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Additive-state-decomposition-based tracking control framework for a class of nonminimum phase systems with measurable nonlinearities and unknown disturbances

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SUMMARY

This paper aims to propose an *additive-state-decomposition-based tracking control framework*, based on which the output feedback tracking problem is solved for a class of nonminimum phase systems with measurable nonlinearities and unknown disturbances. This framework is to 'additively' decompose the output feedback tracking problem into two more tractable problems, namely an output feedback tracking problem for a linear time invariant system and a state feedback stabilization problem for a nonlinear system. Then, one can design a controller for each problem respectively using existing methods, and these two designed controllers are combined together to achieve the original control goal. The main contribution of the paper lies on the introduction of an additive state decomposition scheme and its implementation to mitigate the design difficulty of the output feedback tracking control problem for nonminimum phase nonlinear systems. To demonstrate the effectiveness, an illustrative example is given. Copyright © 2013 John Wiley & Sons, Ltd.

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1. INTRODUCTION

A nonlinear system is a nonminimum phase if its internal or zero dynamics are unstable [1]. Many real world examples are modeled as nonminimum phase nonlinear systems, such as translational oscillators with rotational actuators [2], conventional fixed-wing aircraft [3], and vertical take-off and landing aircraft [4]. Control of nonminimum phase nonlinear systems turns out to be a challenging task. In the literature, it is usually assumed that the internal state is exactly known for control. Based on this assumption, an ideal internal dynamics were derived so that the output tracking problem became a stabilization problem [5–8]. Unfortunately, it is difficult to measure the internal state in real applications. Therefore, the output feedback tracking problem has attracted much research interest.

For the output feedback tracking problem, a commonly used control framework is to estimate the true state and disturbance first, and the estimations are then used to design stabilized control terms and compensated control terms, as shown in Figure 1(a). The major difficulty of this approach lies on the estimation of the true state from the output under disturbances and hence attracted many research efforts. There exist two categories of methods for the estimation problem. The first class of methods is based on the assumption that the disturbance is generated by an autonomous exosystem [9–12].

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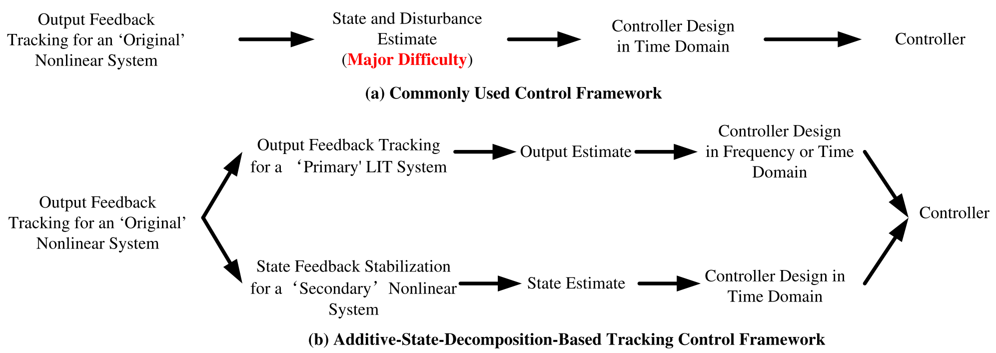

Figure 1. Comparison of two frameworks.

For a class of nonminimum phase systems with measurable nonlinearities and unknown disturbances, stabilization, and disturbance rejection based on the estimated disturbances for single-input single-output (SISO) systems were studied in [9, 10]. The method was further extended to multiple-input multiple-output (MIMO) systems in [11]. In [12], the output feedback tracking problem for SISO systems was investigated for uncertainties in both parameters and order of the exosystem. The considered disturbance is often assumed to be composed of a finite number of sine signals. Based on such an assumption, an extended observer for state and disturbance was proposed. By using the resulting estimates, the final controller was constructed by combining a compensated control term and a stabilized control term. However, it is difficult for this method to handle a general bounded disturbance, which is not easy to describe by an autonomous exosystem. As a consequence, the existing extended observer methods in [9–12] are difficult to apply. Another method is to use a high-gain observer to estimate the true state and disturbance. However, the bound on estimation error depends on the disturbance [13, pp. 610–624]. Moreover, the estimation error is sensitive to the output noise as the high gain is used. As the estimation will be used for subsequent design of the stabilizing controller, this could be problematic to guarantee the closed-loop stability. With the uncertain estimation, it is more difficult to design the tracking controller for uncertain nonminimum phase systems.

The issues earlier motivate us to investigate a new framework, called *additive-state-decomposition-based tracking control framework*, to handle the output feedback tracking problem. Our basic idea is to decompose the considered tracking problem into two easier problems by the additive state decomposition[‡], avoiding the estimation of the true state and disturbance. As shown in Figure 1(b), the output feedback tracking is 'additively' decomposed into an output feedback tracking for a 'primary' linear time invariant (LTI) system including disturbances, and a state feedback stabilization for a 'secondary' nonlinear system absence of disturbances. Because the latter decomposed system is determinate, we can obtain the state of the nonlinear system and the output of the LTI system directly by our proposed observers. Although the state of the secondary nonlinear system is not the true state, it will be enough for the stabilization, which will simplify the whole design and analysis. Then, one can employ some standard design methods in either the frequency domain or the time domain to solve the first problem for LTI systems, which can be considered as a well solved problem. Furthermore, there exist many mature methods for nonlinear system state feedback stabilization [1, 13–15]. Under the proposed framework, we only need to consider state feedback stabilization rather than output feedback stabilization, namely nonminimum phase behavior is avoided for the stabilization problem. This is an important feature because nonminimum phase behavior will restrict the application of basic nonlinear controllers. Hence, the advantage of

[‡]Additive state decomposition [16] is different from the lower order subsystem decomposition methods existing in the literature. Concretely, taking the system $\dot{x}(t) = f(t, x), x \in \mathbb{R}^n$ for example, it is decomposed into two subsystems: $\dot{x}_1(t) = f_1(t, x_1, x_2)$ and $\dot{x}_2(t) = f_2(t, x_1, x_2)$, where $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$, respectively. The lower order subsystem decomposition satisfies $n = n_1 + n_2$ and $x = x_1 \oplus x_2$. By contrast, the proposed additive state decomposition satisfies $n = n_1 = n_2$ and $x = x_1 + x_2$.

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the proposed framework lies on the decomposition of the problem into two well-studied control problems.

This paper can be seen as an extension of our recent work [17], in which a benchmark problem, the tracking (rejection) problem for rotational position of the translational oscillator with a rotational actuator, was investigated also on the basis of additive state decomposition. Compared with [17], this work aims to propose a general framework for a class of nonminimum phase nonlinear systems rather than a special technique for a special system. The contributions of this paper are as follows: (i) a new decomposition to simplify the system analysis and controller design and (ii) a new control framework to solve the output feedback tracking control framework for a class of nonminimum phase nonlinear systems subject to bounded disturbances.

This paper is organized as follows. In Section 2, the problem formulation is given, and the additive state decomposition is introduced briefly. In Section 3, the additive-state-decomposition-based output feedback tracking control framework is presented. In Section 4, an illustrative example is given to demonstrate the effectiveness of the proposed control scheme. Section 5 concludes this paper.

2. PROBLEM FORMULATION AND ADDITIVE STATE DECOMPOSITION

2.1. Problem formulation

Consider a class of SISO uncertain nonlinear systems similar to [9–12]:

$$\begin{aligned} \dot{x} &= Ax + bu + \phi(y) + d, x(0) = x_0 \\ y &= c^T x \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$ is a known stable constant matrix (*Remark 2*), $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ are known constant vectors, $\phi : \mathbb{R} \to \mathbb{R}^n$ is a known nonlinear function vector, $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the control, and $d(t) \in \mathbb{R}^n$ is an unknown bounded disturbance. It is assumed that only $y(t)$ is available from measurement. The reference $r(t) \in \mathbb{R}$ is known and sufficiently smooth, $t \geqslant 0$. In the following, for convenience, we will omit the variable t except when necessary. For system (1), the following assumption is made.

Assumption 1
The pair (A, c) is observable.

Under *Assumption 1*, the objective here is to design a tracking controller u such that $y - r \to 0$ as $t \to \infty$ or with good tracking accuracy, that is, $y - r$ is ultimately bounded by a small value.

Remark 1
The considered SISO nonlinear system (1) is a nonminimum phase system, that is, the transfer function of the linear part (regardless of nonlinear dynamics $\phi(y)$)

$$c^T(sI_n - A)^{-1}b = \frac{N(s)}{D(s)}$$

is a nonminimum phase here, where $N(s)$ has zeros on the right s-plane. It is noticed that the property of nonminimum phase cannot be changed by output feedback.

Remark 2
Because the pair (A, c) is observable under *Assumption 1*, there always exists a vector $p \in \mathbb{R}^n$ such that $A + pc^T$ is stable, whose eigenvalues can be assigned freely. As a result, (1) can be rewritten as $\dot{x} = (A + pc^T)x + bu + [\phi(y) - py] + d$. Therefore, we assume A to be stable without loss of generality.

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Remark 3
In [9–12], only disturbance rejection is considered. Compared with it, the problem considered in this paper is more general. In [9–12], system (1) is required that (i) the disturbance d is generated by an autonomous system, which has $2m + 1$ simple distinct eigenvalues on the imaginary axis and (ii) (A, b, c) is invertible and has no invariant zeros on the imaginary axis. Here, we do not restrict the disturbance d to be composed of sinusoids. This broadens the application of the proposed framework.

2.2. *Additive state decomposition*

In order to make the paper self-contained, additive state decomposition [16, 17] is recalled here briefly. Consider the following 'original' system:

$$f\left(t, \dot{x}, x\right) = 0, x(0) = x_0 \tag{2}$$

where $x \in \mathbb{R}^n$. We first bring in a primary system having the same dimension as (2), according to

$$f_p\left(t, \dot{x}_p, x_p, x\right) = 0, x_p(0) = x_{p,0} \tag{3}$$

where $x_p \in \mathbb{R}^n$. From the original system (2) and the primary system (3), we derive the following secondary system:

$$f\left(t, \dot{x}, x\right) - f_p\left(t, \dot{x}_p, x_p, x\right) = 0, x(0) = x_0 \tag{4}$$

where $x_p \in \mathbb{R}^n$ is given by the primary system (3). Define a new variable $x_s \in \mathbb{R}^n$ as follows:

$$x_s \triangleq x - x_p. \tag{5}$$

Then, the primary system (3) and the secondary system (4) can be further written as follows:

$$\begin{aligned} f_p\left(t, \dot{x}_p, x_p, x_p + x_s\right) &= 0, x_p(0) = x_{p,0} \\ f\left(t, \dot{x}_s + \dot{x}_p, x_s + x_p\right) - f_p\left(t, \dot{x}_p, x_p, x_p + x_s\right) &= 0, x_s(0) = x_0 - x_{p,0}. \end{aligned} \tag{6}$$

From the definition (5), we have

$$x(t) = x_p(t) + x_s(t), t \geqslant 0. \tag{7}$$

Remark 4
By the additive state decomposition, system 2) is decomposed into two systems with the same dimension as the original system. In this sense, our decomposition is 'additive'. In addition, this decomposition is with respect to state. So, we call it 'additive state decomposition'.

As a special case of (2), a class of differential dynamic systems is considered as follows:

$$\begin{aligned} \dot{x} &= f(t, x), x(0) = x_0, \\ y &= h(t, x) \end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. Two systems, denoted by the primary system and (derived) secondary system respectively, are defined as follows:

$$\begin{aligned} \dot{x}_p &= f_p(t, x_p, x_s), x_p(0) = x_{p,0} \\ y_p &= h_p(t, x_p) \end{aligned} \tag{9}$$

and

$$\begin{aligned} \dot{x}_s &= f(t, x_p + x_s) - f_p(t, x_p, x_s), x_s(0) = x_0 - x_{p,0}, \\ y_s &= h(t, x_p + x_s) - h_p(t, x_p) \end{aligned} \tag{10}$$

where $x_s \triangleq x - x_p$ and $y_s \triangleq y - y_p$. The secondary system (10) is determined by the original system (8) and the primary system (9). From the definition, we have

$$x(t) = x_p(t) + x_s(t), y(t) = y_p(t) + y_s(t), t \geqslant 0. \tag{11}$$

3. ADDITIVE-STATE-DECOMPOSITION-BASED TRACKING CONTROL FRAMEWORK

First, based on additive state decomposition, the considered system (1) is decomposed into two systems: an LTI system (12) including all external signals as the primary system, together with the secondary system (13) whose equilibrium point is zero. Because output of the primary system and state of the secondary system can be observed, the original tracking problem for system (1) is correspondingly decomposed into two problems: an output feedback tracking problem for an LTI primary system and a state feedback stabilization problem for the left secondary system. Because the tracking task is only assigned to the LTI system, it is therefore much easier than that for the nonlinear system (1).

3.1. Decomposition

Consider system (1) as the original system. According to the principle mentioned earlier, we choose the primary system as follows:

$$\begin{aligned} \dot{x}_p &= Ax_p + bu_p + \phi(r) + d \\ y_p &= c^T x_p, x_p(0) = x_0. \end{aligned} \tag{12}$$

Then, the secondary system is determined by the original system (1) and the primary system (12) with the rule (10), and we obtain that

$$\begin{aligned} \dot{x}_s &= Ax_s + bu_s + \phi\left(c^T x_p + c^T x_s\right) - \phi(r) \\ y_s &= c^T x_s, x_s(0) = 0 \end{aligned} \tag{13}$$

where $u_s = u - u_p$. According to (11), we have

$$x = x_p + x_s \text{ and } y = y_p + y_s. \tag{14}$$

The secondary system (13) is further written as

$$\begin{aligned} \dot{x}_s &= Ax_s + bu_s + \phi\left(r + c^T x_s + e_p\right) - \phi(r) \\ y_s &= c^T x_s, x_s(0) = 0 \end{aligned} \tag{15}$$

where $e_p \triangleq y_p - r$. If $e_p \equiv 0$, then $(x_s, u_s) = 0$ is an equilibrium point of (15).

Controller design for the decomposed systems (12) and (13) will use their outputs or states as feedback. However, they are unknown. For such a purpose, an observer is proposed to estimate y_p and x_s.

Theorem 1
Suppose that an observer is designed to estimate y_p and x_s in (12) and (13) as follows:

$$\hat{y}_p = y - c^T \hat{x}_s \tag{16}$$

$$\dot{\hat{x}}_s = A\hat{x}_s + bu_s + \phi(y) - \phi(r), \hat{x}_s(0) = 0. \tag{17}$$

Then, $\hat{y}_p \equiv y_p$ and $\hat{x}_s \equiv x_s$.

Proof
Subtracting (17) from (13) results in $\dot{\tilde{x}}_s = A\tilde{x}_s$, $\tilde{x}_s(0) = 0$[§], where $\tilde{x}_s = x_s - \hat{x}_s$. Then, $\tilde{x}_s \equiv 0$. This implies that $\hat{x}_s \equiv x_s$. Consequently, by (14), we have $\hat{y}_p \equiv y - c^T \hat{x}_s \equiv y_p$. □

Remark 5
The measurement y may be inaccurate in practice. In this case, it is expected that small uncertainties still maintain $\hat{x}_s$ close to x_s eventually. Accordingly, the matrix A is required to be stable in the relationship $\dot{\tilde{x}}_s = A\tilde{x}_s$ in the proof earlier. This requirement is easy to be satisfied (*Remark 2*).

[§]Because the initial values $x_s(0)$, $\hat{x}_s(0)$ are both assigned by the designer, they are all determinate.

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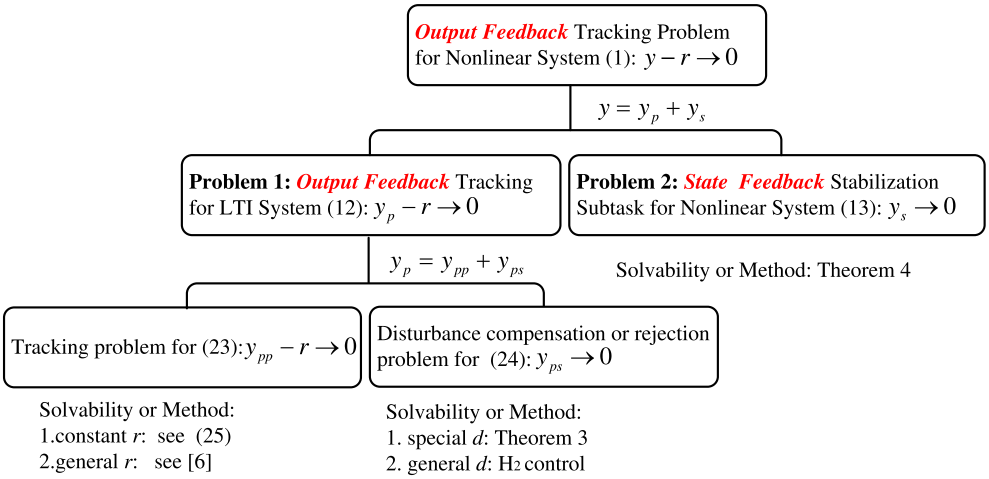

Figure 2. Additive state decomposition flow.

It is clear from (12)–(15) that if the controller u_p drives $y_p - r \to 0$ and the controller u_s drives $y_s \to 0$ as $t \to \infty$, then $y - r \to 0$ as $t \to \infty$. The strategy here is to assign the tracking task to the primary system (12) and the stabilization task to the secondary system (15). Because system (12) is a classical LTI system, standard design methods in either the frequency domain or the time domain can be used to handle the output feedback tracking problem. This is easier than dealing with (1) directly. If $y_p = c^T x_p = r$, then zero is an equilibrium point of the secondary system (15). Because the state of (15) can be obtained by (17), the design is also easier than that for a nonminimum phase system. Notice that x_s is a virtual state not the true state x. According to these, the additive state decomposition offers a way to simplify the original control problem.

3.2. Controller design

So far, we have decomposed the considered system into two systems in charge of corresponding tasks. In this section, we investigate controller design in the form of problems with respect to the two component tasks, respectively. The whole process is shown in Figure 2.

Problem 1
For (12), design a control input

$$\begin{aligned} \dot{z}_p &= \alpha_p(z_p, y_p, r, \cdots, r^{(N)}) \\ u_p &= u_p(z_p, y_p, r, \cdots, r^{(N)}) \end{aligned} \tag{18}$$

such that $e_p = y_p - r \to \mathcal{B}(\delta)$[¶] as $t \to \infty$, where $\delta = \delta(r, d) > 0$ depends on the reference r and disturbance d, and $r^{(k)}$ denotes the kth derivative of r, $k = 1, \cdots, N$.

Remark 6
Because system (12) is a classical LTI system, some standard design methods in the frequency domain, such as the transfer function method, can be used to handle a general bounded disturbance [18]. In this case, the output often cannot track the desired signal asymptotically. So, we have to consider the result $e_p \to \mathcal{B}(\delta)$ besides $e_p \to 0$ in *Problem 1*. If d and $\phi(r)$ are both generated by an autonomous system, then *Problem 1* can be considered as an output regulation problem. In this case, the output can track the reference asymptotically. The techniques in [9–12] of course are still applicable to the problem even if both parameters and the order of the exosystem are uncertain. In

[¶] $\mathcal{B}(\delta) \triangleq \{\xi \in \mathbb{R} \mid \|\xi\| \leqslant \delta\}$, $\delta > 0$; the notation $x(t) \to \mathcal{B}(\delta)$ means $\min_{y \in \mathcal{B}(\delta)} |x(t) - y| \to 0$.

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practice, because the signal $\phi(r)$ depends on r and nonlinear function $\phi(\cdot)$, the signals d and $\phi(r)$ are not often generated by the same autonomous system. For this case, we can design the inversion-based feedforward input [6, 19] to cancel the tracking error caused by the term $\phi(r)$ directly, only leaving disturbances in system (12).

Problem 2
For (15) (or (13)), there exists a controller

$$\begin{aligned} \dot{z}_s &= \alpha_s\left(z_s, x_s, r, \cdots, r^{(N)}\right) \\ u_s &= u_s\left(z_s, x_s, r, \cdots, r^{(N)}\right) \end{aligned} \tag{19}$$

such that (i) the closed-loop system is input-to-state stable with respect to the input e_p, namely

$$\|x_s(t)\| \leqslant \beta(\|x_s(t_0)\|, t-t_0) + \gamma\left(\sup_{t_0 \leqslant s \leqslant t} \|e_p(s)\|\right), t \geqslant t_0, \tag{20}$$

where $r^{(k)}$ denotes the kth derivative of r, $k = 1, \cdots, N$, function β is a class $\mathcal{KL}$ function, and γ is a class $\mathcal{K}$ function [13]. Or (ii) the closed-loop system is asymptotical stable, namely $e_p \to 0$ as $t \to \infty$.

Remark 7
If e_p is nonvanishing, then *Problem 2* is a classical input-to-state stability problem. Readers can refer to [13, 20] for how to design a controller satisfying input-to-state stability. In particular, if $e_p \to 0$ as $t \to \infty$, then $x_s \to 0$ as $t \to \infty$ by (20). In addition, if $e_p \to 0$ as $t \to \infty$, then input-to-state stability can be relaxed as well. Denote $x_s^{e_p}$ and x_s^* as the solution of (15) and the solution of (15) with $e_p \equiv 0$, respectively. The reference [21] discussed under what conditions $\left\|x_s^{e_p}(t) - x_s^*(t)\right\| \leqslant \theta e^{-\eta t}$ is satisfied, where $\theta, \eta > 0$. In this case, only asymptotic stability of (15) with $e_p \equiv 0$ needs to be considered rather than input-to-state stability.

With the solutions of the two problems in hand, we can state

Theorem 2
Under *Assumption 1*, suppose (i) *Problems 1* and *2* are solved and (ii) the controller for system (1) is designed as

Observer:

$$\begin{aligned} \dot{\hat{x}}_s &= A\hat{x}_s + bu_s + \phi(y) - \phi(r), \hat{x}_s(0) = 0. \\ \hat{y}_p &= y - c^T\hat{x}_s \end{aligned} \tag{21}$$

Controller:

$$\begin{aligned} \dot{z}_p &= \alpha_p\left(z_p, \hat{y}_p, r\right), z_p(0) = 0 \\ \dot{z}_s &= \alpha_s\left(z_s, \hat{x}_s, r\right), z_s(0) = 0 \\ u_p &= u_p\left(z_p, \hat{y}_p, r\right), u_s = u_s\left(z_p, \hat{x}_s, r\right) \\ u &= u_p + u_s. \end{aligned} \tag{22}$$

Then, the output of system (1) satisfies $y - r \to \mathcal{B}(\delta + \|c\|\gamma(\delta))$ as $t \to \infty$. In particular, if $\delta = 0$, then the output in system (1) satisfies $y - r \to 0$ as $t \to \infty$.

Proof
See Appendix. □

Remark 8
The proposed control framework has two advantages over the commonly used control framework. First, under the proposed framework, we in fact design a two degree of freedom controller [18]. It

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is well known that there is an intrinsic conflict between performance (trajectory tracking and disturbance rejection) and robustness in the standard feedback framework. So the proposed framework is potential to avoid conflict between tracking performance and robustness. Second, the proposed framework is compatible with various methods designed in both the frequency domain and the time domain.

3.3. Solvability discussion for Problems 1 and 2

For simplicity, the matrices and vectors in this section are with proper dimensions. The framework of solvability discussion is shown in Figure 2.

3.3.1. Solvability discussion for Problem 1. Because d is unknown, the reference r is known a priori. According to this fact, we reuse the additive state decomposition to separate the two signals and then consider the solvability of two easier subproblems. Consider system (12) as the original system and choose the primary system as follows:

$$\begin{aligned} \dot{x}_{pp} &= Ax_{pp} + bu_{pp} + \phi(r) \\ y_{pp} &= c^T x_{pp}, x_{pp}(0) = \bar{x}_{pp} \end{aligned} \tag{23}$$

and the secondary system as

$$\begin{aligned} \dot{x}_{ps} &= Ax_{ps} + bu_{ps} + d \\ y_{ps} &= c^T x_{ps}, x_{ps}(0) = x_0 - \bar{x}_{pp} \end{aligned} \tag{24}$$

where $\bar{x}_{pp}$ is the initial value, which can be specified. Then, $y_p = y_{pp} + y_{ps}$. Similarly to *Theorem 1*, the exact x_{pp} and y_{ps} can be observed. The objective is to drive $y_{pp} - r \to 0$ and $y_{ps} \to \mathcal{B}(\delta)$ *as* $t \to \infty$ so that $y_p - r \to \mathcal{B}(\delta)$ *as* $t \to \infty$. The former is a tracking problem, while the latter is an output feedback disturbance compensation or rejection problem. If the both subproblems are solvable, then *Problem 1* is also solvable by $u = u_{pp} + u_{ps}$. According to this, we will study the two subproblems next.

(i) Tracking problem for (23).

Because the reference r is known a priori, we will design feedforward controllers to drive $y_{pp} - r \to 0$ *as* $t \to \infty$ for a constant r and a general r, respectively.

Consider a constant r first. A solution to (23) satisfies

$$\begin{aligned} 0 &= Ax_{pp}^* + bu_{pp}^* + \phi(r) \\ r &= c^T x_{pp}^*. \end{aligned}$$

If $A_a = \begin{bmatrix} A & b \\ c^T & 0 \end{bmatrix}$ is nonsingular, then the solution can be explicitly written as

$$\begin{bmatrix} x_{pp}^* \\ u_{pp}^* \end{bmatrix} = A_a^{-1} \begin{bmatrix} -\phi(r) \\ r \end{bmatrix}. \tag{25}$$

Because A is stable, $u_{pp} = u_{pp}^*$ can drive $y_{pp} \equiv r$ in (23) by setting $\bar{x}_{pp} = x_{pp}^*$. Therefore, the matrix A_a being nonsingularity is a solvability condition for this scenario.

For a general reference r, we will adopt the inversion-based feedforward input [6, 19]. First, we have to transfer system (23) into a standard form. For this, we apply the additive state decomposition again to (23), obtaining

$$\begin{aligned} \dot{x}_{ppp} &= Ax_{ppp} + bu_{pp} \\ y_{ppp} &= c^T x_{ppp}, x_{ppp}(0) = 0 \end{aligned} \tag{26}$$

and

$$\begin{aligned} \dot{x}_{pps} &= Ax_{pps} + \phi(r) \\ y_{pps} &= c^T x_{pps}, x_{pps}(0) = 0 \end{aligned} \tag{27}$$

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with $y_{pp} = y_{ppp} + y_{pps}$, where $\bar{x}_{pp} = 0$ and the form of (26) are the same to that considered in [6]. The output of (27) is $y_{pps}(t) = c^T \int_0^t e^{A(t-s)} \phi(r(s))ds$, which can be computed a priori. Thus, the objective $y_{pp} - r \to 0$ is equivalent to designing u_{pp} in (26) such that

$$y_{ppp} \to r^* = r - c^T \int_0^t e^{A(t-s)} \phi(r(s))ds.$$

If $r^* \in \mathcal{L}_1 \cap \mathcal{L}_\infty$, then such a problem is a standard stable inversion problem considered in [6]. In practice, we only need to consider $t \in [0, T]$ rather than $t \in [0, \infty)$, where $T > 0$ is a given end time. So, $r^* \in \mathcal{L}_1 \cap \mathcal{L}_\infty$ is easy to satisfy by defining $r^*(t) \equiv 0,\ t > T$. If A has no eigenvalues on the imaginary axis, then the stable inversion problem is solvable.

(ii) Output feedback disturbance compensation or rejection problem for (24).

We first consider a special scenario that disturbance d is generated by

$$\dot{v} = A_d v, d = c_d^T v. \tag{28}$$

Design an output feedback dynamic controller as follows:

$$\begin{aligned} \dot{z}_p &= A_z z_p + b_z y_{ps} \\ u_{ps} &= k_{z_1}^T z_p + k_{z_2} y_{ps}, z(0) = 0. \end{aligned} \tag{29}$$

Furthermore, incorporating the controller (29) into (24) results in

$$\begin{aligned} \begin{bmatrix} \dot{x}_{ps} \\ \dot{z}_p \end{bmatrix} &= \begin{bmatrix} A + k_{z_2} b c^T & b k_{z_1}^T \\ b_z c^T & A_z \end{bmatrix} \begin{bmatrix} x_{ps} \\ z_p \end{bmatrix} + \begin{bmatrix} c_d^T \\ 0 \end{bmatrix} v \\ \dot{v} &= A_d v \\ y_{ps} &= \begin{bmatrix} c^T & 0 \end{bmatrix} \begin{bmatrix} x_{ps} \\ z_p \end{bmatrix}. \end{aligned}$$

By Lemma 1.13 in [22, p. 12], we can obtain the following solvability condition.

Theorem 3

Suppose (i) A_d has no eigenvalues with negative real parts and (ii) matrix $A_c = \begin{bmatrix} A + k_{z_2} b c^T & b k_{z_1}^T \\ b_z c^T & A_z \end{bmatrix}$ is Hurwitz. If there exists a matrix X_c that satisfies the following matrix equations

$$X_c A_d = A_c X_c + \begin{bmatrix} c_d^T \\ 0 \end{bmatrix}, 0 = \begin{bmatrix} c^T & 0 \end{bmatrix} X_c \tag{30}$$

then the controller (29) with gains $\left(A_z, b_z, k_{z_1}, k_{z_2}\right)$ can drive $y_{ps} \to 0$ *as* $t \to \infty$.

Next, we consider a general disturbance d, which often cannot be rejected asymptotically. Because A is stable, y_{ps} is bounded driven by a bounded disturbance d. So, such a problem is always solvable by $u_{ps} = 0$, namely $y_{ps}(t) = c^T e^{At} x_0 + c^T \int_0^t e^{A(t-s)} d(s) ds$ and then $y_{ps} \to \mathcal{B}(\delta)$ as $t \to \infty$ with $\delta = \|c\| \sup_{t \in [0,\infty)} \int_0^t \|e^{A(t-s)}\| ds \|d\|_\infty < \infty$. However, it is not satisfied in practice if δ is large. It is expected δ as small as possible. For such a purpose, we will formulate the problem into an optimization problem. In practice, a disturbance can often be modeled as

$$d(s) = G_d(s) w_d(s)$$

where $G_d(s)$ is a transfer function and w_d is an unknown smaller signal compared with d. In the worst case, $d(s) = w_d(s)$. Then, without considering the initial condition (zero-state response case), system (24) is written as

$$y_{ps}(s) = c^T (sI - A)^{-1} b u_{ps}(s) + c^T (sI - A)^{-1} G_d(s) w_d(s).$$

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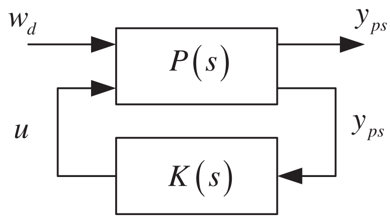

Figure 3. $\mathcal{H}_2$ control problem formulation.

Therefore, as shown in Figure 3, we can cast the design to be an $\mathcal{H}_2$ control problem [23] that is to design $K(s)$ minimizing $\frac{\|y_{ps}\|_\infty}{\|w_d\|_\infty}$:

$$\begin{bmatrix} y_{ps}(s) \\ y_{ps}(s) \end{bmatrix} = P(s) \begin{bmatrix} w_d(s) \\ u_{ps}(s) \end{bmatrix}$$
$$u_{ps}(s) = K(s) y_{ps}(s)$$

where $P(s) = \begin{bmatrix} c^T(sI - A)^{-1} G_d(s) & c^T(sI - A)^{-1} b \\ c^T(sI - A)^{-1} G_d(s) & c^T(sI - A)^{-1} b \end{bmatrix}$. Besides the method earlier, we can also use the method proposed in [24].

3.3.2. Solvability discussion for Problem 2. Because only the state feedback stabilization rather than output feedback stabilization is considered, namely nonminimum phase behavior is avoided, the solvability condition is relaxed. According to this, for a specified structure of nonlinear term ϕ, related results not only for nonminimum phase systems [9–12] but also for minimum phase systems [25, 26] are both helpful for the explicit construction of controllers for the *Problem 2*. However, if the structure of nonlinear term ϕ is not specified, then it will be complex or conservative to establish a solvability condition for *Problem 2* through a constructive method. To avoid discussion case-by-case, we turn to Lyapunov functions to establish a general solvability condition.

Incorporating (19) into (15) results in

$$\dot{x}_a = f(t, x_a, e_p), x_a(0) = 0, \tag{31}$$

where

$$x_a = \begin{bmatrix} z_s \\ x_s \end{bmatrix} \in \mathbb{R}^{m+n},$$
$$f(t, x_a, e_p) = \begin{bmatrix} \alpha_s(z_s, x_s, r, \cdots, r^{(N)}) \\ Ax_s + bu_s(z_p, x_s, r, \cdots, r^{(N)}) + \phi(r + c^T x_s + e_p) - \phi(r) \end{bmatrix},$$

With respect to the input-to-state stability problem for (31), the solvability is stated in the following theorem.

Theorem 4
Let $V : [0, \infty) \times \mathbb{R}^{n+m} \to \mathbb{R}$ be a continuously differentiable function such that

$$\alpha_1(\|x_a\|) \leqslant V(t, x_a) \leqslant \alpha_2(\|x_a\|)$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_a} f(t, x_a, e_p) \leqslant -W(x_a), \forall \|x_a\| \geqslant \rho(\|e_p\|) > 0$$

$\forall (t, x_a, e_p) \in [0, \infty) \times \mathbb{R}^{n+m} \times \mathbb{R}$, where α_1, α_2 are class $\mathcal{K}_\infty$ functions, ρ is a class $\mathcal{K}$ function, and $W(x)$ is a continuous positive definite function on $\mathbb{R}^{n+m}$. Then, the controller (19) makes *Problem 2* solvable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

Proof
It is easy to follow the proof of Theorem 4.19 in [13, p. 176]. □

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4. AN ILLUSTRATIVE EXAMPLE

Consider the following uncertain nonminimum phase system (1) with

$$A = \begin{bmatrix} 1 & 1 \\ -5 & -4 \end{bmatrix}, b = c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \phi(y) = \begin{bmatrix} \phi_{0,1}(y) \\ 0 \end{bmatrix}, \phi_{0,1}(y) = \frac{y^2}{1+y^2}.$$

It is easy to see that *Assumption 1* holds. By setting $y = 0$, $u = 0$, and $d = 0$, the zero dynamics are $\dot{x}_1 = x_1$. Therefore, the considered system is a nonminimum phase system. For simplicity, the objective is to design u to make $y \to r(t) \equiv 1$. To show the effectiveness of the proposed method, we consider two types of disturbance: (i) a step disturbance $d = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$ and (ii) a general bounded disturbance $d(s) = \begin{bmatrix} \frac{0.08}{(s+0.2)(s+0.5)(s+0.8)}\zeta_1(s) & \frac{6}{(s+1)(s+2)(s+3)}\zeta_2(s) \end{bmatrix}^T$, where $\zeta_1, \zeta_2 \sim \mathcal{N}(0,1)$ are normally (Gaussian) distributed random signals.

4.1. Solution to Problem 1

4.1.1. Tracking problem for (23). Because $r(t) \equiv 1$, by (25), we have

$$x_{pp}^* = \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}, u_{pp}^* = -3.5.$$

Consequently, $u_{pp} = u_{pp}^*$ and the initial value of (23) are set as $\bar{x}_{pp} = \begin{bmatrix} -1.5 \\ 1 \end{bmatrix}$. To make the input smooth at $t = 0$, we adopt

$$u_{pp} = -3.5(1 - e^{-0.2t}). \tag{32}$$

4.1.2. Output feedback disturbance compensation or rejection problem for (24). In the following, we consider the two types of disturbances, namely a step disturbance and a general bounded disturbance.

(i) A step disturbance

The step disturbance can be modeled as (28) with

$$A_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, c_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

For system (24), the controller (18) is designed according to (29):

$$\begin{aligned} \dot{z}_p &= y_{ps}, z_p(0) = 0 \\ u_{ps} &= 2y_{ps} + z_p. \end{aligned} \tag{33}$$

The resultant A_c in *Theorem 3* is Hurwitz. With the gains in (33), (30) is further satisfied with

$$X_c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 5 & 1 \end{bmatrix}.$$

By *Theorem 3*, the controller (33) can drive $y_{ps} \to 0$ *as* $t \to \infty$ in (24). Finally, combining (32) and (33) results in the controller u_p for *Problem 1* as

$$\begin{aligned} \dot{z}_p &= y_{ps}, z(0) = 0 \\ u_p &= -3.5(1 - e^{-0.2t}) + 2y_{ps} + z_p \end{aligned} \tag{34}$$

where $y_{ps} = y_p - r$.

(ii) A general bounded disturbance

We will employ the method proposed in [24]. The transfer function of system (24) is

$$y_{ps}(s) = G(s)u_{ps}(s) + c^T(sI_2 - A)^{-1}\left(d(s) + x_0 - \bar{x}_{pp}\right)$$

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where $G(s) = \frac{s-1}{s^2+3s+1}$. Select a minimum phase transfer function to approximate $G(s)$ as

$$G_a = \frac{-1}{s^2+3s+1}.$$

According to [24], the controller for (24) is designed as follows:

$$u_{ps} = -G_a^{-1} Q \hat{d}_l \tag{35}$$

where $\hat{d}_l = y_{ps} - G_a u_{ps}$. The filter $Q(s)$ is designed as $Q(s) = \frac{1}{(0.5s+1)(0.8s+1)}$ to make the controller (35) realizable. For the detailed design procedure, readers can refer to [24]. Finally, combining (33) and (35) results in the controller u_p for *Problem 1* as

$$u_p = -3.5(1 - e^{-0.2t}) - G_a^{-1} Q \hat{d}_l. \tag{36}$$

4.2. *Solution to Problem 2*

In the following, we will design u_s to make (15) input-to-state stable with respect to e_p. For the secondary system (15), the stabilized controller is designed by the backstepping technique [13]. The detailed design is omitted for simplicity. Define two new variables as follows:

$$\begin{aligned} z_1 &= x_{s,1} \\ z_2 &= 2x_{s,1} + x_{s,2} + \phi_{0,1}(r + x_{s,2}) - \phi_{0,1}(r) \end{aligned} \tag{37}$$

where $x_s = [x_{s,1}\ x_{s,2}]^T$. The controller u_s is designed as follows:

$$u_s = 5x_{s,1} + 4x_{s,2} + \frac{1}{1 + \phi'_{0,1}(r + x_{s,2})}[-2(x_{s,1} + x_{s,2}) + z_1 - 5z_2]. \tag{38}$$

By the controller earlier, system (15) is derived as

$$\begin{aligned} \dot{z}_1 &= -z_1 + z_2 + \phi_{0,1}(r + c^T x_s + e_p) - \phi_{0,1}(r + c^T x_s) \\ \dot{z}_2 &= -5z_2 + z_1. \end{aligned} \tag{39}$$

Design a Lyapunov function $V = z_1^2 + z_2^2$, whose derivative along (39) is

$$\begin{aligned} \dot{V} &\leqslant -2z_1^2 - 2z_2^2 - 2z_1[\phi_{0,1}(r + c^T x_s + e_p) - \phi_{0,1}(r + c^T x_s)] \\ &\leqslant -2z_1^2 - 10z_2^2 + \frac{3\sqrt{3}}{4}|z_1||e_p| \\ &\leqslant -2V + \frac{3\sqrt{3}}{4}\sqrt{V}|e_p| \\ &\leqslant -V,\ V \geqslant \frac{27}{16}e_p^2 \end{aligned}$$

where the following properties

$$\|\phi_{0,1}(y)\| \leqslant 1, \phi'_{0,1}(y) = \frac{2y}{(1+y^2)^2}, \|\phi'_{0,1}(y)\| \leqslant \frac{3\sqrt{3}}{8}, \forall y \in \mathbb{R}$$

are used. According to *Theorem 4*, *Problem 2* is solved.

4.3. *Controller combination*

The variables y_p and x_s are estimated by the observers (16) and (17). In the controller combination, the variables y_p and x_s will be replaced with $\hat{y}_p$ and $\hat{x}_s$. According to *Theorem 1*, $\hat{y}_p \equiv y_p$ and $\hat{x}_s \equiv x_s$.

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4.3.1. Controller for a step disturbance. According to (22), the controller in this example is combined by (34) and (38) as

$$\begin{aligned}
\hat{y}_{ps} &= \hat{y}_p - r \\
\dot{z}_p &= \hat{y}_{ps}, z_p(0) = 0 \\
u_s &= 5\hat{x}_{s,1} + 4\hat{x}_{s,2} + \frac{1}{1 + \phi'_{0,1}\left(r + \hat{x}_{s,2}\right)}\left[-2\left(\hat{x}_{s,1} + \hat{x}_{s,2}\right) + \hat{z}_1 - 5\hat{z}_2\right] \\
u &= \left[-3.5(1 - e^{-0.2t}) + 2\hat{y}_{ps} + z_p\right] + u_s
\end{aligned}$$

where $\hat{z}_1 = \hat{x}_{s,1}$ and $\hat{z}_2 = 2\hat{x}_{s,1} + \hat{x}_{s,2} + \phi_{0,1}\left(r + \hat{x}_{s,2}\right) - \phi_{0,1}(r)$. Driven by such a controller, the simulation result is shown in Figure 4. As shown, the system output tracks $r(t) \equiv 1$ asymptotically, meanwhile keeping the internal state bounded. This is consistent with *Theorem 2*.

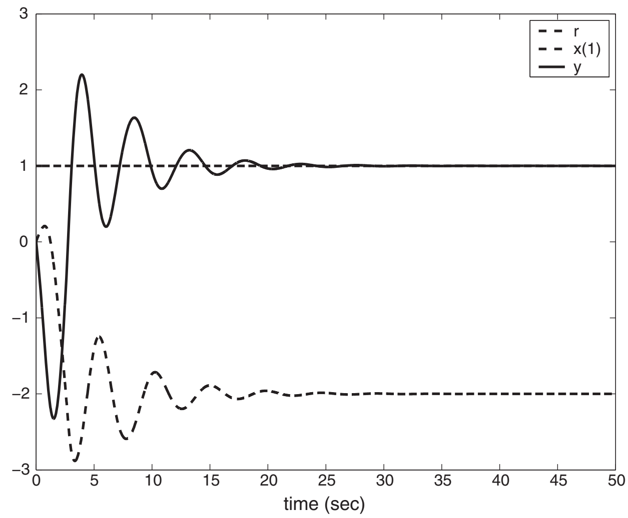

Figure 4. Time response of the nonlinear nonminimum phase system subject to a step disturbance.

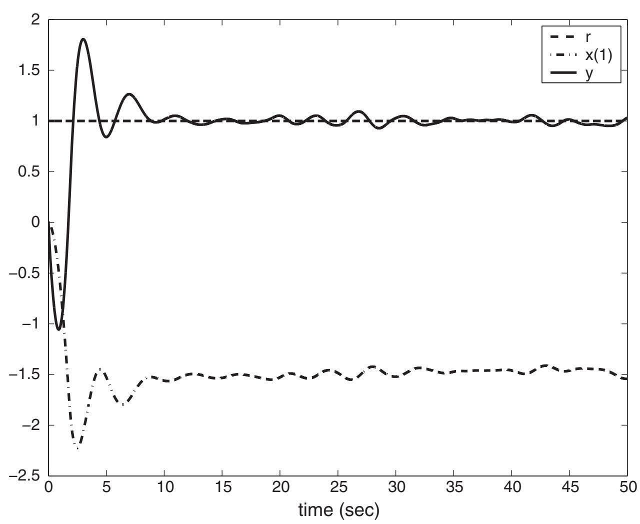

Figure 5. Time response of the nonlinear nonminimum phase system subject to a general bounded disturbance.

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4.3.2. Controller for a general bounded disturbance. According to (22), the controller in this example is combined (35) and (38) as

$$
\begin{aligned}
u_{ps} &= -G_a^{-1} Q \hat{d}_l, \hat{d}_l = \hat{y}_p - G_a u_{ps} \\
u_s &= 5\hat{x}_{s,1} + 4\hat{x}_{s,2} + \frac{1}{1+\phi_{0,1}'\left(r+\hat{x}_{s,2}\right)}\left[-2\left(\hat{x}_{s,1}+\hat{x}_{s,2}\right)+\hat{z}_1-5\hat{z}_2\right] \\
u &= -3.5(1-e^{-0.2t}) + u_{ps} + u_s.
\end{aligned}
$$

where $\hat{z}_1 = \hat{x}_{s,1}$ and $\hat{z}_2 = 2\hat{x}_{s,1} + \hat{x}_{s,2} + \phi_{0,1}\left(r+\hat{x}_{s,2}\right) - \phi_{0,1}(r)$. Driven by such a controller, the simulation result is shown in Figure 5. As shown, the tracking error is ultimately bounded, meanwhile keeping the internal state bounded. This is consistent with *Theorem 2*.

5. CONCLUSIONS

In this paper, the output feedback tracking problem for a class of nonminimum phase nonlinear systems was solved under the additive-state-decomposition-based tracking control framework. Our main contribution lies on the presentation of a new decomposition scheme, named additive state decomposition, which decomposes the original problem into two well-studied control problems. The solvability of the two decomposed problems is further discussed. Based on this decomposition, two controllers corresponding to the two decomposed problems are designed separately, which simplifies the design and also increases the flexibility of the designed controller. This implies that the controller for tracking task can be easily adapted to different references and disturbances. More importantly, existing control methods for nonminimum phase LTI systems can be easily incorporated into the proposed framework.

APPENDIX: PROOF OF THEOREM 2

It is easy to see from the proof in *Theorem 1* that the observer (21) will make

$$\hat{y}_p \equiv y_p \text{ and } \hat{x}_s \equiv x_s. \tag{A.1}$$

The remainder of the proof is composed of two parts: (i) for (12), the controller u_p drives $y_p - r \to \mathcal{B}(\delta)$ as $t \to \infty$ and (ii) based on the result of (i), for (15), the controller u_s drives $y_s \to \mathcal{B}(\|c\|\gamma(\delta))$ as $t \to \infty$. Then, the controller $u = u_p + u_s$ drives $y - r \to \mathcal{B}(\delta + \|c\|\gamma(\delta))$ as $t \to \infty$ in system (1).

(i) Suppose that *Problem 1* is solved. Therefore, by (A.1), the controller (18) can drive $y_p - r \to \mathcal{B}(\delta)$ as $t \to \infty$.
(ii) Let us look at the secondary system (15). Suppose that *Problem 2* is solved. By (A.1), the controller

$$
\begin{aligned}
\dot{z}_s &= \alpha_s\left(z_s, \hat{x}_s, r, \cdots, r^{(N)}\right) \\
u_s &= u_s\left(z_p, \hat{x}_s, r, \cdots, r^{(N)}\right)
\end{aligned}
$$

drives the output y_s such that

$$
\begin{aligned}
\|y_s(t)\| &\leqslant \|c\|\|x_s(t)\| \\
&\leqslant \|c\|\beta(\|x_s(t_0)\|, t-t_0) + \|c\|\gamma\left(\sup_{t_0 \leqslant s \leqslant t}\|e_p(s)\|\right), t \geqslant 0.
\end{aligned}
$$

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On the basis of the result of (i), we obtained $e_p \to \mathcal{B}(\delta)$ as $t \to \infty$. This implies that $\|e_p(t)\| \leqslant \delta + \varepsilon$ when $t \geqslant t_0 + T_1$. Then,

$$\begin{aligned}\|y_s(t)\| &\leqslant \|c\|\beta(\|x_s(t_0+T_1)\|, t-t_0-T_1) + \|c\|\gamma\left(\sup_{t_0+T_1\leqslant s\leqslant t}\|e_p(s)\|\right), t \geqslant t_0+T_1,\\ &\leqslant \|c\|\beta(\|x_s(t_0+T_1)\|, t-t_0-T_1) + \|c\|\gamma(\delta+\varepsilon), t \geqslant t_0+T_1.\end{aligned}$$

Because $\|c\|\beta(\|x_s(t_0+T_1)\|, t-t_0-T_1) \to 0$ as $t \to \infty$ and ε can be chosen arbitrarily small, we can conclude $y_s \to \mathcal{B}(\|c\|\gamma(\delta))$ as $t \to \infty$. Because $y = c^T x_p + c^T x_s$, we can conclude that, driven by the controller (22), the output of system (1) satisfies that $y - r \to \mathcal{B}(\delta + \|c\|\gamma(\delta))$ as $t \to \infty$. In particular, if $\delta = 0$, then the output in system (1) satisfies that $y - r \to 0$ as $t \to \infty$.

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