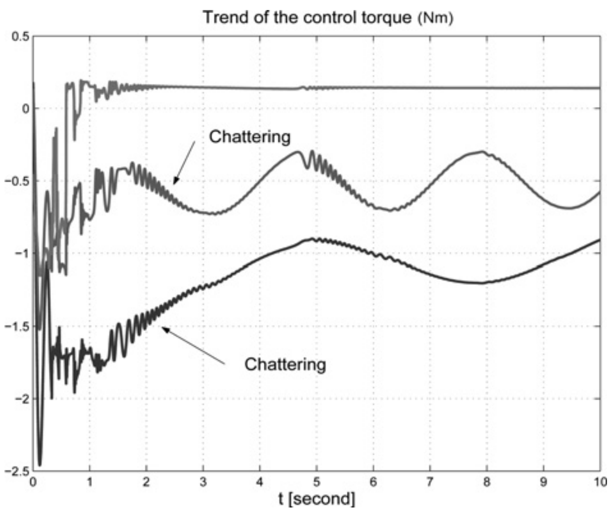

Fig. 4 *Control input in Case 1*

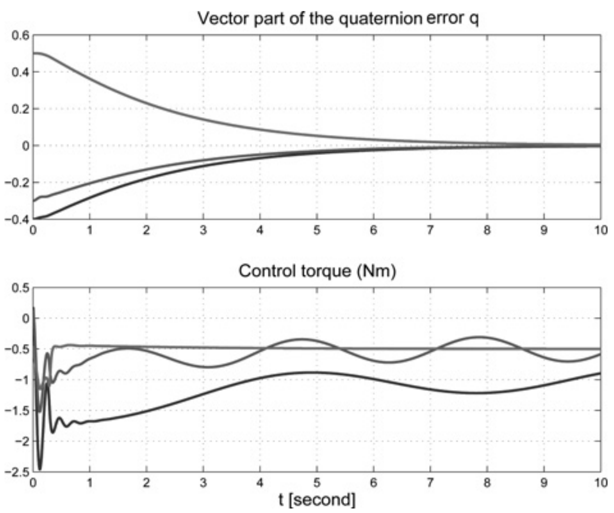

Fig. 5 *Vector part of the quaternion and control input in Case 2*

Given $b_q = 0.2$, parameter ε is chosen to be

$$\varepsilon = 0.1 \leq \frac{(\mu\eta - 1)b_q}{\sqrt{2} + b_q} = 0.12$$

(see Remark 6). As shown in Fig. 5, the control input is continuous without the phenomenon of chattering. Vector part of the quaternion q is ultimately bounded with the ultimate bound $b_q = 0.2$. This is consistent with Theorem 3.

6 Conclusions

This paper studies attitude control of a quadrotor aircraft in the presence of a class of time-varying disturbances. An extended observer is proposed to estimate the disturbance. Then, a controller with a compensation term and a sliding mode term is designed to stabilise the attitude of the quadrotor. In the controller, the compensation term is used to compensate for the disturbance by using the obtained estimate. The remaining disturbance is further attenuated by the sliding mode term. In order to avoid chattering caused by the sliding mode term, a modified feedback controller with continuous output is further designed. Numerical simulations are provided to demonstrate the effectiveness of the proposed controllers.

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