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Reliability Analysis of Multicopter Configurations Based on Controllability Theory

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Abstract: Flight reliability of multicopters is attracting more and more attention these years. There exists a potential risk to civil safety if a multicopter crashes. Since multicopters with various configurations have appeared in the market, users may wonder which kind of configuration has a better reliability in the presence of motor faults or failures. A multicopter's reliability is highly related to its controllability. Based on controllability theory, reliability analyses of different configurations are conducted and the optimal configuration is given.

Key Words: Multicopter, Flight Reliability, Configuration, Controllability Theory

1 Introduction

During recent years, multicopters are very popular for their unique advantages over fixed-wing airplanes and helicopters in many aspects. Firstly, a multicopter has simple structure and therefore is assembled conveniently. Secondly, since basic movements are decoupled from each other, the remote control of a multicopter is simple. Thirdly, a multicopter is easy to maintain. Due to these salient features, multicopters have been widely used in many fields, such as surveillance, search and rescue missions [1]. However, there exists a potential risk to civil safety if a multicopter crashes, especially in an urban area. Therefore, it is significant to consider the reliability of multicopters in the presence of motor faults or failures [2].

For multicopters, there exist various configurations, such as quadcopters, hexacopters, octocopters. Even just for hexacopters, there exist the standard symmetrical configuration and the nonstandard configuration. Different configurations lead to different flight characteristics. In [3], a design optimization process of multicopters with different configurations was presented, and the optimal goals focus on the dynamic performance and the flight time. A new configuration of fixed-pitch multicopter that combines the energetic efficiency of a helicopter and the mechanical simplicity of a quadcopter was described in [4]. In [5], a program was developed which calculates the optimal design vector using the total energy consumption and vehicle diameter as objective function. Although some research has studied the optimization problem of multicoper configuration, the optimal choice of configuration based on the flight reliability remains an open problem.

The flight reliability of a multicopter is highly related to its controllability. If a multicopter is always under control in the air, it can perform its task safely according to users' requirements. However, if a multicopter is out of control, it may accidently attack people around. Also, the multicopter may be damaged. Therefore, each configuration's controllability can be regarded as an index to evaluate the reliability of a multicopter.

Controllability theory has been developed for many years. Many researchers have made contributions to it, such as Kalman [6], Müller and Weber [7], Hughes and Skelton [8], Viswanathan [9], Tarokh [10], Yang [11], Du [12][13][14], and so on. Degree of controllability, which indicates how controllable a system is, is an important concept in controllability theory. If its value is greater than zero, the system is controllable, in other words, it is reliable. According to the degree of controllability, the multicopter reliability can be indirectly obtained, including: i) the reliability of multicopters with similar rotor configurations but different rotor number; ii) the reliability of a kind of multicopter with different rotor configurations; iii) the reliability/price ratio of multicopters with different rotor number.

This paper is organized as follows. In Section 2, the problem of configuration reliability analysis based on controllability theory is presented. Reliability analysis for multicopters with different rotor number is given in Section 3. Reliability analysis for hexacopters with different rotor configurations is described in Section 4. In section 5, reliability/price ratio analysis for multicopters with different rotor number is conducted. Section 6 gives the conclusion of this paper.

2 Problem Formulation

In order to analyze the reliability of a multicopter in the air, for simplicity, four assumptions are given:

Assumption 1: A reliable flight is defined that a multicopter in the air can make a safe landing[1].

Assumption 2: All multicopters discussed in this paper initially hover in the air.

Assumption 3: Only motor failure is considered in this paper, while the other components are all faultless.

Assumption 4: For the multicopters discussed in this paper, all rotors are the same, distributed evenly and coplanar, and the distance from each rotor to the geometric center of the multicopter is equal.

Due to the characteristics of multicopters, rotors can only provide unidirectional lift (upward or downward) in practice, classical controllability theories of linear systems are insufficient to test the controllability of multicopters. The

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[1] Here, a safe landing is confined to the case that a multicopter can land horizontally, in other words, the pitch angle and the roll angle can be kept to zero.

following abstract expression should be considered

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in [a, b]^m \in R^m$ is the control input.

Definition 1: For system (1), the recovery region Ω within time T is defined as follows

$$\Omega = \{x(0) | \exists u(t) \in [a, b]^m, t \in [0, T], st.x(T) = 0\}$$

Definition 2: For system (1), the degree of controllability ρ within time T is defined as follows

$$\rho = \inf \|x(0)\| \qquad \forall x(0) \notin \Omega$$

where $\|\cdot\|$ represents the Euclidean norm. From *Definition 2* it can be seen that the minimal distance from the origin to the boundary of recovery region is considered as system's degree of controllability. The system is uncontrollable when the degree of controllability is equal to zero. The larger the degree of controllability is, the stronger the system control ability is.

In [11], a controllability calculation method discretizing the continuous system is proposed to deal with the problem of constraint on the control input $[a, b]$. Therefore, the theory can be used to analyze the multicopter system. The calculation procedure of degree of controllability is detailed in our lab's website http://rfly.buaa.edu.cn/resources/.

According to the *Assumption 2*, a model for multicopters in the hovering mode should be formulated first. The attitude dynamical model for multicopters is simplified as

$$\begin{gathered} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\ \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = J^{-1} \begin{bmatrix} L \\ M \\ N \end{bmatrix} \end{gathered} \tag{2}$$

where ϕ, θ, φ denote the roll angle, the pitch angle and the yaw angle respectively, p, q, r denote the angular velocity relative to the body axes coordinate system, L, M, N denote the roll torque, the pitch torque and the yaw torque respectively, and J is the inertia matrix. L, M, N are expressed as

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = Hf \tag{3}$$

where $f = [\ f_1 \quad f_2 \quad \cdots \quad f_n\ ]^T$ denotes the produced lift, where n is the rotor number. H denotes the control effectiveness matrix, expressed as

$$H = \begin{bmatrix} db_l \\ db_m \\ k_u b_n \end{bmatrix} \tag{4}$$

where d denotes the distance from the rotor to the geometric center of a multicopter, b_l, b_m, b_n are the mapping coefficients from lift to torque, and k_u is treated as a constant. In this paper, $k_u = 0.1$.

Detailed reliability analysis based on the controllability theory is developed in the following sections. In the controllability analysis, $T = 0.8$s and $\triangle T = 0.2$s are used, where $\triangle T$ denotes the sampling period.

3 Reliability Analysis for Multicopters with Different Rotor Number

Multicopters can be classified by rotor number, and the familiar ones are quadcopters, hexacopters and octocopters. In this section, the above three kinds are being discussed. For these multicopters, all of them have standard symmetrical configurations that the clockwise rotating rotor is adjacent to the counterclockwise rotating rotor. As shown in Fig.1.

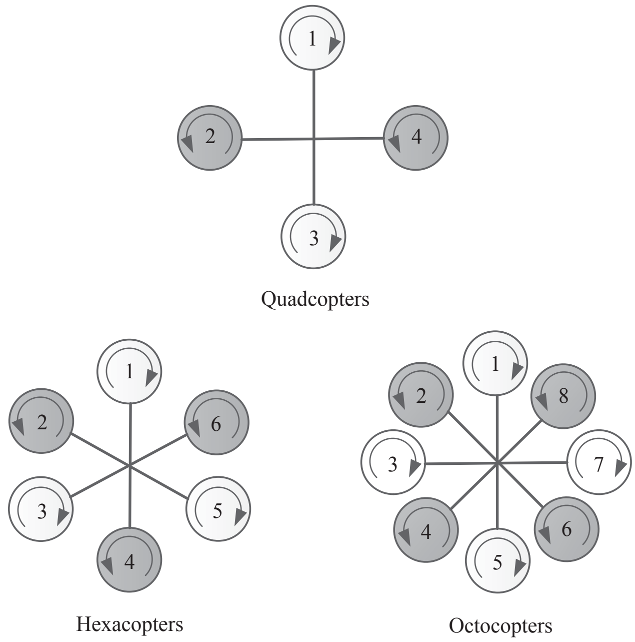

Fig. 1: Multicopters with Different Rotor Number

According to *Assumption 4*, for different configuration multicopters in this paper, $d = 0.28$m, $f_i \in [0, 6\text{N}]$ $(i = 1, 2, \ldots n)$ are uniform. The inertia matrix $J = diag\{0.0411, 0.0478, 0.0599\}$ can be regarded as a constant matrix, whereas b_l, b_m, b_n change with the different configuration. For quadcopters with standard configurations,

$$\begin{aligned} b_l &= [\ 0 \quad -1 \quad 0 \quad 1\ ], \\ b_m &= [\ 1 \quad 0 \quad -1 \quad 0\ ], \\ b_n &= [\ -1 \quad 1 \quad -1 \quad 1\ ]; \end{aligned}$$

for hexacopters with standard configurations,

$$\begin{aligned} b_l &= [\ 0 \quad -\tfrac{\sqrt{3}}{2} \quad -\tfrac{\sqrt{3}}{2} \quad 0 \quad \tfrac{\sqrt{3}}{2} \quad \tfrac{\sqrt{3}}{2}\ ], \\ b_m &= [\ 1 \quad 0.5 \quad -0.5 \quad -1 \quad -0.5 \quad 0.5\ ], \\ b_n &= [\ -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1\ ]; \end{aligned}$$

for octocopters with standard configurations,

$$\begin{aligned} b_l &= [\ 0 \quad -\tfrac{\sqrt{2}}{2} \quad -1 \quad -\tfrac{\sqrt{2}}{2} \quad 0 \quad \tfrac{\sqrt{2}}{2} \quad 1 \quad \tfrac{\sqrt{2}}{2}\ ], \\ b_m &= [\ 1 \quad \tfrac{\sqrt{2}}{2} \quad 0 \quad -\tfrac{\sqrt{2}}{2} \quad -1 \quad -\tfrac{\sqrt{2}}{2} \quad 0 \quad \tfrac{\sqrt{2}}{2}\ ], \\ b_n &= [\ -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1\ ]. \end{aligned}$$

On the basis of the controllability theory, when a quadcopter, hexacopter or octocopter hovers in the air, their system's degree of controllability is shown in Table 1. From Table 1 it can be seen, in spite of quadcopters, hexacopters or

Table 1: Controllability Analysis for Multicopters with Different Rotor Number (No Motor Failure)

Rotor Number	4	6	8
Degree of Controllability	2.6836	3.7361	5.4882

octocopters, all of these aircrafts have the degree of controllability greater than zero in the hovering mode. This implies that they can keep hovering if no motor failure occurs.

In next step, according to *Assumption 3*, motor failure is considered. Since a safe landing is confined to the case that a multicopter can land horizontally on the basis of *Assumption 1*, two situations should be considered when motor failure occurs: i) the roll channel, the pitch channel and the yaw channel are controllable; ii) the roll channel and the pitch channel are controllable, whereas the yaw channel is ignored.

Firstly, a quadcopter is taken into account. When a motor fails, according to the calculation result of the controllability theory, the system's degree of controllability becomes zero. That is to say, it is uncontrollable when a quadcopter is subject to motor failure. Then, if the yaw channel is ignored, only four remaining states φ, θ, p, q are considered in the controllability calculation, the result indicates that the degree of controllability is still zero, in other words, the roll channel and the pitch channel remain uncontrollable. Therefore, when a quadcopter is subject to motor failure, it is impossible to make a safe landing. It is assumed that the reliable rate of each motor is α, the reliability of a quadcopter is expressed as

$$P_4 = \alpha^4. \tag{5}$$

Secondly, a hexacopter is taken into account. When a motor fails, the hexacopter shown in Fig.1 is uncontrollable. However, if the yaw channel is ignored, and the roll channel and the pitch channel are considered only, the result indicates that the degree of controllability is 5.6315. As a result, when a motor fails, although the hexacopter is uncontrollable, the roll channel and the pitch channel are controllable, a hexacopter can make a safe landing.

When two motors fail, the whole system is still uncontrollable. If the yaw channel is ignored, the result is concluded in Table 2. It is shown that nine controllable cases exist.

Table 2: Controllability Analysis for Hexacopters Without Yaw Channel (Two-Motor Failure)

Motor	1	2	3	4	5	6
1	N/A					
2	0	N/A				
3	4.5052	0	N/A			
4	4.5052	4.5052	0	N/A		
5	4.5052	4.5052	5.066	0	N/A	
6	0	5.066	4.5052	4.5052	0	N/A

When three motors fail, this case is equivalent to a quadcopter with one-motor failure, which is uncontrollable. However, if the yaw channel is not considered, as long as the remaining functional motors all rotate clockwise or counterclockwise, the roll channel and the pitch channel are controllable. Hence, the reliability of a hexacopter is

$$\begin{aligned} P_6 &= \alpha^6 + 6\alpha^5(1-\alpha) + 9\alpha^4(1-\alpha)^2 + 2\alpha^3(1-\alpha)^3 \\ &= 2\alpha^6 - 6\alpha^5 + 3\alpha^4 + 2\alpha^3. \end{aligned} \tag{6}$$

At last, an octocopter is taken into account. When a motor fails, the analysis result indicates that the octocopter is controllable. As shown in Table 3.

Table 3: Controllability Analysis for Octocopters (One-Motor Failure)

Motor	1	2	3	4
Degree of Controllability	3.4301	3.4301	3.458	3.4301
Motor	5	6	7	8
Degree of Controllability	3.4301	3.4301	3.458	3.458

When two motors fail, the result illustrated in Table 4 shows that the system is still controllable.

When three motors fail, the uncontrollable cases are shown in Table 5. However, if the yaw channel is ignored, eight motor combinations from Table 5, detailed in Table 6, are controllable. Since the total combinations of three motors can be expressed as C_8^3, there exist $C_8^3 - 8$ controllable combinations.

When four motors fail, the controllable cases are shown in Table 7. In other uncontrollable cases, the motor combinations detailed in Table 8 are controllable by ignoring the yaw channel. Taken together, there totally exist thirty-eight controllable combinations.

When five rotors fail, this case can also be equivalent to a quadcopter with one-motor failure, which is uncontrollable. However, if the yaw channel is not considered, eight combinations are controllable, shown in Table 9.

Therefore, the reliability of an octocopter is

$$\begin{aligned} P_8 &= \alpha^8 + 8\alpha^7(1-\alpha) + 28\alpha^6(1-\alpha)^2 + \\ &\quad 48\alpha^5(1-\alpha)^3 + 38\alpha^4(1-\alpha)^4 + 8\alpha^3(1-\alpha)^5 \\ &= 3\alpha^8 - 16\alpha^7 + 32\alpha^6 - 24\alpha^5 - 2\alpha^4 + 8\alpha^3. \end{aligned} \tag{7}$$

The reliability curves of the three types of multicopters are depicted in Fig.2. It is obvious that octocopters have the highest reliability while quadcopters have the lowest value. From the perspective of flight reliability, octocopters' configuration is the best compared with the other two configurations.

4 Reliability Analysis for Hexacopters with Different Rotor Configurations

Usually, for a kind of multicopter, different rotor configurations may exist. Here, hexacopters are taken as an example. Fig.3 shows two rotor configurations. The former one is used to denote the standard arrangement, where "P" denotes that a rotor rotates clockwise and "N" denotes that a rotor rotates counterclockwise. It is assumed that the physical parameters of the two configurations are the same. For the "PPNNPN" configuration,

$$\begin{aligned} b_l &= [\ 0 \quad -\tfrac{\sqrt{3}}{2} \quad -\tfrac{\sqrt{3}}{2} \quad 0 \quad \tfrac{\sqrt{3}}{2} \quad \tfrac{\sqrt{3}}{2}\ ], \\ b_m &= [\ 1 \quad 0.5 \quad -0.5 \quad -1 \quad -0.5 \quad 0.5\ ], \\ b_n &= [\ -1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1\ ]. \end{aligned}$$

Table 4: Controllability Analysis for Octocopters (Two-Motor Failure)

Motor	1	2	3	4	5	6	7	8
1	N/A							
2	3.0391	N/A						
3	0.7636	3.0341	N/A					
4	2.7441	0.7708	3.0341	N/A				
5	3.3239	2.7441	0.7636	3.0391	N/A			
6	2.7441	3.3546	2.7664	0.7565	3.0391	N/A		
7	0.7636	2.7664	3.3861	2.7664	0.7636	3.0341	N/A	
8	3.0391	2.7565	2.7664	3.3546	2.7441	0.7708	3.0341	N/A

Table 5: Uncontrollable Cases for Octocopters (Three-Motor Failure)

Motor Combination	1	1	1	1	1	1	2	2	2	2	3	3	4	4	5	6
	2	2	3	3	5	7	3	4	4	6	4	5	5	6	6	7
	3	8	5	7	7	8	4	6	8	8	5	7	6	8	7	8

Table 6: Other Controllable Cases Without Yaw Channel (Three-Motor Failure)

Motor Combination	1	1	1	2	2	2	3	4
	3	3	5	4	4	6	5	6
	5	7	7	6	8	8	7	8
Degree of Controllability	6.347	6.347	6.347	5.919	5.919	5.919	6.347	5.919

Table 7: Controllable Cases for Octocopters (Four-Motor Failure)

Motor Combination	1	1	1	1	1	1	1	1	1	1
	2	2	2	2	3	3	3	4	4	5
	4	4	5	6	4	4	6	5	6	6
	5	7	6	7	6	8	8	8	7	8
Degree of Controllability	0.771	0.708	2.315	0.764	0.702	0.764	0.708	2.315	0.702	0.771
Motor Combination	2	2	2	2	2	2	3	3	3	4
	3	3	3	3	4	5	4	4	5	5
	5	5	6	7	5	7	6	7	6	7
	6	8	7	8	7	8	7	8	8	8
Degree of Controllability	0.764	0.702	2.396	0.757	0.708	0.702	0.757	2.396	0.708	0.764

Table 8: Other Controllable Cases Without Yaw Channel (Four-Motor Failure)

Motor Combination	1	1	1	1	1	1	1	1	1
	2	2	3	3	3	3	3	4	4
	4	5	4	5	5	5	6	5	6
	6	7	7	6	7	8	7	7	8
Degree of Controllability	3.925	4.209	4.209	4.209	5.394	4.209	4.209	4.209	3.925
Motor Combination	2	2	2	2	2	2	2	3	3
	3	3	4	4	4	4	5	4	5
	5	6	5	6	6	7	6	6	7
	7	8	8	7	8	8	8	8	8
Degree of Controllability	4.209	4.565	3.925	4.565	5.03	4.565	3.925	4.565	4.209

Table 9: Controllable Cases for Octocopters Without Yaw Channel (Five-Motor Failure)

Motor Combination	1	1	1	1	1	2	2	2
	2	2	3	3	3	3	3	4
	4	4	4	4	5	5	5	5
	5	6	6	6	6	6	7	7
	7	7	7	8	8	8	8	8
Degree of Controllability	3.557	3.814	4.136	3.814	3.557	3.814	4.136	3.814

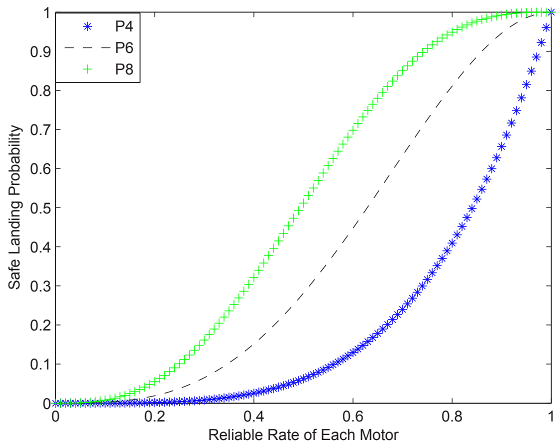

Fig. 2: Safe Landing Probability of Multicopters with Different Rotor Number

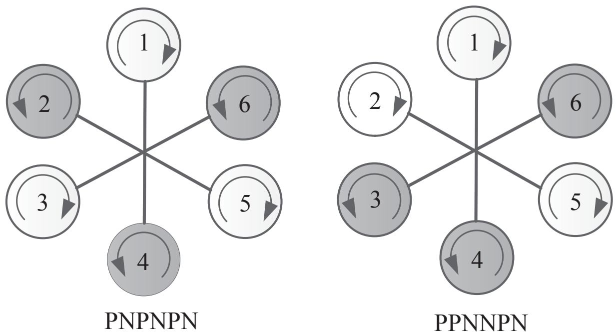

Fig. 3: Hexacopters with Two Configurations

By calculation, the system's degree of controllability with no motor failure is 2.7320, a little smaller than the standard one.

When a motor fails, the corresponding degree of controllability is obtained in Table 10. From Table 10 it can be seen, the system in four cases is still controllable. If the yaw channel is ignored, the other two cases also become controllable.

Table 10: Controllability Analysis for "PPNNPN" Configuration (One-Motor Failure)

Motor	1	2	3
Degree of Controllability	2.6745	1.7075	1.7075
Motor	4	5	6
Degree of Controllability	2.6745	0	0

When two motors fail, there exist three controllable cases, shown in Table 11. If the yaw channel is ignored, the result is as same as the calculation output in Table 2.

When three motors fail, the "PPNNPN" system is uncontrollable. However, if the yaw channel is not considered, as long as the remaining motors all rotate clockwise or counterclockwise, the roll channel and the pitch channel are controllable. Therefore, the reliability of the "PPNNPN" configuration is expressed as

$$\begin{aligned} P_6' &= \alpha^6 + 6\alpha^5(1-\alpha) + 9\alpha^4(1-\alpha)^2 + 2\alpha^3(1-\alpha)^3 \\ &= 2\alpha^6 - 6\alpha^5 + 3\alpha^4 + 2\alpha^3. \end{aligned} \tag{8}$$

Table 11: Controllable Cases for "PPNNPN" Configuration (Two-Motor Failure)

Motor Combination	1 3	1 4	2 4
Degree of Controllability	1.7075	2.4907	1.7075

The result is equal to formula (6). However, through the comparison with the standard-configuration hexacopter, it can be found that for the "PPNNPN" hexacopter, some controllable cases remain exist without ignoring the yaw channel when motor failure occurs. If the yaw channel is considered, the two kinds' reliability are modified as

$$\begin{aligned} P_6 &= \alpha^6 \\ P_6' &= \alpha^6 + 4\alpha^5(1-\alpha) + 3\alpha^4(1-\alpha)^2 \\ &= -2\alpha^5 + 3\alpha^4. \end{aligned} \tag{9}$$

The reliability curves of the two configurations are depicted in Fig.4. Taken together, the "PPNNPN" configuration is of better reliability compared with the standard configuration while the standard one has a higher degree of controllability when no motor failure occurs.

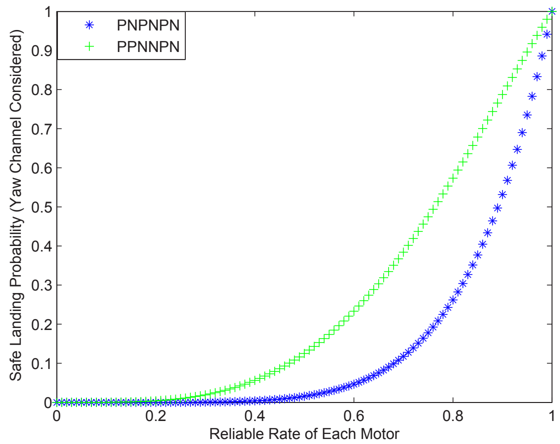

Fig. 4: Safe Landing Probability of Two Configurations

5 Reliability/Price Ratio Analysis for Multicopters with Different Rotor Number

In practice, engineers not only simply consider the reliability of a multicopter, but also care for the reliability/price ratio. Normally, a motor's price increases with its reliability. Assume that the relationship between a motor's price M and its reliable rate α is

$$M = d + k/(1-\alpha) \tag{10}$$

where d and k are constants. Therefore, the price of multicopters with different rotor number is expressed as

$$M_n = C + nM \tag{11}$$

where n is the rotor number, C is the price of other multicopter components, which is regarded as a constant here. In this paper, $d = 100, k = 10, C = 500$ are assumed. According to (5), (6), (7) and (11), the reliability/price ratio

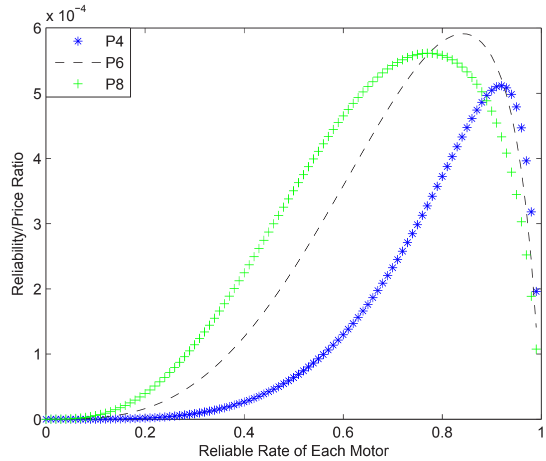

Fig. 5: Reliability/Price Ratio

curves of multicopters with different rotor number are obtained, shown in Fig.5.

This figure provides people with a reference to choose a multicopter with the better reliability/ratio.

6 Conclusion

Reliability is a critical index for a multicopter design. Different configurations have enormous influence on multicopter reliability. In the market, there exist three familiar multicopters, including quadcopters, hexacopters and octocopters. For hexacopters, there exist the "PNPNPN" configuration, the "PPNNPN" configuration, etc. In this paper, a new method is proposed to analyze the multicopter reliability and the optimal design is given. The whole paper studies the following three topics: i) reliability analysis for multicopters with different rotor number; ii) reliability analysis for hexacopters with different rotor configurations; iii) reliability/price ratio analysis for multicopters with different rotor number. All of these are analyzed based on a proper controllability theory. The results indicate that octocopters have the highest reliability of the three types of multicopters, and the "PPNNPN" configuration has a better reliability than the "PNPNPN" configuration for a hexacopter.

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