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Additive-State-Decomposition-Based Dynamic Inversion Stabilized Control of A Hexacopter Subject to Unknown Propeller Damages

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Abstract: In this paper, we study the control problem of a hexacopter subject to unknown propeller damages, namely without fault detection. This problem is first formulated to be a time-varying disturbance rejection problem. Then, a novel control, namely additive-state-decomposition-based dynamic inversion stabilized control, is applied to stabilize the considered system subject to an unknown time-varying disturbance. Neither fault detection and isolation module nor control re-allocation for a hexacopter is required under this framework. Moreover, it can achieve a good rejection performance when damages happen. Finally, a simulation example shows its effectiveness.

Key Words: Hexacopter, Propeller Damage, Fault Tolerant Control, Additive State Decomposition

1 Introduction

Autonomous vertical take-off and landing (VTOL) vehicles capable of quasi-stationary (hover and near hover) aerial vehicles, such as multi-rotor aircraft, which can lift a significant payload while maintaining their flight stability, have been considered in recent years due to their important contribution and cost effective application in several tasks such as surveillance, search, rescue missions, geographic studies, military and security applications. For the considerable intersection between the application environment of VTOL aircraft and urban area, it has potential risk to civil safety if the aircraft crashes. So it is of great importance to consider the safety, reliability and acceptable level of performance of the aircraft not only in normal operation conditions but also in the presence of partial faults or failures in the components of the controlled system.

This paper mainly discusses the fault tolerant control (FTC) problem of a hexacopter (see Fig.1) subject to propeller damage. FTC systems have the ability to accommodate failures or damages automatically in order to maintain system stability and a sufficient level of performance (see [1], [2], [3]). Most FTC methods consist of adjusting the controllers on-line according to the fault magnitude and type, in order to maintain the closed-loop performance of the system. For such a purpose, a reconfigurable control system (such as controller redesign or control re-allocation) is often required. By these control allocation approaches one obvious limitation is the requirement of a fault detection and diagnosis (FDD) module (which designs a model or parameters for only each assumed fault in designing, unexpected faults cannot be detected, see [2]). Furthermore, these systems are often complex due to requiring to characterize several failure modes *a priori*, and solving the resulting optimization problem. It is not suitable for micro unmanned aerial vehicles because of the limitation of the computation load and space. Taking these into account, some research proposed adaptive control method to handle this problem. Adaptive control does not require the knowledge of the failures due to its ability to adjust control parameters according to the existing flight condition and faults and damage to the aircraft. Thus, it is also able to accommodate for parametric uncertainties (see [4], [5], [6]). The key idea is to estimate the uncertainties or disturbance online, and then compensate for them. Since each unknown parameter needs an integrator to estimate, an adaptive controller may require many integrators corresponding to many unknown parameters for an unknown system. This will reduce the stability margin and increase the computation of the hexacopter control system.

In order to overcome these drawbacks, a novel control, namely additive-state-decomposition-based (ASDB) dynamic inversion stabilized control, which is used to stabilize a class of system subject to nonparametric time-varying uncertainties on both state and input, is proposed in [7]. The key idea behind the proposed method is to lump nonparametric time-varying uncertainties on both state and input into one disturbance by additive state decomposition (see [8], [9], [10]), and then to compensate for it. In this paper, we will formulate the propeller damages to be time-varying (as the propeller speed keeps changing) disturbances to the aircraft system. And then, an ASDB dynamic inversion stabilized controller is used to reject the disturbances. To show the effectiveness of the proposed control method, a traditional Proportional-Derivative (PD) controller is compared. The PD controller requires a control re-allocator and a FDD module to deal with the damaged propellers. By contrast, under the framework of the ASDB dynamic inversion stabilized control method, neither control re-allocation nor FDD module is needed when the propellers are damaged.

The main contributions of this paper are that: i)the control problem of a hexacopter subject to propeller damages is formulated as a control problem subject to time-varying disturbances, which avoids control re-allocation and the requirement of a FDD module, and ii) an ASDB dynamic inversion stabilized control method is used to reject the disturbances. Neither controller redesign nor control re-allocation is required after the propellers are damaged during the flight,

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The altitude dynamic of the hexacopter is shown as follows

$$\dot{X}_h = A_0 X_h + b\left(F/m - g\right) \tag{3}$$

where $X_h = \left[\begin{array}{cc} h & V_h \end{array}\right]^T$, h is the altitude of the hexacopter, V_h is the linear velocity going up and down of the origin of S_b expressed in S_I, m denotes the mass of the airframe, g denotes the acceleration due to gravity, F is the total thrust generated by the rotors.

From Fig.1, the mapping between the rotors' lift $f = \left[\begin{array}{cccccc} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \end{array}\right]^T$ and virtual control $u = \left[\begin{array}{cccc} F & L & M & N \end{array}\right]^T$ is expressed by

$$u = Hf \tag{4}$$

where the nominal constant control input matrix

$$H = \left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -\frac{\sqrt{3}d}{2} & -\frac{\sqrt{3}d}{2} & 0 & \frac{\sqrt{3}d}{2} & \frac{\sqrt{3}d}{2} \\ d & \frac{d}{2} & -\frac{d}{2} & -d & -\frac{d}{2} & \frac{d}{2} \\ -k_\mu & k_\mu & -k_\mu & k_\mu & -k_\mu & k_\mu \end{array}\right] \tag{5}$$

and d is the distance from the rotor center to the mass center of the hexacopter. Constant k_μ represents the ratio of reactive torque and lift of used propellers.

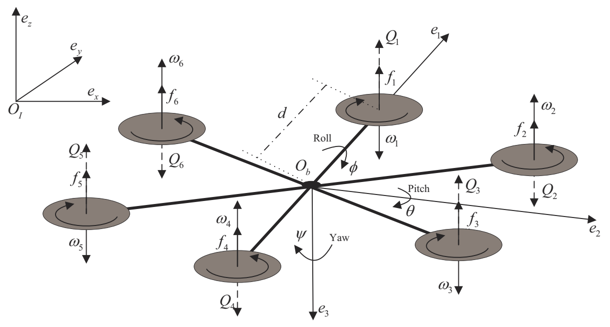

Fig. 1: Kinematic scheme of the hexacopter

which means that we need not to detect the propeller damage level or to check which propeller is damaged.

The outline of this paper is as follows. In Section 2, we derive the mathematical statement of the problem, and system of the hexacopter subjected to propeller damages is formulated as a control system with time-varying disturbances. In Section 3, an ASDB dynamic inversion stabilized controller is used to reject the disturbance caused by the propeller damages. Simulations are provided in Section 4. Finally in Section 5, concluding remarks are stated.

2 Problem Formulation

In this section, the mathematical model of the focused hexacopter subject to propeller damages is approached and formulated to a control system with time-varying disturbances.

2.1 Hexacopter Model

The hexacopter is configured with six counter-rotating rotors which are symmetrically distributed around the center O_b as shown in Fig.1. It also shows the positive directions of rotor (motor-propeller group) rotation ω_i, positive directions of rotor lifts f_i, positive direction of reactive torques Q_i of individual propellers, where $i \in \{1,2,3,4,5,6\}$. The reference frame $S_I = \{e_x, e_y, e_z\}$ denotes an inertial frame, and $S_b = \{e_1, e_2, e_3\}$ denotes a frame rigidly attached to the aircraft. Then according to [11], it is straight forward to derive the linear attitude dynamical model of the hexacopter around hover conditions

$$\begin{aligned} \dot{X}_L &= A_0 X_L + bL/J_x \\ \dot{X}_M &= A_0 X_M + bM/J_y \\ \dot{X}_N &= A_0 X_N + bN/J_z \end{aligned} \tag{1}$$

where $X_L = \left[\begin{array}{cc} \phi & p \end{array}\right]^T$ (in this paper the superscript T denotes the transposition), $X_M = \left[\begin{array}{cc} \theta & q \end{array}\right]^T$, $X_N = \left[\begin{array}{cc} \psi & r \end{array}\right]^T$, ϕ, θ, ψ are roll, pitch and yaw angle as shown in Fig.1, p, q, r are the angular velocities of the airframe expressed in the body-fixed frame, and

$$A_0 = \left[\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right], b = \left[\begin{array}{c} 0 \\ 1 \end{array}\right]. \tag{2}$$

Notations L, M, and N are airframe roll, pitch and yaw torque, J_x, J_y and J_z are the moment of inertia of the hexacopter around e_1, e_2 and e_3 of the body-fixed frame S_b respectively.

2.2 Control Procedure of The Hexacopter

From (1) (3) and (4), engineers do not design the control rule of f directly. As shown in Fig.2, they design the control rule of the virtual control u, based on the control command $r_c = \left[\begin{array}{cccc} \phi_c & \theta_c & \psi_c & h_c \end{array}\right]^T$ and the sensed information $r_s = \left[\begin{array}{cccc} \phi_s & \theta_s & \psi_s & h_s \end{array}\right]^T$ first. And then control allocation is used for hexacopter, which is summarized as follows (see [12]): (a) Give the virtual control u input vector which comes from the control rule of the aircraft, and (b) Find the set of rotors' lift f, such that $u = Hf$. In Fig.2, where f is the desired rotors' lift, λ is the actual output of the rotors.

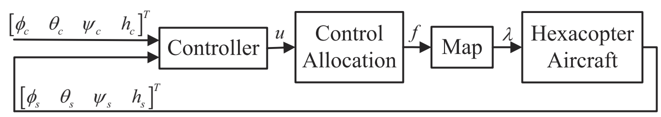

Fig. 2: Feedback controller of hexacopter aircraft

From (4), one way to find the desired rotors' lift f is the pseudo-inverse matrix method which is expressed by

$$f = H^T\left(HH^T\right)^{-1}u. \tag{6}$$

For detailed discussion, please refer to [12] and [13].

2.3 Hexacopter Model of Unknown Propeller Damages

In this paper, we focus on the case that the propellers are damaged when the hexacopter is hovering. After the propellers are damaged, the control input matrix changes to $E = H\left(I_6 - \Gamma\right)$, where

$$\Gamma = diag\left\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\right\} \tag{7}$$

and $0 < \gamma_i < 1, i \in \{1,2,3,4,5,6\}$. The parameter γ_i means that the effectiveness of rotor i is reduced by $100\gamma_i\%$

caused by the damage of propeller i (see [14]). And the following control re-allocation method

$$f = E^T \left(EE^T\right)^{-1} u \tag{8}$$

is needed to replace (6) to find the set of rotor lifts f based on the virtual control u.

But if control re-allocation method such as (8) is not used, because of the propeller damages, the virtual control u cannot be achieved by f, which is calculated by (6). As the control input matrix changes from H to E, we can say that there is a time-varying disturbance $u_d = \left[\begin{array}{cccc} F_d & L_d & M_d & N_d \end{array}\right]^T$ to system (1) and (3), which is expressed by

$$u_d = H\Gamma f. \tag{9}$$

In this paper, we will deal with FTC of the hexacopter subjected to propeller damages from another viewpoint, and consider the disturbances caused by propeller damages directly. From (6) and (9), we get the relationship between u and u_d that

$$\left[\begin{array}{c} F_d \\ L_d \\ M_d \\ N_d \end{array}\right] = H\Gamma H^T \left(HH^T\right)^{-1} \left[\begin{array}{c} F \\ L \\ M \\ N \end{array}\right]$$

We re-formulate the problem as follows

$$\dot{X}_L = A_0 X_L + b\left(\left(L - L_d\right)/J_x + \Delta L\right) \tag{10}$$
$$\dot{X}_M = A_0 X_M + b\left(\left(M - M_d\right)/J_y + \Delta M\right) \tag{11}$$
$$\dot{X}_N = A_0 X_N + b\left(\left(N - N_d\right)/J_z + \Delta N\right) \tag{12}$$

and

$$\dot{X}_h = A_0 X_h + b\left(\left(F - F_d\right)/m - g + \Delta F\right). \tag{13}$$

where L_d, M_d, N_d, F_d are given by (9), and $\Delta L, \Delta M, \Delta N, \Delta F$ represent model errors caused by parameter uncertainties, such as the measure error of the the moment of inertia J_x, J_y and J_z of the hexacopter.

Note $X_{Lc} = \left[\begin{array}{cc} \phi_c & 0 \end{array}\right]^T$, $X_{Mc} = \left[\begin{array}{cc} \theta_c & 0 \end{array}\right]^T$, $X_{Nc} = \left[\begin{array}{cc} \psi_c & 0 \end{array}\right]^T$, $X_{hc} = \left[\begin{array}{cc} h_c & 0 \end{array}\right]^T$ to be the control commands of system (10)-(13), where $\phi_c, \theta_c, \psi_c, h_c$ are constant values. Next we will change the tracking problems of (10)-(13) to stabilizing problems. For (10), let $e_L = X_L - X_{Lc}$, then we have $X_L = e_L + X_{Lc}$ and

$$\dot{e}_L + \dot{X}_{Lc} = A_0 e_L + A_0 X_{Lc} + b\left(\left(L - L_d\right)/J_x + \Delta L\right). \tag{14}$$

Substituting (2) into (14) results in

$$\dot{e}_L = A_0 e_L + b\left(\left(L - L_d\right)/J_x + \Delta L\right) \tag{15}$$

which is a stabilization problem. Similarly, rearranging (11)-(13) results in

$$\dot{e}_M = A_0 e_M + b\left(\left(M - M_d\right)/J_y + \Delta M\right) \tag{16}$$
$$\dot{e}_N = A_0 e_N + b\left(\left(N - N_d\right)/J_z + \Delta N\right) \tag{17}$$
$$\dot{e}_h = A_0 e_h + b\left(\left(F - F_d\right)/m - g + \Delta F\right) \tag{18}$$

where $e_M = X_M - X_{Mc}$, $e_N = X_N - X_{Nc}$, and $e_h = X_h - X_{hc}$.

It is easy to see that the disturbances L_d, M_d, N_d, F_d are zero when the propellers are not damaged. In the next section, an ASDB dynamic inversion stabilized controller is used to reject the disturbances.

3 Additive-state-decomposition-based Dynamic Inversion Stabilized Controller

In this section, the systems (15)-(18) are first transformed into a determinate system but subject to a lumped disturbance by additive state decomposition (see [7] and [8]). Then a dynamic inversion method is applied to this transformed system. We take the controller design of the system (15) as an example first. Then similar controllers for (16)-(18) are obtained directly as they have the same control matrix (A_0, B) to system (15).

3.1 Additive State Decomposition

In order to make the paper self-contained, additive state decomposition in [7] is recalled briefly here. Consider a class of differential dynamic systems as follows:

$$\begin{aligned} \dot{x} &= f\left(t, x\right), x\left(0\right) = x_0, \\ y &= g\left(t, x\right) \end{aligned} \tag{19}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. Two systems, denoted by the primary system and (derived) secondary system respectively, are defined as follows:

$$\begin{aligned} \dot{x}_p &= f_p\left(t, x_p\right), x_p\left(0\right) = x_{p,0} \\ y_p &= g_p\left(t, x_p\right) \end{aligned} \tag{20}$$

and

$$\begin{aligned} \dot{x}_s &= f\left(t, x_p + x_s\right) - f_p\left(t, x_p\right), x_s\left(0\right) = x_0 - x_{p,0}, \\ y_s &= g\left(t, x_p + x_s\right) - g_p\left(t, x_p\right) \end{aligned} \tag{21}$$

where $x_s \triangleq x - x_p$ and $y_s \triangleq y - y_p$. The secondary system (21) is determined by the original system (19) and the primary system (20). From the definition, it has

$$x\left(t\right) = x_p\left(t\right) + x_s\left(t\right), y\left(t\right) = y_p\left(t\right) + y_s\left(t\right), t \geq 0. \tag{22}$$

The key idea of the additive state decomposition is shown by (19)-(22). Next, the additive state decomposition theory is applied to the controller design of the system (15).

As (A_0, b) is controllable, we can always find a vector $\eta_L \in \mathbb{R}^2$ such that $A = A_0 - b\eta_L^T$ is stable. The system (15) can be rewritten to be

$$\dot{e}_L = Ae_L + b\left(\eta_L^T e_L + \left(L - L_d\right)/J_x + \Delta L\right) \tag{23}$$

where $A = A_0 - b\eta_L^T$ and $e_L\left(0\right) = e_{L0}$. Taking (23) as the original system, we choose the primary system as follows

$$\dot{e}_{Lp} = Ae_{Lp} + bL, e_{Lp}\left(0\right) = 0 \tag{24}$$

and then the secondary system is determined by the original system (23) and the primary system (24) with rule (21) as follows

$$\dot{e}_{Ls} = Ae_{Ls} + b\left(-L + \eta_L^T e_L + \left(L - L_d\right)/J_x + \Delta L\right) \tag{25}$$

where $e_{Ls}(0) = e_{L0}$. By the additive state decomposition, we have

$$e_L = e_{Lp} + e_{Ls}. \tag{26}$$

According to the *Theorem 1* in [7], we define a new output $y = c^T e_L$, then rearrange (24)-(26) to be

$$\begin{aligned} \dot{e}_{Lp} &= A e_{Lp} + bL, e_{Lp}(0) = 0 \\ y &= y_p + d_l \end{aligned} \tag{27}$$

where

$$\begin{aligned} d_l &= c^T e_{Ls} \\ &= G\left(-L + \eta_L^T e_L + (L - L_d)/J_x + \Delta L\right) + c^T e^{At} e_{Ls}(0) \end{aligned}$$

and $G = c^T (sI_n - A)^{-1} b$. Here d_l is called the lumped disturbance and $y_p = c^T e_{Lp} = GL$.

3.2 ASDB Dynamic Inversion Stabilized Controller

According to the design procedure in [7], we will design an ASDB dynamic inversion stabilized controller for (23) in four steps:

Step 1. Design a state feedback gain $\eta_L \in \mathbb{R}^2$ such that $A = A_0 - b\eta_L^T$ is stable with $\det(sI_2 - A) = (\alpha s + 1)(\beta_1 s + \beta_0)$. We choose $\eta_L = \begin{bmatrix} 48 & 14 \end{bmatrix}^T$ resulting in

$$A = A_0 - b\eta_L^T = \begin{bmatrix} 0 & 1 \\ -48 & -14 \end{bmatrix}$$

with $\det(sI_2 - A) = \left(\frac{1}{8}s + 1\right)(8s + 48)$.

Remark 1. We notice that the feedback gain η_L should be chosen properly considering the response dynamics of the system (23).

Step 2. Design an output matrix for (27) that

$$c = \left(N^{-1}\right)^T \bar{c} \in \mathbb{R}^2$$

where $\text{adj}(sI_n - A)^{-1} b = N\begin{bmatrix} s & 1 \end{bmatrix}^T$ and $\bar{c} = \begin{bmatrix} \beta_1 & \beta_0 \end{bmatrix}^T$. From step 1, we obtain $\bar{c} = \begin{bmatrix} 8 & 48 \end{bmatrix}^T$ and

$$N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Consequently, the output matrix is designed as

$$c = \left(N^{-1}\right)^T \bar{c} = \begin{bmatrix} 48 & 8 \end{bmatrix}^T.$$

Step 3. From Steps 1-2, we get

$$G = c^T (sI_2 - A)^{-1} b = \frac{1}{\frac{1}{8}s + 1}.$$

As $e_L = X_L - X_{Lc}$, X_L is measurable and X_{Lc} is known, the output $y = c^T e_L$ of the system (27) can be calculated. Since $y_p = c^T e_{Lp} = GL$, the output y is rewritten to

$$y = GL + d_l. \tag{28}$$

and the lumped disturbance d_l can be observed by

$$\hat{d}_l = y - GL = d_l. \tag{29}$$

For system (28), since G is a known minimum phase transfer function, the dynamic inversion tracking controller design is represented as follows

$$L = -G^{-1}\hat{d}_l. \tag{30}$$

Substituting (30) in to (28) results in

$$\begin{aligned} y &= -GG^{-1}\hat{d}_l + d_l \\ &= -\hat{d}_l + d_l = 0. \end{aligned}$$

As a result, perfect tracking is achieved. However, the proposed controller (30) is not realizable. By introducing a low-pass filter Q, the controller (30) is modified as follows

$$L = -QG^{-1}\hat{d}_l \tag{31}$$

where $Q = \frac{1}{\epsilon_L s + 1}$ and an appropriate $\epsilon_L > 0$ should be chosen to achieve a tradeoff between tracking performance and robustness.

Step 4. Choose an appropriate $\epsilon_L > 0$ in practice. For more detailed analysis of the low-pass filter, please refer to [7].

From the ASDB dynamic inversion stabilized controller (31), we get the following roll angle controller (see Fig.3) for the hexacopter

$$L = -\frac{1}{\epsilon_L s + 1} G^{-1}\hat{d}_l \tag{32}$$

where $\epsilon_L > 0$ should be chosen appropriately, and

$$\hat{d}_l = c^T e_L - GL.$$

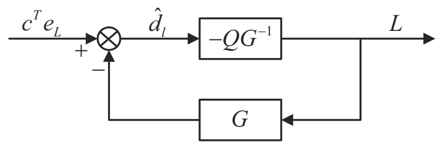

Fig. 3: ASDB dynamic inversion stabilized roll angle controller

In practice, the systems (16) and (17) need nearly the same dynamic response as the system (15). Then from Step 1 to Step 4, the following controller are obtained directly

$$M = -\frac{1}{\epsilon_M s + 1} G^{-1}\hat{d}_m \tag{33}$$

$$N = -\frac{1}{\epsilon_N s + 1} G^{-1}\hat{d}_n \tag{34}$$

where $\epsilon_M > 0, \epsilon_N > 0$ should be chosen appropriately, and

$$\begin{aligned} \hat{d}_m &= c^T e_M - GM \\ \hat{d}_n &= c^T e_N - GN \end{aligned}$$

For the altitude system (18), it usually needs a slower dynamic response than the attitude system (15)-(17). According to Step 1, we choose $\eta_h = \begin{bmatrix} 12 & 7 \end{bmatrix}^T$ resulting in

$$A_h = A_0 - b\eta_h^T = \begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix}$$

Fig. 4: Prototype hexacopter

with $\det(sI_2 - A_h) = \left(\frac{1}{4}s+1\right)(4s+12)$. And then in Step 2, we get the output matrix $c_h = \left[\begin{array}{cc} 12 & 4 \end{array}\right]^T$. At last, we get $G_h = c_h^T(sI_n - A_h)^{-1}b = \frac{1}{\frac{1}{4}s+1}$, and obtain the following controller

$$F = -\frac{1}{\epsilon_F s+1}G_h^{-1}\hat{d}_h \tag{35}$$

where $\epsilon_F > 0$ should be chosen appropriately and

$$\hat{d}_h = c_h^T e_h - G_h F.$$

The set of rotors' lift f is obtained by (6).

When the propellers are damaged, no control re-allocation or FDD module is required, because the controllers (32)-(35) will handle the disturbances caused by the propeller damages automatically.

4 Simulations

In order to show the effectiveness of the proposed controller for the hexacopter subject to propeller damages, simulations of a prototype hexacopter (see Fig.4) are carried out. A PD controller for the attitude and altitude control of the hexacopter is used to compare with the proposed ASDB dynamic inversion stabilized controller. Control re-allocation is used for the PD controller after propeller 2 of the hexacopter is damaged, where we have to detect the effectiveness factors $\gamma_i, i \in \{1,2,3,4,5,6\}$ and use the control re-allocation method shown in (8). But the proposed ASDB dynamic inversion stabilized controller does not need control re-allocation or FDD module after propeller 2 is damaged.

4.1 Simulation Parameters

The physical parameters of the prototype hexacopter are shown in Table 1. Rotor dynamics are considered and modeled by

$$\lambda_i = \frac{1}{t_r s+1}f_i, i \in \{1,2,3,4,5,6\} \tag{36}$$

where λ_i is the actual lift of rotor i. We set the dynamic response parameter $t_r = 0.01s$.

In the simulation, the hexacopter is controlled to 1 meter above the ground ($h_c = 1$), and keeps level ($\phi_c = \theta_c = \psi_c = 0$). Propeller 2 is damaged by 50% at time $t = 5s$, that is, after $t = 5s$, the effectiveness factors (see (7)) $\gamma_1 = 0, \gamma_2 = 0.5, \gamma_3 = 0, \gamma_4 = 0, \gamma_5 = 0, \gamma_6 = 0$. We choose $\epsilon_L = 0.5, \epsilon_M = 0.5, \epsilon_N = 0.5, \epsilon_F = 0.05$ for (32)-(35).

The PD controllers of the systems of (1) and (3) is given by

$$\begin{aligned} L &= -P\phi - Dp \\ M &= -P\theta - Dq \\ N &= -P\psi - Dr \\ F &= P_h(h_c - h) - D_h V_h. \end{aligned} \tag{37}$$

We choose $P = 20, D = 3, P_h = 10, D_h = 6$ in the simulation.

4.2 Simulation Results

Simulation results under propeller damages are shown in Fig.5 and Fig.6 (the dotted line representing the simulation results under the PD controller, and solid line for the ASDB dynamic inversion stabilized controller). In Fig.5, control re-allocation is not used for the PD controller, and the PD controller in Fig.6 has a control re-allocation module. We can see that both the PD controller and the ASDB dynamic inversion stabilized controller can control the hexacopter to the target in a good manner when the propeller 2 is not damaged (before time $t = 5s$). But after propeller 2 is damaged, the PD controller can not control the hexacopter to the target in Fig.5. The PD controller in Fig.6 have a good stabilizing performance, but it needs a control re-allocation module to deal with the disturbances caused by propeller damage, and a FDD module is needed to detect the effectiveness factors of the propellers in practice. From the simulation results, the ASDB dynamic inversion stabilized controller can handle the propeller damage automatically without control re-allocation or FDD module. The control force and torques of the ASDB dynamic inversion stabilized controllers are shown in Fig.7, where we can see that the controller can compensate for the disturbances caused by propeller damage automatically after time $t = 5s$.

Remark 2. It does not mean that all the Proportional-Integral-Derivative (PID) controllers needs control re-allocation to handle the propeller damages. A gain-scheduled PID controller is applied to deal with the propeller damages in [4], which needs no control re-allocation after the propeller damages happen. The PD controller in this paper is only used to show the effectiveness of the proposed ASDB dynamic inversion stabilized controller.

Table 1: HEXACOPTER PARAMETERS

Parameter	Description	Value	Units
m	Mass	1.535	kg
g	Gravity	9.80	m/s^2
d	Rotor to mass center distance	0.275	m
K	Maximum lift of each rotor	6.125	N
J_x	Moment of inertia	0.0411	$kg.m^2$
J_y	Moment of inertia	0.0478	$kg.m^2$
J_z	Moment of inertia	0.0599	$kg.m^2$
k_μ	Ratio of reactive torque and lift	0.1	-

5 Conclusions

In this work, the control problem of a hexacopter subject to unknown propeller damages is studied and formulated as a control problem subject to time-varying disturbances. An

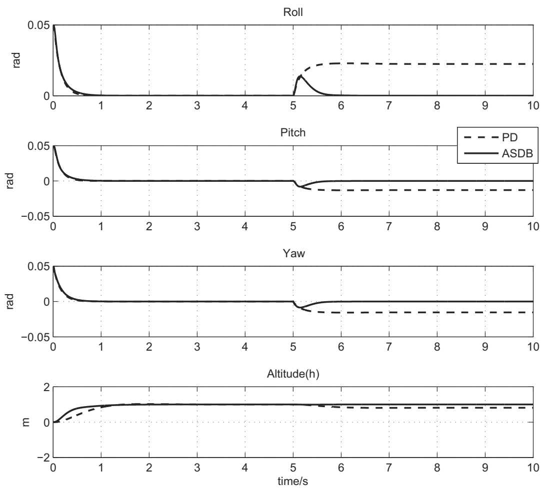

Fig. 5: Attitude and altitude dynamics where controll re-allocation and FDI module are not used for the PD controller

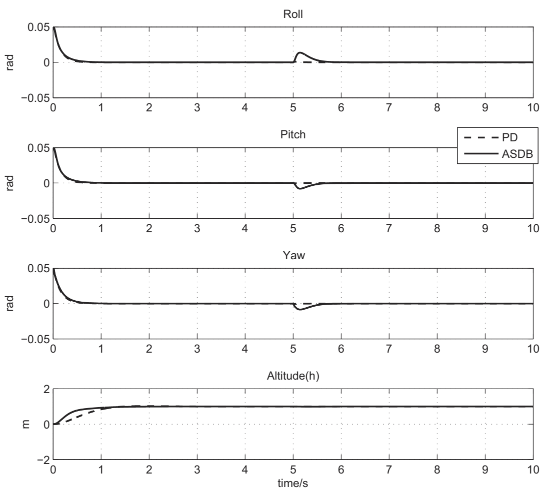

Fig. 6: Attitude and altitude dynamics where controll re-allocation and FDI module are used for the PD controller

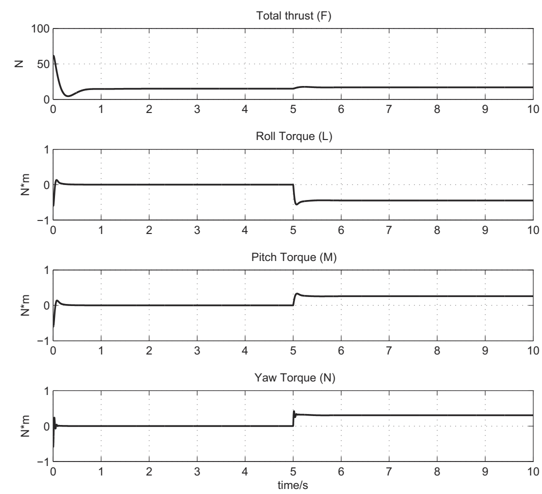

Fig. 7: Control force and torque of the ASDB dynamic inversion stabilized controller

ASDB dynamic inversion stabilized control method is applied to reject the disturbances. When propeller damages happen, a good rejection performance is achieved. Moreover, neither fault detection and diagnosis module nor control re-allocation is required for the hexacopter under this control framework. Finally, a simulation example shows its effectiveness. In our future work, experiments based on the prototype hexacopter (as shown in Fig.4) will be performed to show the practical effectiveness of the ASDB dynamic inversion stabilized controller.

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