Further Results on Additive-State-Decomposition-Based Output Feedback Tracking Control for a Class of Uncertain Nonminimum Phase Nonlinear Systems

Zi-Bo Wei, Jin-Rui Ren, Quan Quan

School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China
E-mail: whisper@buaa.edu.cn; renjinrui@buaa.edu.cn; qq_buaa@buaa.edu.cn

Abstract: This paper shows further results on the output feedback tracking problem for a class of uncertain nonminimum phase nonlinear systems, where a state-based uncertainty is involved. Through a new additive state decomposition process, this uncertainty is divided into two parts which are assigned to the two decomposed subsystems, respectively. Then the controller is designed and its stability is analyzed by the nonlinear small-gain theorem. Finally, to demonstrate the effectiveness of the proposed method, an illustrative example is given.

Key Words: Tracking Control, Nonlinear Systems, Nonminimum Phase Systems, Uncertainty, Additive State Decomposition

1 INTRODUCTION

Nonlinear systems with nonminimum phase exit widely in the real world, such as conventional fixed-wing aircraft [1], vertical take-off and landing aircraft [2], and so on. Theoretically, the internal or zero dynamics are unstable for a nonminimum phase system. Thus, the tracking problem for a nonlinear system with nonminimum phase is much more complex than a general nonlinear system, and it becomes a challenging issue. A large amount of literature solves this issue by using the ideal internal dynamics to transform the output tracking problem to a state tracking problem [3]-[6]. For these methods, the internal states need to be measured exactly. Unfortunately, the internal information is hard to be obtained in practical engineering. That motivates us to propose a method to solve this issue, which only uses the input-output information of the system.

Our basic idea is to decompose a tracking task of a nonlinear system into two subtasks, namely a tracking subtask for a linear time invariant (LTI) system (named as 'primary system') plus a stabilization subtask for a nonlinear system (named as 'secondary system'), through additive state decomposition [7]. Based on the additive state decomposition, a new control scheme, called as *additive-state-decomposition-based tracking control*, is proposed. Furthermore, there exist many mature methods for the two subtasks. Finally, the control actions are integrated together to achieve the original control goal. It should be pointed out that the additive state decomposition is different from the lower-order subsystem decomposition methods existing in the literature [8]. Concretely, taking the system $\dot{x}(t) = f(t,x), x \in \mathbb{R}^n$ for example, it is decomposed into two subsystems: $\dot{x}_1(t) = f_1(t,x_1,x_2)$ and $\dot{x}_2(t) = f_2(t,x_1,x_2)$, where $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$, respectively. The lower-order subsystem decomposition satisfies

$$n = n_1 + n_2 \text{ and } x = x_1 \oplus x_2.$$

By contrast, the proposed additive state decomposition satisfies

$$n = n_1 = n_2 \text{ and } x = x_1 + x_2.$$

In our opinion, lower-order subsystem decomposition aims to reduce the complexity of the system itself, while the additive state decomposition emphasizes the reduction of the complexity of tasks for the system.

The framework of this method and our preliminary results are published in reference [7]. The system considered in reference [7] only contains an uncertainty which is independent of the states of the system. However, in real systems, there also exists state-based uncertainties. These uncertainties may degrade the performance of system. Furthermore, they may even make the system unstale. Thus, on the basis of the model used in reference [7], this paper considers a new uncertainty which is relevant to the states of the system. This uncertainty can be regarded as a disturbance acting on the parameters of the system. Thus, the method used in this paper is more robust than the previous method, which is the main contribution of this paper. Motivated by the consideration of the robustness, a new additive-state-decomposition-based controller is proposed, which is different from that of reference [7]. The main differences are as follows. i) This paper adopts a new additive state decomposition process. Unlike the additive state decomposition process in reference [7], the primary system couples with the secondary system because of the state-based uncertainty. In order to deal with this problem, the primary system is decomposed again and the three subsystems are rearranged to two parts, which are interconnect subsystems. ii) Controllers are designed for the interconnect subsystems, and a rigorous proof of stability is given

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for the controllers based on the nonlinear small-gain theorem for the interconnected input-to-state stable (ISS) system. Finally, an example with disturbance of parameters is employed to illustrate the further results.

This paper is organized as follows. In Section 2, the problem formulation is given and the additive state decomposition is introduced briefly first. In Section 3, a concrete decomposition is demonstrated, and based on it, the controller is designed and the its stability is analyzed. In Section 4, an illustrative example is given to demonstrate the effectiveness of the proposed control scheme. Section 5 concludes this paper.

2 Problem Formulation and Additive State Decomposition

2.1 Problem Formulation

Consider a class of SISO uncertain nonlinear systems similar to [9]-[12]:

$$\begin{aligned} &\dot{x} = Ax + bu + \phi(y) + \Delta\varphi(x) + d, x(0) = x_0 \\ &y = c^T x \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is a known constant matrix, $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ are known constant vectors, $\phi : \mathbb{R} \to \mathbb{R}^n$ is a known nonlinear function vector, $\Delta\varphi : \mathbb{R}^n \to \mathbb{R}^n$ is an unknown uncertain function vector, $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the control, and $d(t) \in \mathbb{R}^n$ is an unknown bounded disturbance. It is assumed that only y is available from measurement. The desired trajectory $r(t) \in \mathbb{R}, t \geq 0$ is known and smooth enough. In the following, for convenience, we will omit the variable t except when necessary. For system (1), the following assumption is made.

Assumption 1. The pair (A, c) is observable.

Under *Assumption 1*, the objective here is to design a tracking controller u such that $y \to r$ as $t \to \infty$ or with good tracking accuracy, i.e, $y - r$ is ultimately bounded by a small value.

Remark 1. As far as the nonminimum phase system (1) is concerned, the system parameters are often assumed to be known accurately except for the disturbance d. Furthermore, in this paper, we assume that the system parameters are subject to the uncertainty $\Delta\varphi(x)$.

Remark 2. Since the pair (A, c) is observable under *Assumption 1*, there always exists a vector $p \in \mathbb{R}^n$ such that $A+pc^T$ is stable, whose eigenvalues can be assigned freely. As a result, system (1) is rewritten as $\dot{x} = \left(A + pc^T\right)x + bu + [\phi(y) - py] + \Delta\varphi(x) + d$. Then $\left(A + pc^T, b\right)$ is stabilizable. Therefore, we can assume that the pair (A, b) is stabilizable with a stable A without loss of generality.

2.2 Additive State Decomposition

In order to make the paper self-contained, a special case of the additive state decomposition [7] is introduced here briefly. Consider the following 'original' system:

$$\begin{aligned} &\dot{x} = f(t, x), x(0) = x_0 \\ &y = h(t, x) \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. We first bring in a 'primary' system having the same dimension as system (2), which is expressed as follows:

$$\begin{aligned} &\dot{x}_p = f_p(t, x_p, x_s), x_p(0) = x_{p,0} \\ &y_p = h_p\left(t, x_p\right) \end{aligned} \tag{3}$$

where $x_p \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$. From the original system (2) and the primary system (3), we derive the following 'secondary' system:

$$\begin{aligned} &\dot{x}_s = f(t, x_p + x_s) - f_p(t, x_p, x_s), x_s(0) = x_0 - x_{p,0} \\ &y_s = h(t, x_p + x_s) - h_p\left(t, x_p\right) \end{aligned} \tag{4}$$

where

$$x(t) = x_p(t) + x_s(t), y(t) = y_p(t) + y_s(t). \tag{5}$$

Remark 3. By the additive state decomposition, the system (2) is decomposed into two subsystems with the same dimension as the original system. This decomposition is introduced in reference [7] completely, so the readers can refer to it for the further details.

3 Tracking Controller Design and Stability Analysis

First, based on additive state decomposition, the considered system (1) is decomposed into two subsystems: an LTI system including all external signals as the primary system, together with the secondary system whose equilibrium point is zero. Since the output of the primary system and the state of the secondary system can be observed, the original tracking task for the system (1) is correspondingly decomposed into two subtasks: an output feedback tracking subtask for an LTI 'primary' system and a state feedback stabilization subtask for the left 'secondary' system, as shown in Fig.1. Since the tracking subtask is only assigned to the LTI system, it is therefore much easier than that for the nonlinear system (1). Then, the controllers are designed for the two systems, and the stability is analyzed. At last, the controllers are intergrated for the original system.

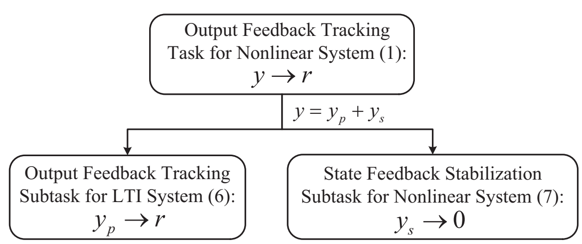

Figure 1: Additive state decomposition flow

3.1 Decomposition

Consider the system (1) as the original system. According to the principle mentioned in Section 2.2, we choose the primary system as follows:

$$\begin{aligned} &\dot{x}_p = Ax_p + bu_p + \phi(r) + \Delta\varphi(x_p + x_s) + d \\ &y_p = c^T x_p, x_p(0) = x_0 \end{aligned} \quad . \tag{6}$$

Then the secondary system is determined by the original system (1) and the primary system (6) with the rule (4), and we obtain that

$$\begin{array}{l}\dot{x}_s = Ax_s + bu_s + \phi\left(c^T x_p + c^T x_s\right) - \phi\left(r\right) \\ y_s = c^T x_s, x_s\left(0\right) = 0\end{array} \tag{7}$$

where $u_s = u - u_p$. According to equation (5), we have

$$x = x_p + x_s \text{ and } y = y_p + y_s. \tag{8}$$

Controller design for the decomposed systems (6)-(7) needs their outputs or states. However, they are unknown. For such a purpose, an observer is proposed to estimate y_p and x_s.

Theorem 1. Suppose that an observer is designed to estimate y_p and x_s in systems (6)-(7) as follows:

$$\hat{y}_p = y - c^T \hat{x}_s \tag{9a}$$

$$\dot{\hat{x}}_s = A\hat{x}_s + bu_s + \phi\left(y\right) - \phi\left(r\right), \hat{x}_s\left(0\right) = 0. \tag{9b}$$

Then $\hat{y}_p \equiv y_p$ and $\hat{x}_s \equiv x_s$.

Proof. Subtracting equation (9b) from equation (7) results in $\dot{\tilde{x}}_s = A\tilde{x}_s$, $\tilde{x}_s\left(0\right) = 0$ (since the initial values $x_s\left(0\right), \hat{x}_s\left(0\right)$ are both assigned by the designer, they are all determinate), where $\tilde{x}_s = x_s - \hat{x}_s$. Then $\tilde{x}_s \equiv 0$. This implies that $\hat{x}_s \equiv x_s$. Consequently, by equation (8), we have $\hat{y}_p \equiv y - c^T \hat{x}_s \equiv y_p$. □

Remark 4. The measurement y may be inaccurate in practice. In this case, it is expected that small uncertainties still lead $\hat{x}_s$ close to x_s. According to this, the matrix A is required to be stable by the relationship $\dot{\tilde{x}}_s = A\tilde{x}_s$ in the proof above. This can be satisfied by the statement in *Remark 2*.

It is clear from systems (6)-(8) that if the controller u_p drives $y_p \to r$ and the controller u_s drives $y_s \to 0$ as $t \to \infty$, then $y \to r$ as $t \to \infty$. The strategy here is to assign the tracking subtask to the primary system (6) and the stabilization subtask to the secondary system (7). Since the system (6) is a classical LTI system, some standard designs in frequency domain, such as the transfer function method, can be used to handle the output feedback tracking problem. This is easier than that for eqation (1) directly. If $y_p = c^T x_p = r$, then zero is an equilibrium point of the secondary system (7). Since its states can be obtained by eqation (9b), the design is also easier than the output feedback stabilization for eqation (1). According to these, the additive state decomposition offers a way to simplify the original control task.

3.2 Controller Design

For (6), design a linear controller in the form:

$$\begin{array}{l}\dot{z}_p = h\left(z_p, y_p, r\right), z_p\left(0\right) = 0 \\ u_p = u^p\left(z_p, y_p, r\right)\end{array} \tag{10}$$

where $z_p \in \mathbb{R}^m$ and functions h, u^p are linear. Substituting controller (10) into system (6) yields

$$\begin{array}{l}\dot{z}_p = h\left(z_p, y_p, r\right) \\ \dot{x}_p = Ax_p + bu^p\left(z_p, y_p, r\right) + \Delta\varphi\left(x_p + x_s\right) + \phi\left(r\right) + d \\ e_p = c^T x_p - r, z_p\left(0\right) = 0, x_p\left(0\right) = x_0.\end{array} \tag{11}$$

The objective is to make $e_p \to 0$ as $t \to \infty$ or with good tracking accuracy, i.e, e_p is ultimately bounded by a small value. Since the term $\Delta\varphi\left(x_p + x_s\right)$ also depends on the secondary system, in order to separate the effect of x_s form the primary, the system (11) is further 'additively' decomposed into

$$\begin{array}{l}\dot{z}_{pp} = h\left(z_{pp}, y_{pp}, r\right) \\ \dot{x}_{pp} = Ax_{pp} + bu^p\left(z_{pp}, y_{pp}, r\right) + \phi\left(r\right) + \Delta\varphi\left(x_{pp}\right) + d \\ e_{pp} = c^T x_{pp} - r, z_{pp}\left(0\right) = 0, x_{pp}\left(0\right) = x_0\end{array} \tag{12}$$

and

$$\begin{array}{l}\dot{z}_{ps} = h\left(z_{ps}, y_{ps}, 0\right) \\ \dot{x}_{ps} = Ax_{ps} + bu^p\left(z_{ps}, y_{ps}, 0\right) \\ \qquad + \left(\Delta\varphi\left(x_{pp} + x_{ps} + x_s\right) - \Delta\varphi\left(x_{pp}\right)\right) \\ y_{ps} = c^T x_{ps}, z_{ps}\left(0\right) = 0, x_{ps}\left(0\right) = 0\end{array} \tag{13}$$

where linearity of h, u^p is utilized. According to equation (5), we have

$$\begin{array}{l}z_p = z_{pp} + z_{ps}, y_p = y_{pp} + y_{ps} \\ x_p = x_{pp} + x_{ps}, e_p = e_{pp} + y_{ps}\end{array}. \tag{14}$$

By the decomposition, zero is an equilibrium point of system (13) when $x_s = 0$.

Remark 5. The controller design for system (6) can be considered as an output regulation problem in [9]-[12], if the reference r and disturbance d are generated by an autonomous system. Since the reference r is known, a feedforward can also be designed to compensate for $\phi\left(r\right)$ and r directly.

For (7), design a controller in the form

$$u_s = u^s\left(x_s, r, \cdots, r^{(N)}\right) \tag{15}$$

where N is a positive integer. Substituting equations (14)(15) into system (7) results in

$$\begin{array}{l}\dot{x}_s = Ax_s + bu^s\left(x_s, r, \cdots, r^{(N)}\right) \\ \qquad + \phi\left(r + e_{pp} + c^T x_{ps} + c^T x_s\right) - \phi\left(r\right) \\ y_s = c^T x_s, x_s\left(0\right) = 0\end{array}. \tag{16}$$

3.3 Stability Analysis

Combining system (13) with system (16) to be

$$\begin{array}{l}\dot{x}_1 = f_1\left(x_1, x_2\right) \\ \dot{x}_2 = f_2\left(x_1, x_2, e_{pp}\right)\end{array} \tag{17}$$

where

$$\begin{array}{l}x_1 = \left[z_{ps}\ x_{ps}\right]^T, x_2 = x_s \\ f_1\left(x_1, x_2\right) = \\ \quad \left[\begin{array}{l}h\left(z_{ps}, y_{ps}, 0\right) \\ Ax_{ps} + bu^p\left(z_{ps}, y_{ps}, 0\right) \\ \quad + \left(\Delta\varphi\left(x_{pp} + x_{ps} + x_s\right) - \Delta\varphi\left(x_{pp}\right)\right)\end{array}\right]. \\ f_2\left(x_1, x_2, e_{pp}\right) = Ax_s + bu_s \\ \quad + \phi\left(r + e_{pp} + c^T x_{ps} + c^T x_s\right) - \phi\left(r\right)\end{array} \tag{18}$$

Remark 6. As shown in Fig.2, we additively decompose system (11) into system (12) and system (13). And then

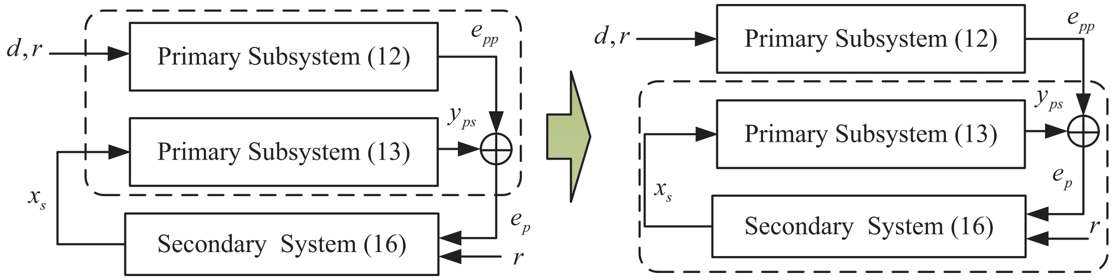

Figure 2: Interconnect subsystems

rearrange the systems (12), (13) and (16), resulting in the system (17) with a zero equilibrium.

Before proceeding further, we have the following preliminary results.

Definition 1 [13]. A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$. It is of class $\mathcal{K}_\infty$ if, in addition, it is unbounded. A function β: $\mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$ if, for each fixed t, the function $\beta(\cdot, t)$ is of class $\mathcal{K}$ and, for each fixed s, the function $\beta(s, \cdot)$ is decreasing and tends to zero at infinity.

Definition 2 [13]. A system $\dot{x} = f(x, u)$ is said to be ISS if there exist a function β of class $\mathcal{KL}$, a function γ of class $\mathcal{K}$ and a nonnegative constant d such that for each initial condition $x(t_0)$ and each measurable essentially bounded control $u(\cdot)$ defined on $[t_0, \infty)$, the solution $x(\cdot)$ of the system exists on $[0, \infty)$ and satisfies:

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t) + \gamma\left(\|u\|_{[t_0,t]}\right) \ \forall t \geq 0. \quad (19)$$

where $\|u\|_{[t_0,t]} \triangleq \sup_{[t_0,t]} \|u(t)\|$.

Definition 3 [14]. Smooth functions $V_i, i = 1, 2$ are said to be ISS-Lyapunov functions (the ISS-Lyapunov function here is a special case of input-to-state practically stable Lyapunov functions in [14]) for system (17) if

1) there exist functions $\psi_{i1}, \psi_{i2} \in \mathcal{K}_\infty$ so that

$$\psi_{i1}(\|x_i\|) \leq V_i(x_i) \leq \psi_{i1}(\|x_i\|) \quad (20)$$

2) there exist functions $\alpha_i \in \mathcal{K}_\infty$, $\chi_i, \gamma_i \in \mathcal{K}$ so that $V_1(x_1) \geq \chi_1(V_2(x_2))$ implies

$$\nabla V_1 f_1(x_1, x_2) \leq -\alpha_1(V_1) \quad (21)$$

and $V_2(x_2) \geq \max\left\{\chi_1(V_1(x_1)), \gamma_2\left(|e_{pp}|_{[t_0,\infty)}\right)\right\}$ implies

$$\nabla V_2 f_2(x_1, x_2, e_{pp}) \leq -\alpha_2(V_2). \quad (22)$$

Lemma 1. For $i = 1, 2$, the x_i-subsystem (17) has ISS-Lyapunov functions V_i satisfying equations (20)-(22). If $\chi_1 \circ \chi_2(r) < r, \forall r > 0$, then the x_i-subsystem (17) is ISS with respect to e_{pp}. In particular, if $e_{pp} \to 0$ as $t \to \infty$, then the system (17) is globally asymptotically stable, namely $e \to 0$ as $t \to \infty$.

Proof. Since for $i = 1, 2$, the x_i-subsystem (17) has ISS-Lyapunov functions V_i satisfying equations (20)-(22), moreover $\chi_1 \circ \chi_2(r) < r, \forall r > 0$, we can conclude that the system (17) is ISS with respect to e_{pp} by Theorem 3.1 in [14], namely

$$\|x_c(t)\| \leq \beta(\|x_c(t_0)\|, t) + \gamma\left(|e_{pp}|_{[t_0,t]}\right), \ \forall t \geq 0$$

where $x_c = [x_1 \ x_2]^T$. In particular, if $e_{pp} \to 0$ as $t \to \infty$, then for an arbitrary real $\epsilon = \gamma^{-1}(\varepsilon) > 0$, $|e_{pp}|_{[t_0,\infty)} \leq \epsilon$ when $t_0 > T$. This further implies that $\|x_c(t)\| \leq \beta(\|x_c(t_0)\|, t) + \varepsilon, t_0 > T$. Since ε can be chosen arbitrarily small, we have $x_c \to 0$ as $t \to \infty$, namely x_i-subsystem (17) is globally asymptotically stable. Since the tracking error can be represented as $e = e_{pp} + [c^T \ c^T]x_c$, we have $|e(t)| \leq 2\|c\| \|x_c(t)\| + |e_{pp}(t)|$. Therefore $e \to 0$ as $t \to \infty$. □

Remark 7. The uncertainty $\Delta\varphi$ will determine the size of function χ_1 in *Lemma 1*. If the uncertainty $\Delta\varphi$ is large, then the size of function χ_1 will increase, namely it is more difficult to satisfy $\chi_1 \circ \chi_2(r) < r$, vice versa. Without $\Delta\varphi$, the x_i-subsystem (17) degenerates to

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, 0) \\ \dot{x}_2 &= f_2(x_1, x_2, e_{pp}). \end{aligned}$$

For such a case, χ_1 is zero and then $0 \equiv \chi_1 \circ \chi_2(r) < r$ is always satisfied.

Remark 8. In the analysis, we do not need to know the variables $z_{pp}, z_{ps}, y_{pp}, y_{ps}, x_{pp}, x_{ps}$ because they will not appear in both the controllers and the conditions. The decomposition of system (11) is only for analysis.

3.4 Control Integration

So far, we have designed two controllers with respect to the two decomposed subtasks respectively. With *Lemma 1* in hand, we can finally integrate them together to achieve the original control task.

Theorem 2. Under *Assumption 1*, suppose i) the controller for system (1) is designed as

$$\begin{aligned} \dot{z}_p &= h(z_p, \hat{y}_p, r), z_p(0) = 0 \\ u &= u^p(z_p, \hat{y}_p, r) + u^s\left(\hat{x}_s, r, \cdots, r^{(N)}\right) \end{aligned} \quad (23)$$

where $\hat{y}_p$ and $\hat{x}_s$ are obtained from observer (9); ii) the conditions of *Lemma 1* hold. Then the tracking error e is ultimately bounded if e_{pp} is ultimately bounded. In particular, if $e_{pp} \to 0$ as $t \to \infty$, then $e \to 0$ as $t \to \infty$.

Proof. Since $\hat{y}_p$ and $\hat{x}_s$ are obtained from observer (9), $\hat{y}_p \equiv y_p$ and $\hat{x}_s \equiv x_s$ by *Theorem 1*. Incorporating controller (23) into system (1), we can get system (17) by the additive state decomposition. Furthermore, by *Lemma 1*, we can conclude this proof. □

4 An Illustrative Example

Consider the following uncertain nonminimum phase system (1) with

$$\begin{array}{l} A=\left[\begin{array}{cc} 1 & 1 \\ -5 & -4 \end{array}\right], b=c=\left[\begin{array}{c} 0 \\ 1 \end{array}\right], d=\left[\begin{array}{c} 0.5 \\ -0.5 \end{array}\right] \\ \Delta\varphi(x)=\Delta A x, \Delta A=\left[\begin{array}{cc} 0.05 & 0.05 \\ 0.05 & 0.05 \end{array}\right] \\ \phi(y)=\left[\begin{array}{c} \phi_{0,1}(y) \\ 0 \end{array}\right], \phi_{0,1}(y)=\dfrac{y^2}{1+y^2} \end{array}. \tag{24}$$

It is easy to see that *Assumption 1* holds. For the nonlinear function $\phi_{0,1}(y)$, we have the following properties

$$\begin{array}{l} \|\phi_{0,1}(y)\| \leq 1, \phi_{0,1}'(y)=\frac{2y}{(1+y^2)^2} \\ \|\phi_{0,1}'(y)\| \leq \frac{3\sqrt{3}}{8}, \forall y \in \mathbb{R} \end{array}. \tag{25}$$

For simplicity, the objective is to design u to make $y \rightarrow r(t) \equiv 1$. By setting $y=0, u=0$ and $d=0$, the zero dynamics are $\dot{x}_1 = 1.05 x_1$. Therefore, the considered system is a nonminimum phase system. The controller design and analysis are divided into the following five steps.

(1) $e_{pp} \rightarrow 0$ as $t \rightarrow \infty$. The uncertain nonlinear system (1) with the parameters (24) is first additively decomposed into the primary system (6) and the secondary system (7). For the primary system (6), the controller (10) is designed as

$$\begin{array}{l} \dot{z}_p = y_p - r, z_p(0)=0 \\ u_p = 2y_p + z_p. \end{array} \tag{26}$$

It is easy to verify that the system matrix of the closed-loop system (11) is

$$A_a=\left[\begin{array}{cc} 0 & c^T \\ b & A+2bc^T+\Delta A \end{array}\right] \tag{27}$$

which is stable, namely max $\mathrm{Re}\lambda(A_a) < 0$. The system (11) is further decomposed and then rearranged with system (16), resulting in system (17).

Since the system (12) without external signals is exponentially stable, $e_{pp}(t)$ and $z_p(t)$ will tend to constants as $t \rightarrow \infty$ driven by constant signals $r, \phi(r)$ and d. By the integral term, the relationship between e_{pp} and z can be written as

$$e_{pp} = c^T x_p - r = \dot{z} \tag{28}$$

which implies that $e_{pp}(t) = \dot{z}(t) \rightarrow 0$ as $t \rightarrow \infty$ by (28). For the explanation, readers can also refer to [15].

(2) ISS-Lyapunov function V_1. In the following, we will design ISS-Lyapunov functions for system (17) to satisfy equations (20), (21), and (22). In this example, the system (13) can be written as

$$\left[\begin{array}{c} \dot{z}_{ps} \\ \dot{x}_{ps} \end{array}\right] = A_a \left[\begin{array}{c} z_{ps} \\ x_{ps} \end{array}\right] + \left[\begin{array}{c} 0 \\ \Delta A \end{array}\right] x_s. \tag{29}$$

Since max $\mathrm{Re}\lambda(A_a) < 0$, there exist matrices $0 < P, Q \in \mathbb{R}^{3\times 3}$ such that $PA_a + A_a^T P = -Q$. Design a Lyapunov function $V_1 = x_1^T P x_1$, where $x_1 = [z_{ps}\ x_{ps}]^T$. Its derivative along equation (29) is

$$\begin{array}{l} \dot{V}_1 \leq -\left[\underline{\sigma}(Q) - 2\overline{\sigma}(P)\overline{\sigma}(\Delta A)\right]\|x_1\|^2 \\ \quad + 2\overline{\sigma}(P)\overline{\sigma}(\Delta A)\|x_1\|\|x_s\| \\ \quad \leq -\frac{\underline{\sigma}(Q)-2\overline{\sigma}(P)\overline{\sigma}(\Delta A)}{\overline{\sigma}(P)} V_1 \\ \quad + \frac{2\overline{\sigma}(\Delta A)\overline{\sigma}(P)}{\sqrt{\underline{\sigma}(P)}}\sqrt{V_1}\|x_s\| \end{array}. \tag{30}$$

(3) ISS-Lyapunov function V_2. In the following, we will design u_s to make system (17) be ISS. For the secondary system (7), the stabilizing controller is designed by the idea of backstepping technique [13]. The design is omitted for simplicity. Define two new variables as follows

$$\begin{array}{l} z_1 = x_{s,1} \\ z_2 = 2x_{s,1} + x_{s,2} \\ \quad + \phi_{0,1}(r + x_{s,2}) - \phi_{0,1}(r) \end{array} \tag{31}$$

where $x_s = [x_{s,1}\ x_{s,2}]^T$. The controller u_s is designed as follows

$$\begin{array}{l} u_s = 5x_{s,1} + 4x_{s,2} \\ \quad + \frac{1}{1+\phi_{0,1}'(r+x_{s,2})}\left[-2(x_{s,1}+x_{s,2}) + z_1 - 5z_2\right] \end{array} \tag{32}$$

By the controller above, the system (16) is derived as

$$\begin{array}{l} \dot{z}_1 = -z_1 + z_2 + \phi_{0,1}\left(r + c^T x_s + e_{pp} + c^T x_{ps}\right) \\ \quad - \phi_{0,1}\left(r + c^T x_s\right) \\ \dot{z}_2 = -5z_2 + z_1. \end{array} \tag{33}$$

Design a Lyapunov function $V_2 = z_1^2 + z_2^2$, whose derivative along equation (33) is

$$\begin{array}{l} \dot{V}_2 \leq -2z_1^2 - 2z_2^2 \\ \quad - 2z_1\left[\phi_{0,1}\left(r + c^T x_s + e_{pp} + c^T x_{ps}\right) - \phi_{0,1}\left(r + c^T x_s\right)\right] \\ \quad \leq -2z_1^2 - 10z_2^2 + \dfrac{3\sqrt{3}}{4}|z_1|\left(|x_1| + |e_{pp}|\right) \\ \quad \leq -2V_2 + \dfrac{3\sqrt{3}}{4}\sqrt{V_2}\left(\dfrac{\sqrt{V_1}}{\sqrt{\underline{\sigma}(P)}} + |e_{pp}|\right) \end{array} \tag{34}$$

where the properties in equation (25) are used. This implies that

$$\nabla V_2 f_2(x_1, x_2, e_{pp}) \leq -\epsilon V_2, \epsilon > 0 \tag{35}$$

if $V_2 \geq \max\left(\chi_2 V_1, \frac{27}{16(2-\epsilon)^2}|e_{pp}|^2_{[t_0,\infty)}\right)$, where $\chi_2 = \frac{27}{16(2-\epsilon)^2 \underline{\sigma}(P)}$.

(4) Condition $\chi_1\chi_2 < 1$. By the properties in equation (25), the relationship (31) can be written as

$$\begin{array}{l} z = F x_s, \lambda \in [0,1] \\ F = \left[\begin{array}{cc} 1 & 0 \\ 2 & 1 + \phi_{0,1}'(r + \lambda x_{s,2}) \end{array}\right] \end{array}. \tag{36}$$

Since $\left|\phi_{0,1}'(r + \lambda x_{s,2})\right| \leq \frac{3\sqrt{3}}{8} < 1$, the matrix F is nonsingular. So we have $\|x_s\| \leq \frac{1}{\underline{\sigma}(F)}\|z\|$. Substituting it into equation (30) results in

$$\dot{V}_1 \leq -\frac{\underline{\sigma}(Q) - 2\overline{\sigma}(P)\overline{\sigma}(\Delta A)}{\overline{\sigma}(P)} V_1 + \frac{2\overline{\sigma}(\Delta A)\overline{\sigma}(P)}{\sqrt{\underline{\sigma}(P)}\underline{\sigma}(F)} \sqrt{V_1}\sqrt{V_2}. \quad (37)$$

This implies that

$$\nabla V_1 f_1(x_1, x_2) \leq -\epsilon V_1, \epsilon > 0 \quad (38)$$

if $V_1 \geq \chi_1 V_2$, where $\chi_1 = \frac{4\overline{\sigma}(\Delta A)^2 \overline{\sigma}(P)^4}{(1-\epsilon)^2 \underline{\sigma}(P)\underline{\sigma}(F)^2 [\underline{\sigma}(Q) - 2\overline{\sigma}(P)\overline{\sigma}(\Delta A)]^2}$. Note that $e_{pp}(t) \to 0$ as $t \to \infty$ in this example. If $\chi_1\chi_2 < 1$, then $x_1 \to 0$ and $x_s \to 0$ as $t \to \infty$ by *Lemma 1*. This further implies that $y \to r$ as $t \to \infty$. From the inequality above, if the uncertainty $\|\Delta A\|$ is small enough, then $\chi_1\chi_2 < 1$ holds. For this example, $\chi_1\chi_2 < 1$ holds.

(5) Controller Integration. According to the controller (23), the controller in this example is integrated as

$$\begin{aligned} &\dot{z}_p = \hat{y}_p - r, z_p(0) = 0 \\ &u_p = 2\hat{y}_p + z_p + 5\hat{x}_{s,1} + 4\hat{x}_{s,2} \\ &\quad + \frac{1}{1 + \phi'_{0,1}(r + \hat{x}_{s,2})} \left[-2(\hat{x}_{s,1} + \hat{x}_{s,2}) + \hat{z}_1 - 5\hat{z}_2\right] \end{aligned} \quad (39)$$

where $\hat{z}_1 = \hat{x}_{s,1}, \hat{z}_2 = 2\hat{x}_{s,1} + \hat{x}_{s,2} + \phi_{0,1}(r + \hat{x}_{s,2}) - \phi_{0,1}(r)$. The estimate $\hat{y}_p$ and $\hat{x}_s$ are observed by equation (9). Driven by such a controller, the simulation result is shown in Fig.3. As shown, the system output tracks $y_d(t) \equiv 1$ asymptotically, meanwhile keeping the internal state bounded. This is consistent with *Theorem 2*.

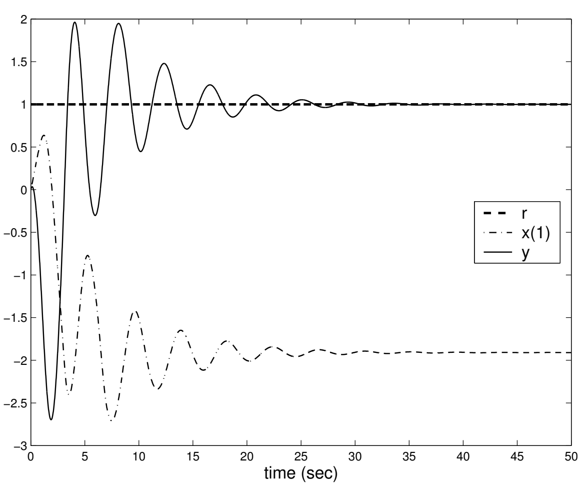

Figure 3: Time response of the nonlinear nonminimum-phase system.

5 CONCLUSIONS

In this paper, the output feedback tracking problem for a class of uncertain nonminimum phase nonlinear systems was solved by the additive-state-decomposition-based tracking control. In comparison to our previous work, the main contribution of this paper is the robustness of the system is increased by employing the proposed controller, that means the system can tolerate the uncertainty which is relevant to the states. In order to suppress this uncertainty, a new additive state decomposition process is given and the controller is designed based on it. Finally, an example with this uncertainty is employed to illustrate the effectiveness of the proposed method. In this paper, only a class of nonlinear nonminimum phase systems is considered. Thus, part of our future work is to investigate more general tracking problems by the additive state decomposition.

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