New Transition Method of a Ducted-Fan Unmanned Aerial Vehicle

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In this research, a winged version of the ISTAR (from “intelligence, surveillance, target acquisition, and reconnaissance”) unmanned aerial vehicle is modeled first, and then a new transition method is proposed, which enables the vehicle to fly sideways at low speed for drag reduction. A nonlinear trajectory-following logic is adopted to correct altitude loss and horizontal drift during the transition. By simulation, the proposed transition method shows better acceleration performance. The comparison of the induced drag shows that the vehicle should roll to resume fixed-wing lift as soon as its minimum level-flight speed is achieved to optimize its acceleration performance.

I. Introduction

Among various configurations of unmanned aerial vehicles (UAVs), the ducted-fan tail-sitter vertical takeoff and landing (VTOL) UAV is a simple configuration to realize VTOL function. This advantage has led to the development of several of these vehicles, such as the GoldenEye, Kestrel, and ISTAR (from “intelligence, surveillance, target acquisition, and reconnaissance”). The ISTAR is the most widely used one. Its general layout consists of a ducted fan and deflectable vanes at the end of the duct.

Different battlefield situations may require the vehicle to fly at high speed (e.g., ground-fire evasion) or at low speed (e.g., battlefield surveillance and reconnaissance). In hover and low-speed level flight, the vehicle’s longitudinal axis is vertical, or with a small angle of inclination [1]. In transition mode, it tilts its body forward to accelerate. In high-speed level-flight mode, the vehicle’s longitudinal axis is almost parallel to the horizon [2,3].

ISTAR has many types of different sizes and weights. Despite such differences, an axisymmetric configuration, as shown in Fig. 1, is shared by all types, which makes it easy to give an aerodynamic model so as to simplify control laws [4], but it also has an evident disadvantage in high-speed level flight because the lift-to-drag ratio is relatively low, thus reducing level-flight efficiency.

One solution is to add fixed wings. Figure 2 shows a winged version of an ISTAR UAV. Fixed wings enhance its high-speed flight efficiency but generate tremendous drag at a high angle of attack, when the UAV flies at low speed, or when it initializes its transition flight, which is very steep. An existing solution is to use a variable-incidence wing, for example the GoldenEye, as shown in Fig. 3. However, its pitching mechanism increases the weight and the complexity. To surmount this difficulty, the UAV may fly sideways at low speed for drag reduction, because of a smaller windward side area, and transfer directly from flying sideways to high-speed mode after a roll maneuver, as shown in Fig. 4. In fact, before the roll maneuver is executed, the UAV flies like a helicopter, or a wingless version of ISTAR, because the fixed wing creates no lift at zero angle of attack. In this case, the control law could be based on that developed for the wingless version of ISTAR. But after the roll maneuver, fixed-wing lift is resumed, and a new control law is required.

The UAV will gain two advantages by flying sideways. First of all, greater acceleration will be achieved because of the drag reduction. Second, the variable-incidence wing is waived off and fixed wing is enough. As described previously, a special control law is required to tilt its body in a relatively complicated way, while eliminating altitude loss and horizontal drift caused by the roll maneuver. In this research, a nonlinear control law is developed and proved, which allows posing the UAV at the desired attitude and maintaining the original trajectory.

One possible disadvantage of flying sideways is increased difficulty of holding a position in a horizontal varying wind. This is equivalent to an ordinary fixed-wing aircraft flying at a bank angle of 90 deg. Because this preliminary study is for illustrating a new concept, such a problem will be discussed in future work.

The UAV under consideration has a layout similar to that of the winged version of ISTAR. It consists of a ducted fan, a rotor, four vanes at the end of the duct, and two fixed wings, as shown in Fig. 5. The vanes attached under the slipstream of the propeller always produce control moments at all points of the flight envelope.

This paper is organized as follows. A mathematical model is given in Sec. II, an attitude-control law is developed in Sec. III, a trajectory-following method is adopted to correct horizontal and vertical deviation in Sec. IV, and the simulation results are given in Sec. V.

II. Mathematical Model

A model of the UAV was created for simulation to verify controller behavior. The following sections detail the formulation of this model, with numerical values listed in Appendix A.

Let \( \{e_1, e_2, e_3\} \) denote an inertial frame and \( \{e_x, e_y, e_z\} \) denote a body frame rigidly attached to the aircraft whose origin is attached to the center of gravity, where \( e_1, e_2, e_3, e_x, e_y, \) and \( e_z \) are the unit vectors of axis \( x, y, z \).
In the following, all vectors are expressed in the body frame, unless otherwise specified.

The vane system has four control surfaces, and each operates independently. They are numbered 1, 2, 3, and 4, and their deflection input is shown in Fig. 7.

Equivalent vane deflections ($\delta_x$, $\delta_y$, and $\delta_z$) of roll ($z$ axis), yaw ($y$ axis), and pitch ($x$ axis) are given as follows:

\[
\begin{align*}
\delta_z &= \delta_1 + \delta_2 + \delta_3 + \delta_4 \\
\delta_y &= \delta_2 - \delta_4 \\
\delta_x &= \delta_3 - \delta_1
\end{align*}
\]

where $\delta_1$, $\delta_2$, $\delta_3$, and $\delta_4$ denote the deflection of vane numbers 1, 2, 3, and 4, respectively. The maximum deflection angle of the vanes is set to 28 deg.

To overcome singularity points, a quaternion attitude description is adopted as a global attitude reference [5,6]. This globally nonsingular representation of the orientation is given by the vector $[q_0 \quad q^T]^T$ subjected to the constraint $q_0^2 + q^Tq = 1$. $q$ is a three-dimensional vector presented as $q = [q_1 \quad q_2 \quad q_3]^T$.

Note that the quaternion describing the vehicle in hover is $[1 \quad 0 \quad 0 \quad 0]^T$. In this case, the axes $x$, $y$, and $z$ in the body frame coincide with the axes $x'$, $y'$, and $z'$ in the inertial frame.
The direction cosine matrix $R$ that transfers vectors in the inertial frame to the body frame can be written as \cite{7}

$$R(q_0q) = I_{3 \times 3} + 2S(q)v^2 - 2q_0S(q) \tag{2}$$

where $S(\cdot)$ is the skew-symmetric matrix such that $S(v) = u \times v$, where $\times$ denotes the vector cross product and $u, v \in \mathbb{R}^3$.

It was of primary importance to model the major forces and moments acting on the vehicle. Simplifying assumptions were made in certain respects.

The basic dynamic equations are given as

$$\dot{p} = v$$

$$\dot{v} = \frac{R^T F}{m}$$

$$J\dot{\Omega} = -\Omega \times J\Omega + M$$

$$\dot{q} = \frac{1}{2} (q \times \Omega + q_0 \Omega)$$

$$\dot{q}_0 = -\frac{1}{\Omega} \Omega q$$ \tag{3}

where $p$ denotes the position vector of the UAV in the inertial frame. The vector $v = [v_1 \ v_2 \ v_3]^T$ denotes the velocity components in the inertial frame along axes $x'$, $y'$, and $z'$, respectively. The vector $\Omega = [\Omega_x \ \Omega_y \ \Omega_z]^T$ denotes the angular velocity of the vehicle expressed in the body frame, where $\Omega_x$, $\Omega_y$, and $\Omega_z$ are, respectively, the pitch, yaw, and roll rates. Here, $J$ denotes the inertia matrix of the airframe, while $F$ and $M$ represent the sum of external forces and moment vectors acting on the vehicle, expressed in the body frame. The force and moment vectors can be expressed as

$$F = F_{\text{aero}} + F_{\text{rotor}} + F_{\text{cw}} + F_{\text{cs}} + F_{\text{grav}} + F_d \tag{4}$$

$$M = M_{\text{aero}} + M_{\text{rotor}} + M_{\text{cw}} + M_{\text{cs}} + M_{\text{gyro}} + M_d \tag{5}$$

These component forces and moments are discussed next.

A. Vehicle Aerodynamics

In the following section, all vectors are projected to the body frame for simplification. The projection of the velocity vector to the body frame can be written as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = R \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{6}$$

The aerodynamic force consists of the lift, the drag, and the side force. Note that the resultant of duct consists of two contributions: one from aerodynamics, and one from the crosswind effect of the duct body \cite{4}. Here, the aerodynamical contributions of duct and fixed wing are considered altogether as a lifting body, and the contribution of crosswind effect will be discussed separately in Sec. II.C. Because the vehicle can maneuver at high angle of attack $\alpha$ and high angle of side slip $\beta$, these two angles are of primary importance. They can be expressed as follows:

$$\alpha = \begin{cases} 0 & \text{if } v_z = 0 \\ -\tan^{-1}(\frac{v_y}{v_x}) & \text{if } v_z \geq 0 \text{ and } v_y \neq 0 \\ -\tan^{-1}(\frac{v_y}{v_x}) + \pi & \text{if } v_z < 0 \text{ and } v_y \neq 0 \end{cases} \tag{7}$$

$$\beta = \begin{cases} 0 & \text{if } v_x = 0 \\ \tan^{-1}(\frac{v_x}{v_z}) & \text{if } v_z \geq 0 \text{ and } v_x \neq 0 \\ \tan^{-1}(\frac{v_x}{v_z}) + \pi & \text{if } v_z < 0 \text{ and } v_x \neq 0 \end{cases} \tag{8}$$

Note that Eqs. (7) and (8) are well defined when $v_z = 0$ because $\tan^{-1}(\pm \infty) = \pm \frac{\pi}{2}$. Here, $S_{\text{wing}}$ is the reference area of the lift and the drag, and $S_{\text{fix}}$ is that of the side force. Let $C_{d,\beta}$ and $C_{\beta}$ denote the lift curve slope and the side-slip force curve slope of the lifting body. Given the fact that the symmetric airfoil is adopted, the coefficients of lift, drag, and side force can be expressed approximately by the sinusoidal expressions written as Eq. (9). The same method is adopted in \cite{4}:

$$C_l = \max \left\{ -C_{l,\max} \min \left\{ 1, \frac{1}{2} C_{\beta} \sin(2\alpha) \right\} \right\}$$

$$C_d = C_{d,\beta} \sin(2\alpha) + C_{d,0}$$

$$C_s = \max \left\{ -C_{s,\max} \min \left\{ 1, \frac{1}{2} C_{\beta} \sin(2\beta) \right\} \right\} \tag{9}$$

In \cite{4}, this modeling technique is used for an airfoil, but here we assume that it is also acceptable for the entire lifting body. Such simplified modeling causes some errors, especially at high angle of attack and angle of side slip, thus the disturbing moment $M_d$ and disturbing force $F_d$ could not be neglected, which have negative effects on attitude and position control. Such disturbance will be compensated by a sliding-mode control and a trajectory-following logic, as discussed in following sections.

Remark 1: Although $C_l$ is the lift coefficient of the whole lifting body, including the aerodynamical contributions from the duct and the fixed wing, it does not take the crosswind effect of the duct into account. This effect will be described separately in Sec. II.C.

The minimum level-flight speed could be derived from $C_{l,\max}$ as follows:

$$v_{\text{min}} = \sqrt{\frac{mg}{\frac{1}{2} C_{l,\max} \rho_{\infty} S_{\text{wing}}}} \tag{10}$$

By plugging the numerical values into Eq. (10), one can get a minimum level-flight speed of 23.1 m/s.

The lift $L$, drag $D$, and side force $Y$ in the body frame can be expressed as follows:

$$L = \begin{bmatrix} 0 \\ \frac{1}{2} C_{l,\rho}(v_x^2 + v_y^2) S_{\text{wing}} \cos \alpha \\ \frac{1}{2} C_{l,\rho}(v_x^2 + v_y^2) S_{\text{wing}} \sin \alpha \end{bmatrix} \tag{11}$$

$$D = \begin{bmatrix} 0 \\ -\frac{1}{2} C_{d,\rho}(v_x^2 + v_y^2) S_{\text{wing}} \cos \alpha \\ -\frac{1}{2} C_{d,\rho}(v_x^2 + v_y^2) S_{\text{wing}} \sin \alpha \end{bmatrix} \tag{12}$$

$$Y = \begin{bmatrix} -\frac{1}{2} C_{s,\rho}(v_x^2 + v_y^2) S_{\text{side}} \cos \beta \\ 0 \\ \frac{1}{2} C_{s,\rho}(v_x^2 + v_y^2) S_{\text{side}} \sin \beta \end{bmatrix} \tag{13}$$

Figure 8 shows the axes of the body frame with respect to $\alpha$ and $\beta$. The density of air $\rho_{\infty}$ is supposed to be constant.

Because the vehicle may experience high angle of side slip, additional drag is generated (the induced drag of side force) and is perpendicular to the side force. By analogy to the discussion of the components of $D$, the additional drag can be expressed as

$$C_{d}' = C_{d,\beta} \sin(2\beta) \tag{14}$$

$$D' = \begin{bmatrix} 0 \\ -\frac{1}{2} C_{d,\rho}(v_x^2 + v_y^2) S_{\text{side}} \sin \beta \\ -\frac{1}{2} C_{d,\rho}(v_x^2 + v_y^2) S_{\text{side}} \cos \beta \end{bmatrix} \tag{15}$$

In terms of aerodynamic moment, given the symmetric layout of the vehicle, let the vector $[0 \ 0 \ -z_1]^T$ denote the position vector of the
where \( M \) is the term representing the rotor torque vector, given as

\[
M = M_{\text{aero1}} + M_{\text{aero2}}
\]

where

\[
M_{\text{aero1}} = [0 \ 0 \ -z_1] \times (L + D)
\]

\[
M_{\text{aero2}} = [0 \ 0 \ -z_2] \times (Y + D')
\]

B. Rotor

The term \( F_{\text{aero}} \) representing thrust vector can now be expressed as

\[
T = \bar{b}u_i^2 \quad F_{\text{aero}} = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}
\]

The term \( M_{\text{rotor}} \), representing the rotor torque vector, is given as follows:

\[
Q = \kappa \omega_r^2 \quad M_{\text{rotor}} = \begin{bmatrix} 0 \\ 0 \\ Q \end{bmatrix}
\]

where \( b \) and \( \kappa \) are two positive parameters depending on the density of air, the size and twist of blades, and other factors [8]. Here, \( \omega_r \) represents the angular velocity of the rotor.

The airflow and the induced velocity through the rotor are negative because the airflow is accelerated in the negative \((-z)\) direction. The induced velocity can be expressed as follows [9]:

\[
v_i = \frac{1}{2} \left( v_i - \sqrt{v_i^2 + \frac{2T}{\rho r^2}} \right)
\]

where \( r \) represents the radius of blades.

C. Crosswind Effect of Duct

In the presence of crosswind, the duct is subjected to additional force and moment vector, which can be expressed as [4]

\[
F_{\text{cw}} = v_i \rho_{\infty} r^2 \begin{bmatrix} 0 \\ v_i \\ 0 \end{bmatrix}
\]

\[
M_{\text{cw}} = C_{\text{duc}} \rho_{\infty} r^4 \begin{bmatrix} -v_i |v_i| \\ v_i |v_i| \\ 0 \end{bmatrix}
\]

Note that this model is approximate. The vector \( M_{\text{cw}} \) measured by [10] and other references such as [1] shows that \( M_{\text{cw}} \) varies linearly with crosswind velocity, not in a quadratic manner as described by Eq. (21). This quadratic model is simple and has modeling errors, which contributes to the disturbing moment. But [4] shows that this error could be compensated if the control law is properly designed. A robust sliding mode control law will be shown in Sec. III, and its robustness will be proved by assuming that the disturbing moment is bounded.

D. Control Surfaces

The control surfaces (vanes) are exposed in the downwash of the rotor. The dynamic pressure \( q_{\text{cs}} \) can be written as follows:

\[
q_{\text{cs}} = \frac{1}{2} \rho_{\infty} (v_z - v_i)^2
\]

Let \( C_{\text{cs}} \) denote the lift curve slope of the vanes. It is assumed that the distance between the aerodynamic center of a vane and the \( z \) axis, \( y \) axis, and \( x \) axis of the body frame are respectively \( l_{e, z} \), \( l_{e, y} \), and \( l_{e, x} \). Note that \( l_{e, x} = l_r \) due to symmetry. By adding a constraint,

\[
\delta_z = -\delta_4
\]

one can express \( [\delta_1 \ \delta_2 \ \delta_3 \ \delta_4 \ \delta_z]^T \) with \( [\delta_x \ \delta_y \ \delta_z]^T \) as follows:

\[
\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_z \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & -0.5 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}
\]

The constraint [Eq. (23)] is necessary because the system [Eq. (1)] has more unknowns than equations; thus, Eq. (1) alone is not enough to express vane deflections as a function of control inputs.

The terms \( F_{\text{cs}} \) and \( M_{\text{cs}} \) representing the force and the moment of vanes can be written as

\[
F_{\text{cs}} = q_{\text{cs}} S_{\text{cs}} C_{\text{cs}} \begin{bmatrix} -\delta_z + \delta_1 \\ -\delta_y - \delta_1 \\ 0 \end{bmatrix}
\]

\[
M_{\text{cs}} = q_{\text{cs}} S_{\text{cs}} C_{\text{cs}} \begin{bmatrix} I_e \ 0 \ 0 \\ 0 \ I_e \ 0 \\ 0 \ 0 \ I_u \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}
\]

Note that, for vanes, the effect of the drag forces has been neglected.

Once the control input \( M_{\text{cs}} \) is obtained from the control law, vane deflections could be derived by inverting Eq. (26) and then plugging in to Eq. (24).

E. Gravity

Gravitational forces are accounted for as follows, with \( m \) as the total mass of the UAV and \( g \) the acceleration due to gravity:
F. Gyroscopic Moment

The rotor rotates counterclockwise when seen from the front view. By assuming that the rate of change of the rotor’s angular velocity is negligible, the expression for this moment vector is given next [4]:

\[
M_{\text{gyro}} = n_bl_o \Omega \begin{bmatrix}
-\Omega_z \\
\Omega_x \\
0 
\end{bmatrix}
\]  

where \( \Omega \) is the equilibrium point of system (29) to compensate.

III. Attitude-Control Design

In this section, a feedback control is developed to stabilize the vehicle at any attitude; the vehicle dynamics of Eq. (3) are used. To avoid the sign ambiguity described in [3], the Euler angles and the equivalent rotation angle are taken between \(-\pi \) and \( \pi \), which implies \( q_0 \in \{0, 1\} \). Let \([q_0, q^T_1]\) denote the quaternion describing the objective attitude. The vector of control signals is \( M_{cs} \).

The error quaternion, which is part of the state vector of the error system, is defined as

\[
[q_{e0}, q^T_e] = [q_0, q^T_1] \ast [q_0, q^T_1] = [q_{e0}, -q^T_e] \ast [q_0, q^T_1]
\]

where \( \ast \) denotes quaternion multiplication.

Because \([q_{e0}, q^T_e]\) is, in fact, the expression of \([q_0, q^T_1]\) in another inertial frame, the fourth and fifth equation of Eq. (3) are also verified for \([q_{e0}, q^T_e]\). The error system can be expressed as

\[
\dot{q}_e = \frac{1}{2}(q_e \times \Omega + q_0 \dot{\Omega}) \\
\dot{q}_{e0} = -\frac{1}{2}\Omega^T q_e \\
J\ddot{\Omega} = -\Omega \times J\dot{\Omega} - M_{\text{com}} + M_{cs} + M_d
\]  

where \( M_{\text{com}} = -M_{\text{aero}} - M_{\text{rotor}} - M_{cw} - M_{\text{gyro}} \) denotes the moment to compensate.

Given the presence of \( M_d \), a proportional-derivative sliding mode controller is proposed to neutralize the disturbance.

**Theorem 1:** Under the following control law:

\[
M_{cs} = J\left[-\frac{1}{2}s - K_s \text{sign}(s) + K_{\text{com}}\right] + M_{\text{com}}
\]

where

\[
s = q_e + \Omega \\
\text{sign}(s) = \begin{bmatrix}
\text{sign}(s_1) \\
\text{sign}(s_2) \\
\text{sign}(s_3)
\end{bmatrix} \\
K_{\text{com}} = J^{-1}\Omega \times J\dot{\Omega} - \frac{1}{2}(q_e \times \Omega + q_0 \dot{\Omega})
\]

the equilibrium point of system (29)

\[
[q_{e0}, q^T_e] = [1, 0, 0, 0]^T
\]

is globally asymptotically stable.

Note that there is an implied gain of 1 with units in the definition of \( s \), because one cannot add quaternion (with no units) to angular velocity (with units).

The proof of this control law could be found in Appendix B.

Figure 9 shows the simulated result of attitude step change command of \( [q_0, q^T_1] \) from \([1, 0, 0, 0]^T\) to \([0.6, 0.3, 0.7, 0.245]^T\), with excellent speed of convergence. It takes 1–2 s to converge to desired attitude. This controller will also be effective in tracking a slowly changing reference, as will be shown in the following sections.

IV. Trajectory-Following Logic

In this section, our goal is to accomplish the transition without altitude loss or horizontal drift. Any deviation from \( \gamma' = 0 \) or \( z' = 0 \) should be compensated by a lateral acceleration. In this article, the following trajectory-following logic is adopted [11], as shown in Fig. 10.

\[
a_{\text{cmd}} = 2V^2 \sin \eta
\]  

The lateral acceleration \( a_{\text{cmd}} \) is perpendicular to velocity vector, pointing toward the desired trajectory. The reference point is on the desired path at a distance \( L_1 \) forward of the vehicle. In this research, \( L_1 \) is set to a value significantly greater than the deviation so that \( \eta \) is a small angle. Note that Fig. 11 is a simple schema in two dimensions, while the UAV maneuvers in three dimensions. A detailed schema with coordinate frame as reference will be shown in Figs. 11–13.

A significant property of this guidance logic is that the lateral acceleration changes sign at some points, which guarantees a smooth convergence to the desired path. Besides, considering the fact that the
UAV may fly in narrow indoor circumstances, such a control law offers quick convergence to a curved path, which allows avoiding indoor obstacles\cite{11}. With small-angle approximation, this guidance logic is equivalent to a proportional-integral-derivative (PID) controller\cite{12}, which eliminates static error of trajectory tracking despite the presence of disturbing force. As mentioned in Sec. 1, the transition consists of two phases, during which the lateral acceleration is generated in respectively different ways. In the first phase, the vehicle flies sideways. The gravity $G$ is balanced by the vertical component of side force $Y$, and thus the lateral acceleration is generated in the $x$-$y$ plane. Such components are generated by the resultant of $G, Y$, and $T$. As it accelerates, it inclines forward, and in view of Eq. (13), Eq. (35) gives

$$T \sin \beta + Y = mg \cos \sigma - m\alpha_{cmd}$$

where

$$\alpha_{cmd} = \frac{v_x}{v_z} \sin \sigma_{total}$$

From force balance:

$$ma_{cmd} = mg \cos \sigma - T \cos (\gamma + \sigma) - Y$$

Given that $\beta = \frac{\pi}{2} - \gamma - \sigma$ and in view of Eq. (13), Eq. (35) gives

$$\frac{T}{2} \sin(2\beta)$$

Once $\alpha_{cmd}$ is calculated by Eq. (32), $\beta$ can be obtained from Eqs. (36) and (37) iteratively. Thus, the objective angle of inclination $\gamma$ is given by $\gamma = \frac{\pi}{2} - \beta - \sigma$. The quaternion representing the desired attitude can be written as follows:

$$q_{\ell, \text{phase} 1} = \left[ \cos \frac{\gamma}{2}, 0, \sin \frac{\gamma}{2} \right]^T$$

When the first phase ends after the vehicle reaches a threshold speed, the vehicle executes a roll maneuver to level flight.

Remark 2: For the conventional transition method, the vehicle inclines forward, and the angle of inclination could be obtained with a similar method as described previously. The only difference is that the vehicle uses lift $L$ instead of side force $Y$ to balance its gravity.

B. Second Phase

The altitude loss and horizontal drift caused by the roll will be compensated by pitch and yaw maneuvers. Note that the pitch angle $\phi$ (Fig. 13) and the yaw angle $\theta$ (Fig. 12) are small, and thus these two maneuvers are commutative and can be considered independently. The lateral acceleration in the $x$-$z'$ plane and in the $x$-$y'$ plane are noted $\alpha_{cmd}$ (Fig. 12) and $\alpha_{cmd}$ (Fig. 13), respectively. In this case, the direction cosine matrix $R$ is simplified and can be expressed as follows:

$$R = \begin{bmatrix} -\theta & \phi & 0 \\ \phi & 0 & 1 \\ 0 & 1 & -\phi \end{bmatrix}$$

The velocity expressed in the body frame is thus given by

$$v_x = R \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\theta v_1 + v_2 \\ \phi v_1 + v_3 \\ v_1 + \theta v_2 - \phi v_3 \end{bmatrix}$$
Similar to the discussion in the first phase, the schema of lateral acceleration and velocity are drawn in the \(x'-y'\) plane and the \(x'-z'\) plane, respectively.

1. \(x'-y'\) Plane

\[
a_{cmd1} = 2 \frac{v_y^2 + v_z^2}{\sqrt{(x' - x_{obj})^2 + (y' - y_{obj})^2}} \sin \sigma_{total1} \tag{41}
\]

where

\[
\sigma_1 = \tan^{-1}\left(\frac{v_y}{v_z}\right) \quad \sigma_{total1} = \tan^{-1}\left(\frac{y' - y_{obj}}{x_{obj} - x'}\right) + \sigma_1
\]

When expressed in the inertial frame, the vector form of \(a_{cmd1}\) is

\[
a_{cmd1vec} = \begin{bmatrix} a_{cmd1} \sin \sigma_1 \\ -a_{cmd1} \cos \sigma_1 \\ 0 \end{bmatrix}
\]

2. \(x'-z'\) Plane

\[
a_{cmd2} = 2 \frac{v_x^2 + v_z^2}{\sqrt{(x' - x_{obj})^2 + (z' - z_{obj})^2}} \sin \sigma_{total2} \tag{42}
\]

where

\[
\sigma_2 = \tan^{-1}\left(\frac{v_x}{v_z}\right) \quad \sigma_{total2} = \tan^{-1}\left(\frac{z' - z_{obj}}{x_{obj} - x'}\right) + \sigma_2
\]

When expressed in the inertial frame, the vector form of \(a_{cmd2}\) is

\[
a_{cmd2vec} = \begin{bmatrix} a_{cmd2} \sin \sigma_2 \\ -a_{cmd2} \cos \sigma_2 \\ 0 \end{bmatrix}
\]

To calculate the desired attitude, the objective aerodynamic forces (lift and side force) are essential. Because \(a_{cmd1}\) and \(a_{cmd2}\) only have lateral components that are perpendicular to velocity, but the gravity \(G\) and the thrust \(T\) also have such lateral components, these two forces must be compensated. Given small-angle approximation, and in view of Eqs. (11) and (13), the first and the second components of aerodynamic force (expressed in body frame) denote respectively the desired side force and the lift, which is equivalent to the following (other contributions to \(F\) are neglected, which are accounted for in the disturbing force):

\[
L = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} R F_{cmd} \quad Y = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix} R F_{cmd} \tag{43}
\]

where

\[
F_{cmd} = m(a_{cmd1vec} + a_{cmd2vec}) - G - R^T T
\]

The desired angle of attack and angle of side slip are expressed as

\[
\alpha = -\frac{v_y}{v_z} \quad \beta = \frac{v_x}{v_z} \tag{45}
\]

In view of Eqs. (40) and (45), and given that \(v_4\) is significantly greater than \(v_1\) and \(v_3\):

\[
\alpha = -\frac{\phi v_1 + \theta \phi v_2 + v_3}{v_1} \quad \beta = \frac{-\theta v_1 + v_2}{v_1} \tag{46}
\]

By using the small-angle approximation, the desired pitch angle and yaw angle can be obtained from Eq. (46):

\[
\phi = -\alpha - \frac{v_3}{v_1} \\
\theta = -\beta + \frac{v_2}{v_1} \tag{47}
\]

Here, the three components of velocity are assumed to be derived from onboard sensors. Once the desired angle of attack and angle of side slip are obtained from Eq. (44), the desired yaw angle and pitch angle (\(\phi\) and \(\theta\)) are calculated from Eq. (47) accordingly.

Considering that the reference attitude quaternion of the roll maneuver is \(\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T\) (the \(z\) axis and the \(x\) axis in the body frame coincide respectively with the \(x'\) axis and the \(y'\) axis in inertial frame), the quaternion representing the desired attitude of the second phase can be written as follows:

\[
q_{r,phase2} = \left\{\begin{bmatrix} 1 \\
\frac{\theta}{2} \\
0 \\
0 \end{bmatrix} * q_{yaw} * q_{pitch}\right\}^T \tag{48}
\]

where

\[
q_{yaw} = \begin{bmatrix} \cos \frac{\theta}{2} \sin \frac{\phi}{2} \\
0 \\
\sin \frac{\theta}{2} \cos \frac{\phi}{2} \\
0 \end{bmatrix} \quad q_{pitch} = \begin{bmatrix} \phi \cos \frac{\theta}{2} \\
0 \\
\phi \sin \frac{\theta}{2} \\
0 \end{bmatrix}
\]

Figure 14 shows the control implementation. This diagram is inclusive of phases 1 and 2, except the roll maneuver, during which only the attitude is under control (closed-loop), but the position is open-loop.

V. Simulation Result

A. Results of New Transition Method

The animation of this simulation has been uploaded online.5

The transition from hover to forward flight begins at \(t = 0\). There is a roll maneuver between phases 1 and 2. During the first step of this simulation, the roll maneuver is executed at \(t = 15\) s, which is long enough for the vehicle to reach its minimum level-flight speed \(v_{min}\) derived from Eq. (10); \(v_{min}\) is reached at \(t = 9\) s, and so other conversion times will be tested in the following sections. The roll maneuver lasts for \(1\) s, with the desired attitude quaternion \(\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T\). This time interval must be compatible with the dynamic characteristics of the UAV. For a UAV as agile as this one, \(1\) s is enough for it to finish the roll maneuver. But for other vehicles, such as a heavier ISTAR model, this time interval could be longer.

Figure 15 shows the displacement over time in three directions in the inertial frame. Figure 16 shows the corresponding velocity, which is the time derivative of Fig. 15. The vehicle inclines relatively slowly. The simulation shows that it takes \(10\) s to achieve a pitch angle of about 90 deg. This is the result of the control law because it requires the vehicle to maintain altitude, as described in Eq. (36). If it inclines too fast, there will not be enough dynamic pressure for the vehicle to balance its gravity with side force.

5Data available online at http://www.youtube.com/user/zwzzwz/videos [retrieved 01 July 2012].
At the beginning, the vehicle does not maintain altitude, but ascends slowly until \( t = 10 \) s. This is because the attitude control is time-delayed, as shown in Fig. 17; therefore, the angle of inclination is smaller than the command, thus making the vertical component of thrust greater than needed. This could be treated as a disturbing force in the vertical plane. Despite such disturbance, the vehicle descends back to its original altitude at \( t = 12 \) s because the trajectory-following logic is equivalent to a PID controller [12] with small-angle approximation. In this case, the duct axis of symmetry is nearly...
B. Comparison Between Conventional and New Transition Method

Because it takes 9 s to reach the minimum level-flight speed, in the second step, the conversion times are set to 9, 12, 15, and 18 s. The conventional transition method is also simulated for the same vehicle, with the strategy described in Remark 2. A comparison of displacement over time is simulated as shown in Fig. 19. Significant initial acceleration advantage is demonstrated by the new transition method. This advantage could be ascribed to the low drag due to a smaller windward side by flying sideways, while the conventional method requires the vehicle to fly at an angle of attack of near 90 deg, thus causing tremendous drag.

Remark 3: This advantage will disappear at higher speed, as the conventional method has a higher lift/drag ratio than flying sideways in level flight. The simulation also shows that the sooner it rolls, the faster it accelerates. This is because, when flying sideways at high speed, it is equivalent to a wingless version of ISTAR UAV shown in Fig. 1, and thus it has a lower lift/drag ratio in level flight and generates more induced drag. Generally, flying sideways are more efficient at low speed but less efficient at high speed.

The induced drag for different conversion times is plotted in Fig. 20. Note that the oscillations on the curves are due to the roll maneuver.

VI. Conclusions

The new method of transition flight, which tilts the ducted winged unmanned aerial vehicle (UAV) to fly sideways, enables the UAV to achieve faster acceleration compared to conventional transition flight, due to its smaller windward side area. The simulation shows that, by flying sideways, the UAV generates less induced drag at low speed but more induced drag at higher speed. Thus, to accelerate faster, the UAV should roll and convert to high-speed flight mode as soon as its minimum level-flight speed is achieved. The control laws designed are robust against disturbance. This property is demonstrated by the elimination of altitude loss and horizontal drift despite the disturbance. The simulation shows that the skid-to-turn is effective in trajectory following.

A more accurate model will be developed, including the variation of thrust, the dynamics of rotor, time delays, and revised aerodynamical coefficient curves. The negative effects of the horizontal varying wind will also be simulated. Subsequently, flight tests on real a UAV model will be carried out.

Appendix A: Numerical Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass and inertia</td>
<td></td>
</tr>
<tr>
<td>( m )</td>
<td>100 kg</td>
</tr>
<tr>
<td>( J )</td>
<td>diag[25, 30, 28] kg \cdot m(^2)</td>
</tr>
<tr>
<td>Duct</td>
<td></td>
</tr>
<tr>
<td>( C_{\text{duct}} )</td>
<td>0.2 m(^2)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Fuselage aerodynamics</td>
<td></td>
</tr>
<tr>
<td>( C_{\text{vmax}} )</td>
<td>1.2</td>
</tr>
<tr>
<td>( C_{\text{ve}} )</td>
<td>5</td>
</tr>
<tr>
<td>( S_{\text{wing}} )</td>
<td>2.5 m(^2)</td>
</tr>
<tr>
<td>( C_{\text{vmax}} )</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_{\text{r}} )</td>
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</tr>
<tr>
<td>( S_{\text{cone}} )</td>
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<tr>
<td>( C_{\text{gain}} )</td>
<td>1.9</td>
</tr>
<tr>
<td>( C_{\text{gain}} )</td>
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</tr>
<tr>
<td>( \kappa )</td>
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</tr>
<tr>
<td>( z_1 )</td>
<td>0.05 m</td>
</tr>
<tr>
<td>( z_2 )</td>
<td>0.06 m</td>
</tr>
<tr>
<td>Control surfaces</td>
<td></td>
</tr>
<tr>
<td>( l_1 )</td>
<td>0.3 m</td>
</tr>
<tr>
<td>( l_3 )</td>
<td>0.3 m</td>
</tr>
<tr>
<td>( l_4 )</td>
<td>0.25 m</td>
</tr>
<tr>
<td>( S_{\text{cone}} )</td>
<td>0.06 m</td>
</tr>
<tr>
<td>Rotor</td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>0.0045 N \cdot s(^2)</td>
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<tr>
<td>( \kappa )</td>
<td>0.0015 N \cdot m \cdot s(^{-1})</td>
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<tr>
<td>( \sigma_{\text{b}} )</td>
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<tr>
<td>( n_b )</td>
<td>4</td>
</tr>
<tr>
<td>( i_b )</td>
<td>0.0027 kg \cdot m(^2)</td>
</tr>
<tr>
<td>Miscellaneous</td>
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<tr>
<td>( \rho_{\text{m}} )</td>
<td>1.225 kg \cdot m(^{-3})</td>
</tr>
<tr>
<td>( g )</td>
<td>9.8 N \cdot kg(^{-1})</td>
</tr>
<tr>
<td>( L_1 )</td>
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</tr>
<tr>
<td>( K_r )</td>
<td>51 cm</td>
</tr>
<tr>
<td>( M_{\text{d}} )</td>
<td>( 10 \sin(0.2\theta)[111]^T ) N \cdot m</td>
</tr>
</tbody>
</table>
Appendix B: Proof of the Controller

First of all, by deriving the following Lyapunov function candidate:

\[ V = s^T s \]

in view of Eqs. (3), (5), and (33), one gets

\[ \dot{V} = 2s^T s \]
\[ = 2s^T \left[ -\frac{1}{2} s - K_i \text{sign}(s) + J^{-1} M_d \right] \] (B1)

Let

\[ H = [H_x H_y H_z]^T = J^{-1} M_d \] (B2)

and

\[ K_i = \text{diag}[K_1 K_2 K_3] \quad (K_1 > |H_x|, K_2 > |H_y|, K_3 > |H_z|) \]

Therefore, we get

\[ \dot{V} = -s^T s + 2s^T \]
\[ \begin{bmatrix} H_x & 0 & 0 \\ H_y & 0 & 0 \\ H_z & 0 & 0 \end{bmatrix} \]
\[ = -s^T s + 2s^T \]
\[ \begin{bmatrix} H_x & 0 & 0 \\ H_y & K_1 \text{sign}(s_1) & 0 \\ H_z & 0 & K_3 \text{sign}(s_3) \end{bmatrix} \]
\[ - 2(K_1 |s_1| + K_2 |s_2| + K_3 |s_3|) \]
\[ \leq -s^T s + 2(H_x |s_1| + H_y |s_2| + H_z |s_3|) \]
\[ - 2(K_1 |s_1| + K_2 |s_2| + K_3 |s_3|) \]
\[ \leq -s^T s - 2\Delta K(|s_1| + |s_2| + |s_3|) \]
\[ \leq 0 \] (B4)

where

\[ \Delta K = \min\{K_1 - |H_x|, K_2 - |H_y|, K_3 - |H_z|\} \]

From inequality (B4), one gets that \( s = 0 \) is the only invariant set such that \( \dot{V} = 0 \). By using La Salle’s invariance theorem, we conclude that there exists a time \( t_f \) such that \( s = 0 \) for \( t \geq t_f \) (i.e., \( q_e = -\Omega \)), and the equilibrium point is globally asymptotically stable.

When the sliding surface \( q_e + \Omega = 0 \) is achieved, it then follows from Eq. (29) that

\[ \dot{q}_e = -\frac{1}{2} q_{e0} q_e \]
\[ \dot{q}_{e0} = \frac{1}{2} q_e^T q_e \]

The time derivative of the Lyapunov function candidate \( W = q_e^T q_e + (1 - q_{e0})^2 \) gives

\[ \dot{W} = -q_e^T q_e < 0 \]

which implies that \( \Omega(t) \to 0 \) and \( q_e(t) \to 0 \) as \( t \to \infty \). The equation

\[ \begin{bmatrix} q_0 & q_e \end{bmatrix}^T = \begin{bmatrix} q_{e0} & q_e \end{bmatrix} \]
\[ \] * \begin{bmatrix} q_0 & q_e \end{bmatrix} \]

shows that \( [q_0 q_e]^T \to [q_{e0} q_e]^T \) and the equilibrium point is globally asymptotically stable.

Acknowledgments

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References