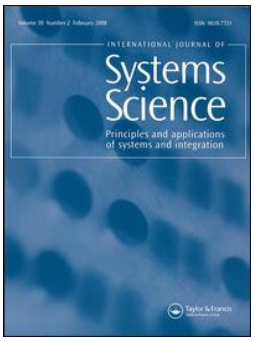

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Transient tracking performance improvement for nonlinear nonminimum phase systems: an additive-state-decomposition-based control method

Jinrui Ren[a], Zhiyu Xi[a], Li-Bing Zhao[b] and Quan Quan[a]

[a]School of Automation Science and Electrical Engineering, Beihang University, Beijing, China; [b]China Academy of Aerospace Standardization and Product Assurance, Beijing, China

ABSTRACT
To study the problem of improving transient tracking performance for nonlinear systems, this paper proposes an additive-state-decomposition-based control method for a class of nonlinear nonminimum phase (NMP) systems. This method 'additively' decomposes the original problem into two more tractable problems, namely a tracking problem for a linear time-invariant NMP 'primary' system and a state stabilisation problem for a certain nonlinear 'secondary' system. Then, controller for each system is designed respectively by employing existing methods, i.e. the linear quadratic regulator (LQR) method for the primary system and the backstepping method for the secondary system. Next, these two controllers are combined together to achieve the original tracking goal. Furthermore, the adjustment of weighting matrix Q in the LQR regulates the transient response of the closed-loop nonlinear system. Finally, two illustrative examples are provided to demonstrate the effectiveness of the proposed method.

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Transient performance; nonlinear nonminimum phase systems; tracking control; additive state decomposition; LQR

1. Introduction

Tracking control problem has been extensively investigated (Isidori, 2013; Xu, Yang, & Pan, 2015). In an output tracking controller design, time-domain indices mainly include three aspects: stability, steady-state error and transient response. Among the three aspects, the basic research around stability and steady-state error has been conducted widely. However, the more advanced control problem about transient response has received relatively little attention. In general, the transient performance of a control system consists of overshoot, undershoot, convergence speed, monotonicity and oscillation, which directly affect the quality of task performing and operation safety. Unsatisfactory transient performance will degrade the system efficiency, or even lead to failures and safety issues. Therefore, it is very meaningful to study how to improve the system transient performance, especially for nonlinear nonminimum phase (NMP) systems, whose transient response is restricted by NMP behaviour.

The objective of transient control is to achieve transient response specifications under the premise of guaranteeing that stability and steady-state error meet requirements. Representative methods investigating transient performance improvement can be generally divided into five classes. The first class of method is to construct the structure of closed-loop systems, which mainly consists of pole-zero configuration (Schmid & Ntogramatzidis, 2012), and eigenstructure assignment (Schmid & Ntogramatzidis, 2010). A linear multivariable feedback controller is designed by pole-zero configuration to achieve a nonovershooting and nonundershooting step response in Schmid and Ntogramatzidis (2012). This kind of method is based on the relationship between system structure and transient performance. There are many relevant findings for linear systems, but few results exist for nonlinear systems because of the complex nonlinear system structure. The second method is linear quadratic regulator (LQR) (Li & Wang, 2010; Liao & Xu, 2015), which pushes system outputs track reference signals in an optimal way. Substantially, it is an optimal way to find an appropriate state-feedback controller. By adjusting weighting matrices in the cost function, one can get satisfying time-domain transient response. This method is simple as well as robust. Li and Wang (2010) show that LQR is capable of improving transient response which includes short rise time and small overshoot in the track-seeking of hard disk drive servo systems. Unfortunately, LQR is just designed by using linear state-space models. For a nonlinear system, there are two ways to utilise LQR. The first one is to linearise the nonlinear system (Belkheiri, Rabhi, Hajjaji, & Pegard, 2012), and the second one is to combine nonlinear open-loop control with LQR feedback control. A third method is composite nonlinear feedback (CNF) (Wang & Zhao, 2016; Zhang, Huang, Wang, Li, & Peng, 2012), which is a nonlinear technique to improve the transient performance for tracking control problem of linear systems with input saturation. It involves a linear feedback part and a nonlinear feedback part without any switching elements. The linear part builds a closed-loop system with a small damping ratio to obtain a quick response, and the nonlinear part increases the damping ratio of the closed-loop system when the system output approaches the reference signal to obtain a small overshoot. The desired transient performance with quick response and small overshoot is achieved simultaneously in Wang and Zhao (2016). The fourth method is switching control scheme (Beker, Hollot, & Chait, 2001; Zhu & Cai, 2012), which includes

CONTACT Jinrui Ren renjinrui@buaa.edu.cn

gain scheduling, gain switching and reset control, etc. Switching control can overcome the drawbacks of conventional smooth feedback control. It allows for the design of suitable controllers that autonomously adjust themselves to modifications of the desired operating conditions or to varying external inputs. A simple switching control scheme is utilised to avoid overshoot in Zhu and Cai (2012). However, switching control may lead to discontinuous or rough control signals, which may give rise to chattering effect. The fifth method is intelligence techniques, such as neural networks, fuzzy logic theory and evolutionary computational techniques. They are often incorporated into adaptive control or optimal control to approximate unknown information or seek optimal parameters, and finally improve the transient response (Chen, Liu, & Wen, 2014; Liu, Gao, Tong, & Li, 2016).

On the whole, in contrast with linear systems, transient performance improvement for nonlinear systems is much harder and relevant research is rare and complicated. What is worse, engineering implementations of most methods are also difficult. On the contrary, a large number of methods can be used to improve transient performance for linear systems, and related controller designs and implementations are simple. However, it is unfortunate that these mature methods for linear systems generally cannot apply to nonlinear systems directly. So, a thinking arises that: is it possible to use existing control methods for linear systems to improve the corresponding transient performance for nonlinear systems? If the answer is yes, it will provide a new way to improve transient performance for nonlinear systems and it will be convenient to use familiar methods for linear systems to handle nonlinear problems. Attempting to answer this question, the problem of improving transient tracking performance for a class of nonlinear NMP systems subject to unknown disturbances and whose state couples with output is studied in this paper. For this problem, an additive-state-decomposition-based control (ASDBC) is proposed. The technique is based on *additive state decomposition* (*ASD*) (Quan, Cai, & Lin, 2015; Quan, Du, & Cai, 2016) of the given nonlinear NMP system. The original nonlinear system is decomposed into a 'primary' LTI NMP system including all the disturbances and a 'secondary' nonlinear system with zero initial state and no disturbance. The primary linear system describes the dynamics of the original system in the neighbourhood of the desired output. Then, two different controllers are designed, one for each system. The linear one aims at achieving the tracking goal through LQR optimal state feedback augmented with an integral part, and the nonlinear one achieves just the stabilisation by the backstepping control (BC) method (Isidori, 2013). What is more, the transient response is regulated by the adjustment of the weighting matrices in the LQR. This work is quite different from the past ASD-based control in three aspects: different problem, different decomposition procedure and different design method.

The rest of this paper is organised as follows. In Section 2, the problem formulation is given, and ASD is introduced briefly. The additive-state-decomposition-based tracking control is presented in Section 3. In Section 4, two illustrative examples are given and related simulations are carried out. Section 5 concludes this paper.

2. Problem formulation and additive state decomposition

2.1. Problem formulation

Consider a class of nonlinear NMP systems as follows:

$$\begin{aligned} \dot{x}(t) &= A\left(y(t)\right)x(t) + Bu(t) + d(t) \\ y(t) &= Cx(t) + Du(t),\ x(0) = x_0, \end{aligned} \tag{1}$$

where $A:\mathbb{R}^m \to \mathbb{R}^{n\times n}$ is a known nonlinear function matrix, $B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$ are known constant matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^m$ is the output, $u(t) \in \mathbb{R}^m$ is the input, and $d(t) \in \mathbb{R}^n$ is an unknown bounded disturbance vector. The reference output is a known constant vector, denoted by $y_r \in \mathbb{R}^m$. For simplicity, denote $A_r = A(y_r)$, and the variable t will be omitted except when necessary hereinafter. For simplicity, the following two assumptions on system (1) are made.

Assumption 2.1: *The pair* (A_r, B) *is controllable.*

Assumption 2.2: *The state* x *is available.*

Remark 2.1: Assumption 2.1 can be determined by $\mathrm{rank}(B, A_rB, \ldots, A_r^{n-1}B) = n$. Then, since the pair (A_r, B) is controllable under Assumption 2.1, there always exists a matrix $K \in \mathbb{R}^{m\times n}$ such that $A_r + BK$ is stable with eigenvalues which can be assigned freely. Therefore, without loss of generality, it can be assumed that A_r is stable. The following design is based on a stable A_r. Assumption 2.2 indicates that the state can be measured. Assumptions 2.1 and 2.2 are very common, and the practicability of the research will not be affected.

Remark 2.2: It should be noticed that the input and output have the same dimensions. Otherwise, tracking may not be achieved. The special system structure is mainly the result of the fact that the state couples with the output in the form of $A(y)$, which is a nonlinear function of y and has effect on the later ASD. If $A(y)$ is replaced by $A(x)$ in the system model, then A_r is unobtainable because of the unknown x_r, which is the value of x when $y = y_r$.

Remark 2.3: By linearising system (1) around the steady state of the reference, $A_{L,r}$ is obtained, and the transfer function from u to y becomes

$$W(s) = C\left(sI_n - A_{L,r}\right)^{-1}B + D,$$

which has zeros in the right-half s-plane. If the linearised system is NMP, then the original nonlinear system is NMP. Therefore, system (1) is an NMP system.

Under Assumptions 2.1 and 2.2, the objectives in this paper are (i) to design a tracking controller u for system (1) such that $y - y_r \to 0$ as $t \to \infty$; (ii) to improve the transient tracking performance of the closed-loop system.

2.2. Additive state decomposition

The following controller design is grounded in additive state decomposition, which is a method to decompose a system. In order to make this paper self-contained, the additive state decomposition (Quan et al., 2015) is recalled here briefly. A class

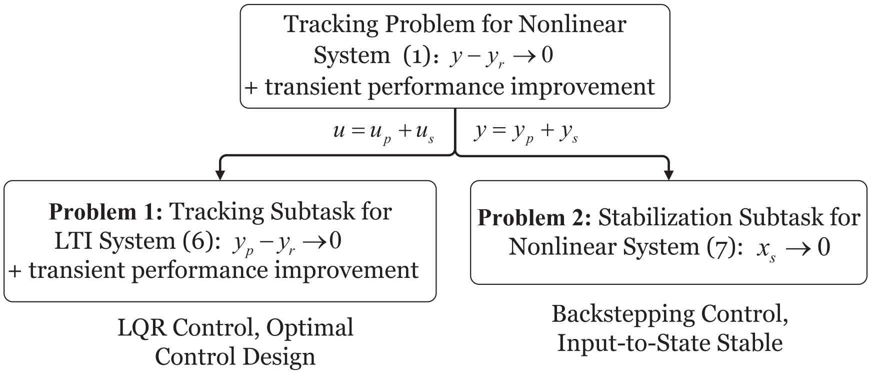

Figure 1. Additive state decomposition.

of differential dynamic systems, which can be viewed as the original system, is considered as follows:

$$\begin{aligned} \dot{x} &= f(t, x, u), \\ y &= h(t, x, u), x(0) = x_0, \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ and $y, u \in \mathbb{R}^m$. Two systems with the same dimensions as the original system, denoted by the primary system and secondary system, respectively, are defined as follows:

$$\begin{aligned} \dot{x}_p &= f_p(t, x_p, u_p), \\ y_p &= h_p(t, x_p, u_p), x_p(0) = x_{p,0}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{x}_s &= f(t, x_p + x_s, u_p + u_s) - f_p(t, x_p, u_p) \\ y_s &= h(t, x_p + x_s, u_p + u_s) - h_p(t, x_p, u_p), x_s(0) = x_0 - x_{p,0}, \end{aligned} \tag{4}$$

where $x_s \triangleq x - x_p, y_s \triangleq y - y_p, u_s \triangleq u - u_p, f_p$ and h_p are abstract functions which can be designed according to the original system and control problem. The secondary system (4) is determined by subtracting the primary system (3) from the original system (2). According to (2)–(4), one has

$$x = x_p + x_s, y = y_p + y_s \text{ and } u = u_p + u_s. \tag{5}$$

3. Additive-state-decomposition-based tracking control

First, based on ASD, system (1) is decomposed into two systems: an LTI system (6) including all external disturbances as the primary system, and a nonlinear secondary system (7) whose equilibrium point is the origin. Because the state and output of the two subsystems can be observed, the original tracking problem for system (1) is correspondingly decomposed into two problems: a tracking problem for the LTI primary system and a stabilisation problem for the nonlinear secondary system.

3.1. Decomposition

Consider system (1) as the original system. According to the principle mentioned in (3), the primary system is designed as follows:

$$\begin{aligned} \dot{x}_p &= A_r x_p + B u_p + d, \\ y_p &= C x_p + D u_p, x_p(0) = x_0. \end{aligned} \tag{6}$$

Then, the secondary system is determined by subtracting the primary system (6) from the original system (1) with rule (4)

$$\begin{aligned} \dot{x}_s &= A_r x_s + A\left(y_p + y_s\right)\left(x_p + x_s\right) - A_r\left(x_p + x_s\right) + B u_s, \\ y_s &= C x_s + D u_s, x_s(0) = 0, \end{aligned} \tag{7}$$

which is further written as

$$\begin{aligned} \dot{x}_s &= A_r x_s + \left(A\left(y_r + y_s + e_p\right) - A_r\right) x + B u_s, \\ y_s &= C x_s + D u_s, x_s(0) = 0, \end{aligned} \tag{8}$$

where $e_p = y_p - y_r$. If $e_p \equiv 0$, then $(x_s, u_s) = 0$ is an equilibrium point of (8). It is obvious that the unknown disturbance affects only the primary system, leaving the secondary system deterministic.

Controller design for the decomposed systems (6) and (7) will use their states and outputs as feedback variables. However, they are virtual variables, which are unknown. For such a reason, an observer is proposed in Theorem 3.1 to estimate x_s, x_p and y_p.

Theorem 3.1: *Suppose that an observer is designed to estimate x_s, x_p and y_p in (6) and (7) as follows:*

$$\dot{\hat{x}}_s = A_r \hat{x}_s + A\left(y\right) x - A_r x + B u_s, \hat{x}_s(0) = 0 \tag{9}$$

$$\hat{x}_p = x - \hat{x}_s \tag{10}$$

$$\hat{y}_p = C \hat{x}_p + D u_p. \tag{11}$$

Then, $\hat{x}_s \equiv x_s$, $\hat{x}_p \equiv x_p$ and $\hat{y}_p \equiv y_p$.

Proof: Subtracting (9) from (7) results in $\dot{\tilde{x}}_s = A_r \tilde{x}_s$, $\tilde{x}_s(0) = 0$, where $\tilde{x}_s \triangleq x_s - \hat{x}_s$. Because A_r is stable, $\tilde{x}_s \equiv 0$. This implies that $\hat{x}_s \equiv x_s$. Consequently, it can be obtained that $\hat{x}_p \equiv x - \hat{x}_s \equiv x_p$. Additionally, $\hat{y}_p = C\hat{x}_p + Du_p = y_p$. ■

It is clear from (6) and (7) that if the controller u_p drives $y_p - y_r \to 0$ and the controller u_s drives $x_s \to 0$ as $t \to \infty$, then $y - y_r \to 0$ as $t \to \infty$. The strategy here is to assign the tracking subtask to the primary system (6) and the stabilisation subtask to the secondary system (7), which is shown in Figure 1. Since (6) is a classical LTI system, standard design methods in both frequency domain and time domain are applicable to the tracking problem. This is easier than dealing with the tracking problem for the nonlinear system (1) directly. On the other hand, the stabilising controller for the nonlinear system (7) can be designed by BC. According to these, ASD offers a way to simplify the original control problem.

Furthermore, because the secondary system is just a system with zero initial state and zero equilibrium point to be stabilised, the tracking performance mainly depends on the primary LTI system, to which the tracking task is assigned. Thus, the task of transient response improvement is mainly allocated to the primary system, to which LQR is introduced to achieve the tracking objective and adjust the transient response simultaneously. The transient performance adjustment process will be conducted in the latter simulation.

3.2. Controller design

So far, the considered system has been decomposed into two subsystems in charge of corresponding subtasks. The controller design for each subtask is proposed in the form of a problem. The two designed controllers are then combined together to achieve the original control target for system (1).

Problem 3.1: *For (6), design an LQR-based tracking controller*

$$u_p = u_p\left(x_p, \int_0^t \left(y_p(s) - y_r\right) \mathrm{d}s\right) \tag{12}$$

such that $e_p = y_p - y_r \to 0$as $t \to \infty$, meanwhile keeping $x_p$bounded.

Intuitively, to remove the tracking error, an integral action must be employed in the controller. Because y_r is a known constant vector, $\dot{y}_r = 0$. Then, the derivative of e_p is

$$\dot{e}_p = C\dot{x}_p + D\dot{u}_p. \tag{13}$$

Similarly, when d is constant, $\dot{d} = 0$. Differentiating Equation (6), one has

$$\frac{d}{dt}\dot{x}_p = A_r\dot{x}_p + B\dot{u}_p. \tag{14}$$

By combining (13) with (14), the manipulated augmented system of (6) is

$$\underbrace{\frac{d}{dt}\begin{bmatrix} e_p \\ \dot{x}_p \end{bmatrix}}_{\dot{X}_a} = \underbrace{\begin{bmatrix} 0 & C \\ 0 & A_r \end{bmatrix}}_{A_a}\underbrace{\begin{bmatrix} e_p \\ \dot{x}_p \end{bmatrix}}_{X_a} + \underbrace{\begin{bmatrix} D \\ B \end{bmatrix}}_{B_a}\underbrace{\dot{u}_p}_{u_a}. \tag{15}$$

According to Ogata (2001), for the given system

$$\dot{X}_a = A_aX_a + B_au_a,$$

an LQR stabilising controller can be designed as

$$u_a = -G_aX_a, \tag{16}$$

where $G_a \in \mathbb{R}^{m\times(m+n)}$ is a constant feedback gain matrix. Then, the tracking controller for the primary system (6) is designed as

$$u_p = \int_0^t u_a(s)\, ds = -G_a\int_0^t X_a(s)\, ds. \tag{17}$$

Let $G_a = [\, G_{a_1}\ G_{a_2}\,]$, where $G_{a_1} \in \mathbb{R}^{m\times m}$, $G_{a_2} \in \mathbb{R}^{m\times n}$. Controller (17) can be further written as

$$\begin{aligned} &u_p\left(x_p, \int_0^t \left(y_p(s) - y_r\right) \mathrm{d}s\right) \\ &= -G_{a_1}\int_0^t \left(y_p(s) - y_r\right) \mathrm{d}s - G_{a_2}x_p. \end{aligned} \tag{18}$$

Theorem 3.2: *Under Assumptions 2.1 and 2.2, for (6), suppose (i) d is constant; (ii) u_p is designed as (18). Then, $y_p - y_r \to 0$ as $t \to \infty$ and x_p is bounded.*

Proof: It is well known that the LQR controller (16) can guarantee that $X_a \to 0$ as $t \to \infty$ (Ogata, 2001), which means $e_p \to 0$ and $\dot{x}_p \to 0$ as $t \to \infty$. Thus, $y_p - y_r \to 0$ as $t \to \infty$ and x_p tends to be constant. Furthermore, $x_p(0) = x_0$ is bounded, so x_p is bounded. ■

Remark 3.1: The stability conditions of system (15) together with controller (17) are: (i) (A_a, B_a) is completely controllable or at least stabilisable; (ii) $Q > 0$, $R > 0$ or $Q \geq 0$, $R > 0$, (A_a, S) is completely observable or at least detectable, where $SS^T = Q$. Condition (i) can guarantee the performance index is finite, and condition (ii) can guarantee the optimal feedback system is asymptotically stable.

Remark 3.2: It should be noticed that the constant d is very common in practical systems. Furthermore, if d is a slow-varying signal, which can be described as a random walk signal, then the Linear Quadratic Gaussian (LQG) control method can be used to replace LQR.

Remark 3.3: In order to use mature LQR method in nonlinear systems, most existing literature is based on models obtained by approximate linearisation in given flight conditions or feedback linearisation technique (Belkheiri et al., 2012). However, approximate linearisation is valid only locally and often leads to worse control effect because of ignoring the nonlinearity directly. Feedback linearisation requires accurate nonlinear models, which are difficult to obtain in practice. Besides, it may cause singularity problem (Ren & Quan, 2016). In our ASD-based method, the nonlinearity of the original system is compensated by the secondary system and the remaining primary system is an LTI system, to which LQR can apply and perform well. ASD can be considered as a novel kind of linearisation method, which can avoid the drawbacks of the existing two linearisation methods.

Problem 3.2: *For (7) (or (8)), design a controller*

$$u_s = u_s\left(x_s, y_r\right) \tag{19}$$

such that (i) the closed-loop system is input-to-state stable with respect to the input e_p, namely

$$\|x_s(t)\| \leq \gamma\left(\sup_{t_0 \leq s \leq t} \left\|e_p(s)\right\|\right), t \geq t_0, \tag{20}$$

where γ is a class $\mathcal{K}$function (Khalil & Grizzle, 1996)*; or (ii) the closed-loop system is asymptotically stable as $e_p \to 0$.*

Remark 3.4: Suppose e_p is nonvanishing, inequality (20) will guarantee that for any bounded input $e_p(t)$, the state $x_s(t)$ will be bounded. Furthermore, the state $x_s(t)$ will be ultimately bounded by a class $\mathcal{K}$ function of $\sup_{t\geq t_0}\left\|e_p(t)\right\|$, then Problem 3.2 is a classical input-to-state stability problem. Khalil and Grizzle (1996) provide methods about how to design a controller satisfying input-to-state stability or ways to prove that the designed controller ensures input-to-state stability. Especially, if $e_p \to 0$ as $t \to \infty$, then $x_s \to 0$ as $t \to \infty$.

With the solutions to the two problems in hand, one is ready to claim

Theorem 3.3: *Under Assumptions 2.1 and 2.2, suppose (i) Problems 3.1 and 3.2 are solved; (ii) $\|u_s(x_s, y_r)\| \leq k_s\|x_s\|$, $k_s > 0$; (iii) the controller for system (1) is designed as*

Observer:

$$\begin{aligned}
\dot{\hat{x}}_s &= A_r\hat{x}_s + A\left(y\right)x - A_r x + Bu_s,\ \hat{x}_s(0) = 0\\
\hat{x}_p &= x - \hat{x}_s\\
\hat{y}_p &= C\hat{x}_p + Du_p
\end{aligned} \tag{21}$$

Controller:

$$u = u_p\left(\hat{x}_p, \int_0^t \left(\hat{y}_p(s) - y_r\right) ds\right) u_p + u_s\left(\hat{x}_s, y_r\right). \tag{22}$$

Then, the output of system (1) satisfies $y - y_r \to 0$ as $t \to \infty$.

Proof: See Appendix 1. ■

4. Two illustrative examples

To show the effectiveness of the methods proposed above, ASDBC is applied to two control objects: a practical one of Bank-to-Turn (BTT) aerial vehicle and another strongly nonlinear process.

4.1. Example 1: BTT

4.1.1. System model

The dynamic model of a BTT aerial vehicle in a flight condition of Mach 2 and 30,000 ft is given by Schumacher (1994). This model comprised six state variables $x = [\alpha\ \beta\ \phi\ p\ q\ r]^T$, three control inputs $\delta = [\delta_p\ \delta_q\ \delta_r]^T$ and three control outputs $y = [\phi\ A_y\ A_z]^T$, where $\alpha \in \mathbb{R}$ is the angle of attack, $\beta \in \mathbb{R}$ is the sideslip angle, $\phi \in \mathbb{R}$ is the roll angle, $p \in \mathbb{R}$ is the roll rate, $q \in \mathbb{R}$ is the pitch rate and $r \in \mathbb{R}$ is the yaw rate. The inputs δ_p, δ_q, $\delta_r \in \mathbb{R}$ represent three control surface deflections that influence the roll, pitch and yaw moments, respectively. The outputs A_y and A_z represent the pitch and yaw acceleration of the vehicle. In order to facilitate the controller design, the full model is simplified as

$$\begin{aligned}
\dot{x} &= A_0\left(y\right)x + B_0\delta + d,\\
y &= C_0x + D_0\delta,\ x(0) = x_0,
\end{aligned} \tag{23}$$

where x_0 is the initial state and all the simplified and ignored quantities in the full model are lumped into the unknown disturbance vector d, which is assumed to be constant. For more information about this model, please refer to Appendix 2.

Remark 4.1: As shown in Appendix 2, A_0 is with respect to p. Although p is not an explicit output, considering the relationship that $\dot{\phi} = p$, it can also be regarded as an output variable. Thus, the state couples with the output in the form of $A_0(y)$.

The objective here is to design a tracking controller δ based on the simplified model (23) such that $y - y_r \to 0$ as $t \to \infty$, where y_r is constant. The concrete controller design is put into Appendix 3. Finally, the overall closed-loop system is depicted in Figure 2 .

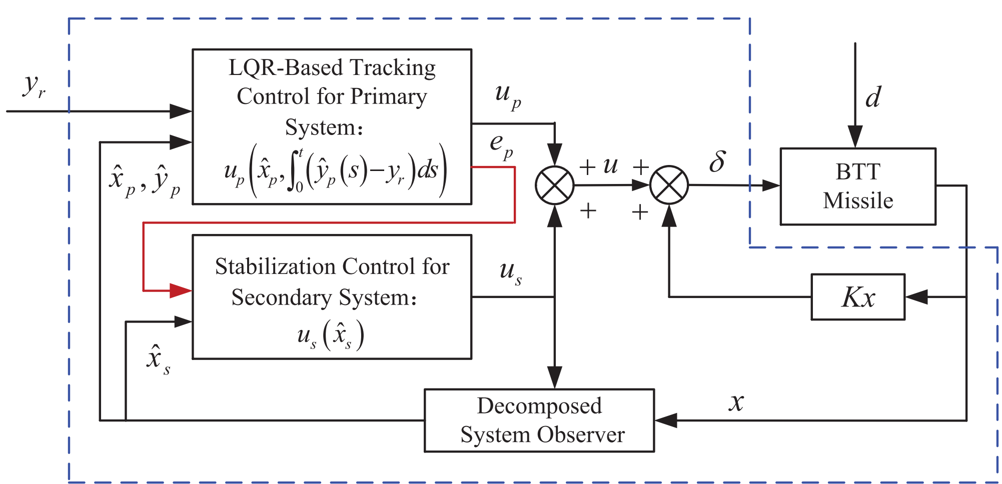

Figure 2. The overall closed-loop system of Bank-to-Turn (BTT).

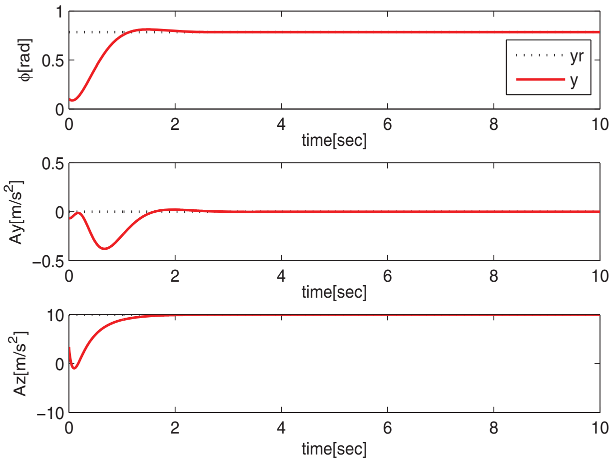

Figure 3. The tracking responses of ϕ, A_y and A_z.

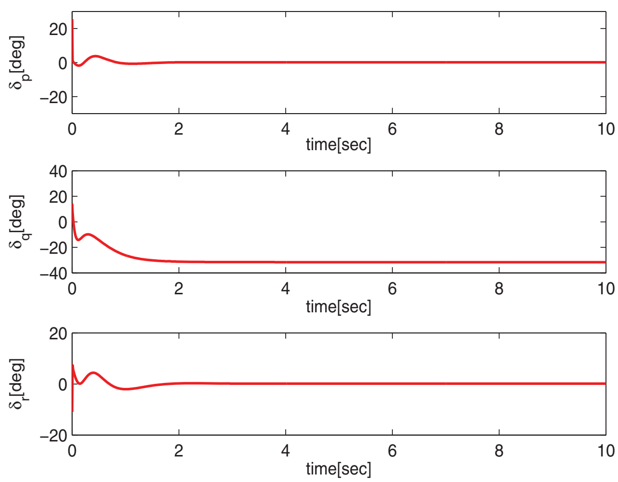

Figure 4. Control inputs.

4.1.2. Simulation results

Let the reference signal be $y_r = [45^\circ\ 0\,\mathrm{m/s^2}\ 10\,\mathrm{m/s^2}]^T$ and the initial states be $\alpha_0 = 1.2^\circ$, $\beta_0 = 0.17^\circ$, $\phi_0 = 5.73^\circ$, $p_0 = 0^\circ$/s, $q_0 = 0^\circ$/s, $r_0 = 0^\circ$/s. The disturbance vector is given as $d = [0.01, 0.01, 0, 0.01, 0.01, 0.01]^T$ rad/s.

The tracking responses are presented in Figure 3, and the corresponding control inputs are presented in Figure 4. As shown, ASDBC provides fine tracking and satisfactory transient performance. The system responses for the primary and secondary systems are depicted separately in Figure 5, which indicate that the primary system takes charge of the main tracking task and dominates the transient tracking response of the closed-loop system.

In order to test the inherent robustness of the design, additional simulations are carried out in the following three situations: (1) with constant and time-varying disturbance simultaneously; (2) with system parameter perturbation; (3) with input time delay. Selecting $d_1 = ([0.5, 0.5, 0, 0.5, 0.5, 0.5]^T + [0.1\sin t, 0.05\sin t, 0, 0.1\cos t, 0.05\cos t, 0.2\exp(-t)]^T)$ rad/s, the corresponding system response is presented in Figure 6, which is satisfying. From Figure 7, when the aerodynamic coefficient C_{m_α} is perturbed by 20%, the tracking effect is nearly unchanged. As shown in Figure 8, when adding time delay $\tau = 0.015$ in the input channel, through changing R to $10I_3$, the simulation result is still good.

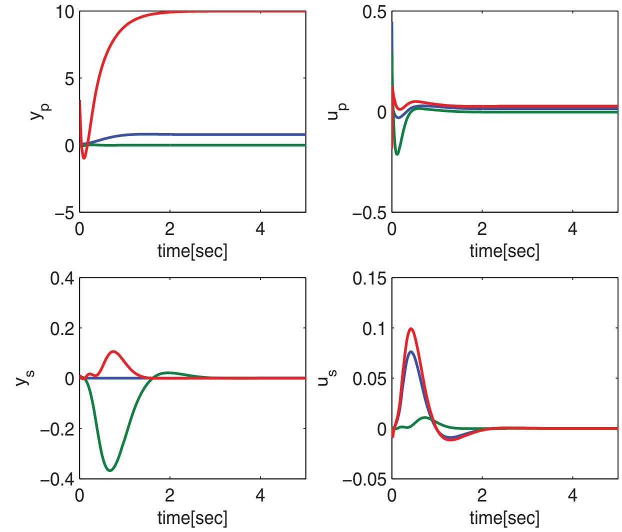

Figure 5. The system responses for primary and secondary systems.

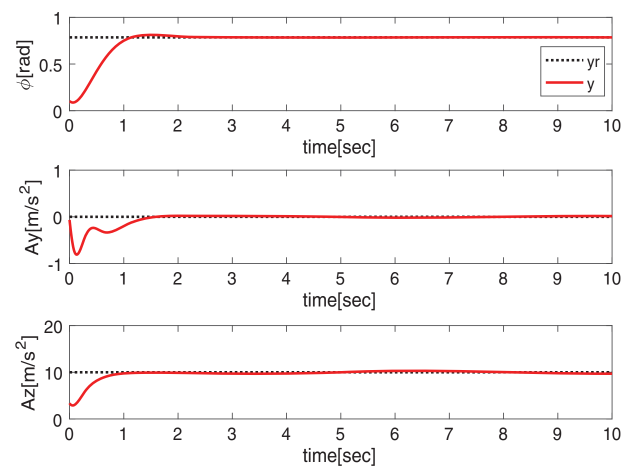

Figure 6. The tracking response with constant and time-varying disturbance simultaneously.

Here, the transient performance adjustment is conducted through simulation. In general, when a certain diagonal element in Q increases, the corresponding state variable converges more quickly. When a certain diagonal element in R increases, the corresponding input variable converges more quickly. In order to analyse the influence of weighting matrices Q and R on output transient response, the following six cases (Cases 1–6) are studied: $Q_1 = I_9$, $Q_2 = diag(10I_3, I_6)$, $Q_3 = diag(100I_3, I_6)$, $R = I_3$; $Q = diag([100, 100, 0.5, 1, 1, 1, 1, 1, 1])$, $R_1 = I_3$, $R_2 = 10I_3$, $R_3 = 100I_3$. Indeed, according to Figure 9, as the diagonal elements in Q increase, the overshoot range is extended, and the rise time and settling time are reduced. It can be seen from Figure 10 that the change of the weighting matrix R impacts little on the output transient behaviour.

Remark 4.2: The effect caused by adjusting Q and R here is exactly consistent with that of linear systems. Therefore, the primary system is indeed a linear system, and LQR method can improve the transient response of the BTT aerial vehicle. Apart from LQR, any other existing methods to improve transient performance of linear systems, for example pole assignment, can be adopted for the primary system, and finally improve the

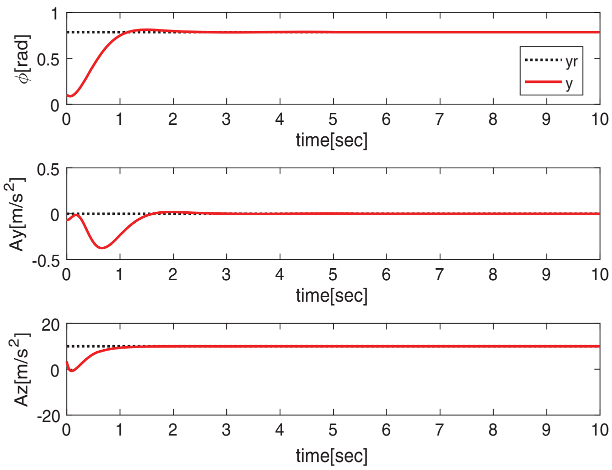

Figure 7. The tracking response with parameter perturbation.

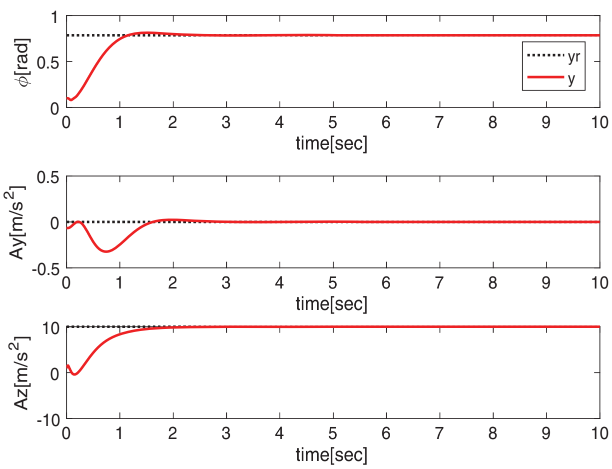

Figure 8. The tracking response with input time delay.

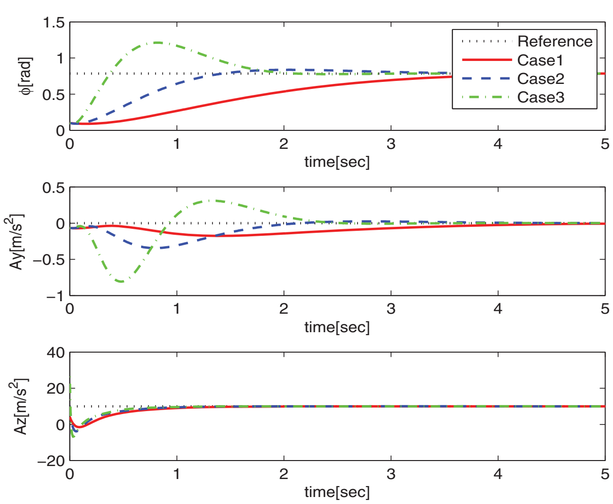

Figure 9. The tracking responses of ϕ, A_y and A_z affected by weighting matrix Q.

transient behaviour of the original nonlinear system. As for CNF and intelligence techniques, their corresponding controllers are somewhat complex, and it is difficult to prove the system stability. What is worse, long learning time is often needed. As for switch control, the discontinuous or rough control signals may give rise to chattering effect.

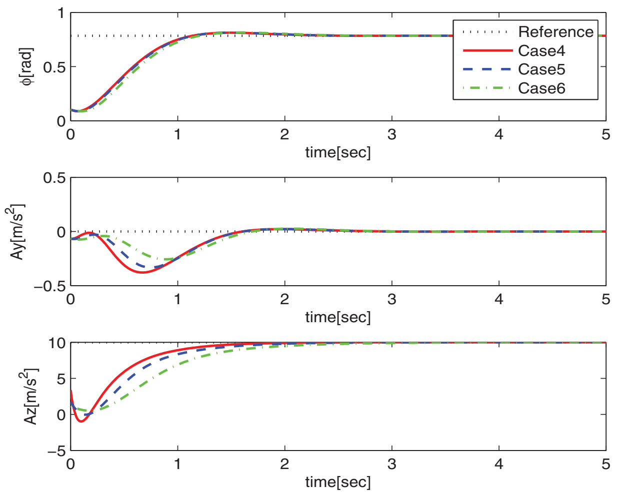

Figure 10. The tracking responses of ϕ, A_y and A_z affected by weighting matrix R.

4.2. Example 2: A strongly nonlinear system

4.2.1. System model

A strongly nonlinear NMP system is also considered here

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= x_2 + x_2^2 + \upsilon, x(0) = x_0 \end{aligned} \tag{24}$$

where $x_1, x_2 \in \mathbb{R}$ are the states, $\upsilon \in \mathbb{R}$ is the input.

The objective here is to design a stabilising controller for (24) such that $x_1, x_2 \to 0$ as $t \to \infty$, which can be viewed as a special case of tracking control. The ASDBC design process is the same as that of Example 1, hence omitted here. To highlight the superiority of ASDBC further, a competitive controller is designed in parallel, which is denoted as direct linearisation-based control (DLBC).

4.2.2. DLBC

First, system (24) is directly linearised at the origin as

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2, \\ \dot{x}_2 &= x_2 + \upsilon. \end{aligned} \tag{25}$$

Then, an LQR-based stabilising controller is also designed as

$$\upsilon = -G_a x.$$

4.2.3. Simulation results

For fair comparison, the parameters in LQR are selected as the same values for ASDBC and DLBC. Let $x_0 = [3\ 3]^T$, $c_1 = c_2 = 10$, and the following three cases (Cases i–iii) are studied: $Q_1 = 100I_2$, $Q_2 = 10I_2$, $Q_3 = I_2$, $R = 1$. The corresponding state response is depicted in Figures 11, 12 and 13, respectively. As shown, in Cases i and ii, although the undershoot of x_2 in ASDBC is a bit larger, ASDBC has a higher state convergence rate than DLBC. And, in Case iii, ASDBC guarantees the system stability when DLBC cannot. Furthermore, when the initial state is changed to $x_0 = [4\ 4]^T$ in Case ii, which is denoted as Case iv, it can be found from Figure 14 that system divergence occurs for DLBC, while ASDBC still performs well.

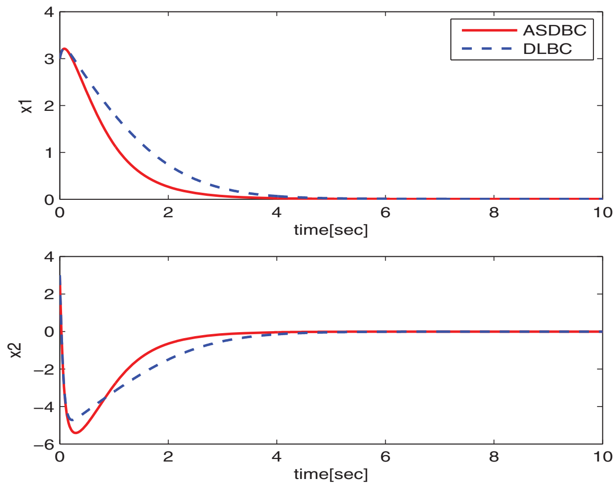

Figure 11. The system state of Case i.

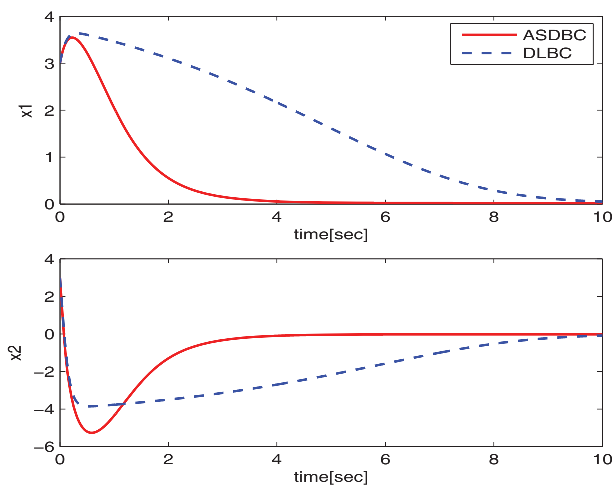

Figure 12. The system state of Case ii.

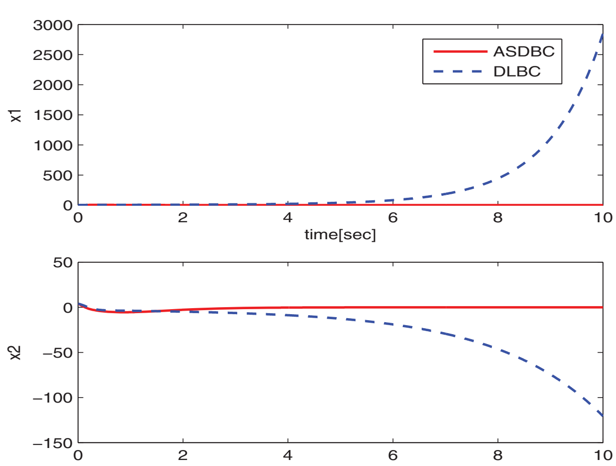

Figure 13. The system state of Case iii.

Remark 4.3: For this strongly nonlinear NMP system, the nonlinear term x_2^2 also plays an important role in the transient process, which should be considered with the aim to improve the transient performance. DLBC adopts the conventional approximate linearisation method, which directly ignores x_2^2 here, while ASDBC uses ASD to implement linearisation, which considers

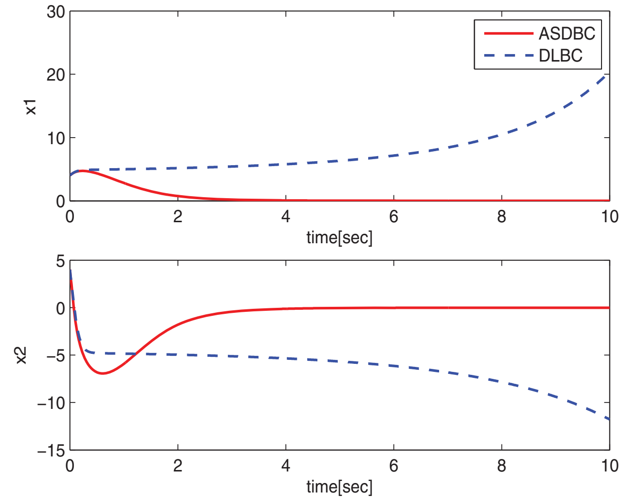

Figure 14. The system state of Case iv.

x_2^2 in the secondary system (see Remark 3.3 for details). Thus, ASDBC shows better transient response than DLBC by faster convergence and better stability. On the basis of linearity of the primary system, LQR method can regulate the transient performance further just like that in Example 1.

4.3. *Discussion*

In Example 1, it is verified that tracking and transient performance mainly depend on that of the primary system. Moreover, adjusting the weighting matrix Q in LQR improves the transient response of the closed-loop nonlinear system, which answers the question proposed in Section 1, namely ASDBC makes it possible to use existing control methods (LQR here) for linear systems to improve the corresponding transient performance for nonlinear systems. In Example 2, compared to DLBC, ASDBC is more powerful in terms of the system stability, fine tracking and transient response. The parameter stability region of ASDBC is larger than that of DLBC, and the selection of the initial state is more limited in DLBC. These are substantially because of ASDBC taking nonlinearity into consideration. On the whole, the first point is that adjusting the weighting matrix Q in LQR can regulate the transient response of the closed-loop nonlinear system. The second point is that nonlinearity is considered in linearisation by ASD, which also improves the transient performance, especially for strongly nonlinear system.

5. Conclusions

In this paper, the tracking control for a class of nonlinear NMP systems is solved by ASDBC, and the transient performance is improved by LQR. Two examples are provided to show the concrete design procedure and simulation results. ASDBC outperforms the direct linearisation and the methods that pay little attention to regulating the transient response. The contributions of this paper come from three aspects: (1) an ASD-based controller design method is proposed to solve the tracking control for a class of nonlinear NMP systems. Following ASD, the primary and secondary systems are controlled separately, which simplifies the design and also increases the flexibility of controller design; (2) through ASD, the nonlinearity of the original system is compensated by the secondary system and the remaining primary system is an LTI system, to which LQR can apply. ASD can be viewed as a novel kind of linearisation method, which can avoid the drawbacks of the existing two linearisation methods; (3) the transient performance of tracking process is also taken into consideration. LQR for linear systems can be easily incorporated into the proposed control framework and has a great effect on the transient behaviour of the closed-loop nonlinear system. Based on this framework, some other control methods that are applicable to improve the transient performance for linear systems can also be employed to solve the corresponding problem for nonlinear systems.

The transient performance improvement for linear systems has been studied for many years. There are many results, which also can be a valuable asset for nonlinear systems. It is beneficial to introduce these valuable achievements into nonlinear field through the ASD method. ASD establishes a bridge from the mature methods that can improve transient performance for linear systems to nonlinear systems. In the future research, some other kinds of nonlinear systems and some other control methods could also be considered. Based on the work in this paper, transient performance improvement for nonlinear systems will be studied in a more strictly mathematical way in the next step

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Notes on contributors

Jinrui Ren received the B.S. degree from Northwestern Polytechnical University, Xi'an, China, in 2014. Currently, She is a Ph.D. candidate of School of Automation Science and Electrical Engineering at Beihang University, Beijing, China. Her main research interests include nonlinear control, flight control, and aerial refueling.

Zhiyu Xi obtained her M.Eng. and Ph.D degrees from the University of New South Wales, Australia in 2007 and 2011 respectively. She was a Postdoc Teaching Fellow/Associate Lecturer at School of Electrical Engineering & Telecommunications, University of New South Wales, Australia, 2011 – 2014. She has been with the School of Automation Science and Electrical Engineering, Beihang University, China since 2015. Her research interest covers sliding mode control, fuzzy control systems, multi-agent systems and networked control systems.

Li-Bing Zhao obtained his master degree from Beihang University. He is now an engineer at China Academy of Aerospace Standardization and Product Assurance. His research interest covers nonlinear control, electronic products reliability simulation.

Quan Quan received the B.S. and Ph.D. degrees from Beihang University (Beijing University of Aeronautics and Astronautics), Beijing, China, in 2004, and 2010, respectively. He has been an Associate Professor with the School of Automation Science and Electrical Engineering, Beihang University since 2013. His research interests include reliable flight control, vision-based navigation, repetitive learning control, and time-delay systems.

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Appendix 1. Proof of Theorem 3.3

It is easy to see from the proof in Theorem 3.1 that observer (21) will make

$$\hat{x}_s \equiv x_s, \hat{x}_p \equiv x_p \text{ and } \hat{y}_p \equiv y_p. \tag{A1}$$

The remainder of the proof is composed of two parts: (i) for (6), the controller u_p drives $y_p - y_r \to 0$ as $t \to \infty$, and (ii) based on the result of (i), for (7), the controller u_s drives $y_s \to 0$ as $t \to \infty$. Then, the controller $u = u_p + u_s$ drives $y - y_r \to 0$ as $t \to \infty$in system (1).

(i) Suppose that Problem 3.1 is solved. Under (A1), controller (12) drives $y_p - y_r \to 0$ as $t \to \infty$ (Theorem 3.2).
(ii) Suppose that Problem 3.2 is solved. Under (A1), controller (19) drives $y_s$such that

$$\begin{aligned}\left\|y_s(t)\right\| &\le \|C\|\,\|x_s(t)\| + \|D\|\,\|u_s(t)\| \\ &\le (\|C\| + k_s\|D\|)\,\|x_s(t)\| \ \text{(condition (ii))} \\ &\le (\|C\| + k_s\|D\|)\,\gamma\left(\sup_{t_0\le s\le t}\left\|e_p(s)\right\|\right), t \ge t_0.\end{aligned}$$

Based on the result of (i), it can be achieved that $e_p \to 0$ as $t \to \infty$, which implies that $\|e_p(t)\| \le \varepsilon$ when $t \ge t_0 + T_1$. Then,

$$\begin{aligned}\left\|y_s(t)\right\| &\le (\|C\| + k_s\|D\|)\,\gamma\left(\sup_{t_0+T_1\le s\le t}\left\|e_p(s)\right\|\right) \\ &\le (\|C\| + k_s\|D\|)\,\gamma(\varepsilon), t \ge t_0 + T_1.\end{aligned}$$

Since ε can be chosen arbitrarily small, it can be concluded that $y_s \to 0$ as $t \to \infty$.

Thus, driven by controller (22), the output of system (1) satisfies that $y - y_r \to 0$ as $t \to \infty$.

Appendix 2. Parameters of Example 1

$$A_0(y) = \begin{bmatrix} -0.092 & -p & 0 & 0 & 1 & 0 \\ p & -0.0375 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -6.4354 & 102.2269 & 0 & -0.2223 & 0 & 0 \\ -25.2112 & 0.4018 & 0 & 0.01p & -0.0621 & 0.9922p \\ 0 & 9.8096 & 0 & 0.0039p & -0.9771p & -0.0618 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -126.9688 & -3.1941 & 97.9748 \\ -0.4956 & -12.5134 & 0.3847 \\ 1.7892 & 0.02882 & 7.7577 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -22.2 & 0 & -0.00109 & 0 & 0.024 \\ 54.534 & 0 & 0 & 0 & 0.022 & 0 \end{bmatrix},$$

$$D_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 8.96 & 0 \end{bmatrix}.$$

Appendix 3. Controller design for Example 1

C.1. State feedback

Since $(A_0(y_r), B_0)$ is controllable, choose

$$K = \begin{bmatrix} 0.0134 & -0.0727 & -0.0165 & -0.0114 & 0.0075 & -0.1603 \\ -1.9380 & 0.0073 & -0.0217 & -0.0166 & 0.1702 & -0.0081 \\ 0.0228 & -1.1335 & -0.0333 & -0.0364 & 0.0176 & -0.2120 \end{bmatrix}$$

to make $A_r = A_0(y_r) + B_0K$ stable with eigenvalues ($-0.5, -0.7, -0.9, -1.2, -1.5, -1.8$). After introducing the state feedback $\delta = u + Kx$, system (23) can be formulated as Equation (1) with $A(y) = A_0(y) + B_0K$, $B = B_0$, $C = C_0 + B_0K$, $D = D_0$.

Remark C.1: Since $A_0(y_r)$ depends on y_r, the gain K is time-varying and can be designed online. However, in this example, $A_0(y_r)$ is constant because the desired roll rate $p_r \equiv 0$ for any given constant y_r. Therefore, K is degenerated to a constant matrix.

Remark C.2: Note that the transfer function of the linearised system of BBT has right-half s-plane zeros, and the input–output dynamic characteristic of it is NMP (Remark 2.2).

C.2. Solution to Problem 3.1

For the primary system (6), the weighting matrices Q and R are selected as $Q = diag([100, 100, 0.5, 1, 1, 1, 1, 1, 1])$, $R = I_3$. Then, the optimal feedback gain matrix is calculated as

$$G_a = \begin{bmatrix} -8.0199 & 5.9714 & -0.0107 & -0.0377 & -23.4664 & -3.7573 & -0.8439 & 0.0123 & 1.9453 \\ 0.2184 & 0.0407 & -0.7069 & -14.3168 & -0.1077 & 0.0837 & -0.0113 & -2.1763 & -0.0085 \\ 5.9693 & 8.0213 & 0.0115 & 0.1262 & -30.6686 & 2.7223 & 0.5673 & 0.0301 & 2.4944 \end{bmatrix}.$$

Hence, the controller for Problem 3.1 is

$$u_p = -G_{a_1}\int_0^t \left(y_p(s) - y_r\right) ds - G_{a_2}x_p. \tag{C1}$$

C.3. Solution to Problem 3.2

In the following, u_s will be designed to make the secondary system (8) input-to-state stable. Problem 3.2 is solved based on the following system:

$$\begin{aligned}\dot{x}_s &= A_r x_s + Bu_s + [A(y_s) - A_r]x_s + d_p \\ &= A(y_s)x_s + Bu_s + d_p,\end{aligned} \tag{C2}$$

where $d_p = (A(y) - A_r)x - (A(y_s) - A_r)x_s \approx 0$, which is a fast dynamic, and can be ignored in the following design. For (C2), the stabilising controller} can be designed by the backstepping technique (Khalil & Grizzle, 1996). To apply the backstepping method, Equation (C2) is rewritten in a 'strict-feedback' form as

$$\dot{x}_{1,s} = g_1(x_{1,s})x_{2,s} + f_1(x_{1,s}), \tag{C3}$$
$$\dot{x}_{2,s} = g_2u_s + f_2(x_{1,s}, x_{2,s}), \tag{C4}$$

where $x_{1,s} = [\alpha_s\ \beta_s\ \phi_s]^T$, $x_{2,s} = [p_s\ q_s\ r_s]^T$, $f_1(x_{1,s}) \in \mathbb{R}^3$ and $g_1(x_{1,s}) \in \mathbb{R}^{3\times 3}$ are function matrices about state $x_{1,s}$, $f_2(x_{1,s}, x_{2,s}) \in \mathbb{R}^3$ is a function matrix about states $x_{1,s}$and $x_{2,s}$, $g_2 \in \mathbb{R}^{3\times 3}$ is a constant matrix, and $x_{2,s}$ is treated as a fictitious control input to (C3). The concrete design procedure for BC is routine, and hence omitted here. Finally, the controller for Problem 3.2 is

$$u_s(x_s) = -g_2^{-1}\Bigg(c_2 z_2 + f_2(x_{1,s}, x_{2,s}) - \frac{\partial \psi_1(x_{1,s})}{\partial x_{1,s}^T}(g_1(x_{1,s})x_{2,s} + f_1(x_{1,s})) + g_1^T(x_{1,s})\, x_{1,s}\Bigg), \tag{C5}$$

where $\psi_1(x_{1,s}) = -g_1^{-1}(x_{1,s})(f_1(x_{1,s}) + c_1 x_{1,s})$, $z_2 = x_{2,s} - \psi_1(x_{1,s})$, $c_1, c_2 \in \mathbb{R}_+$, and they are selected as $c_1 = c_2 = 10$.

Remark C.3 The reason why BC technique is adopted here is that system (C2) can be transformed into the 'strict-feedback' form. Otherwise, another nonlinear stabilisation control method needs to be considered to replace BC.

C.4. Controller integration

The variables x_s, x_p and y_p are estimated by observer (21). Then, the controller for BTT is combined as

$$\delta = u_p\left(\hat{x}_p, \int_0^t (\hat{y}_p(s) - y_r)\, ds\right) + u_s(\hat{x}_s) + Kx. \tag{C6}$$