Terminal Iterative Learning Control for Autonomous Aerial Refueling Under Aerodynamic Disturbances

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I. Introduction

AERIAL refueling has demonstrated significant benefits to aviation by extending the range and endurance of aircraft [1]. The development of autonomous aerial refueling (AAR) techniques for unmanned aerial vehicles (UAVs) makes new missions and capabilities possible [2], like the ability for long-range or long-time flight. As the most widely used aerial refueling method, the probe-drogue refueling (PDR) system is considered to be more flexible and compact than other refueling systems. However, a drawback of PDR is that the drogue is passive and susceptible to aerodynamic disturbances [3]. Therefore, it is difficult to design an AAR system to control the probe on the receiver to capture the moving drogue within centimeter level in the docking stage.

It used to be thought that the aerodynamic disturbances in the aerial refueling mainly include the tanker vortex, wind gust, and atmospheric turbulence. According to NASA Autonomous Aerial Refueling Demonstration (AARD) project [2], the forebody flow field of the receiver may also significantly affect the docking control of AAR, which is called “the bow wave effect” [4]. As a result, the modeling and simulation methods for the bow wave effect were studied in our previous works [5,6]. Because the obtained mathematical models are somewhat complex and there may be some uncertain factors in practice, this paper aims to use a model-free method to compensate for the docking error caused by aerodynamic disturbances, including the bow wave effect.

Most of the existing studies on AAR docking control do not consider the bow wave effect. In [7–9], the drogue is assumed to be relatively static (or oscillates around the equilibrium) and not affected by the flow field of the receiver forebody. However, in practice, the receiver aircraft is affected by aerodynamic disturbances, and the drogue is affected by both the wind disturbances and the receiver forebody bow wave. As a major difficulty in the control of AAR, the aerodynamic disturbances, especially the bow wave effect, attract increasing attention in these years. In [10,11], the wind effects from the tanker vortex, the wind gust, and the atmospheric turbulence are analyzed, and in [5,6,12], the modeling and simulation methods for the receiver forebody bow wave effect are studied, but no control methods are proposed. In [4], simulations show that the bow wave effect can be compensated by adding an offset value to the reference trajectory, but the method for obtaining the offset value is not given.

Because the accurate mathematical models for the aerodynamic disturbances are usually difficult to obtain [5], iterative learning control (ILC) is a possible choice for the docking control of AAR. According to [13], the ILC is a model-free control method that can improve the performance of a system by learning from the previous repetitive executions or iterations. ILC methods have been proved to be effective to solve the control problems for complex systems with no need for the exact mathematical model [14]. For an actual AAR system, the relative position between the probe and the drogue is usually measured by vision localization methods [15] whose measurement precision depends on the relative distance (higher precision in a closer distance). Therefore, compared with the trajectory data, the terminal positions of the probe and the drogue are usually easier to measure in practice. As a result, terminal iterative learning control (TILC) methods are suitable for AAR systems because TILC methods need only the terminal states or outputs instead of the whole trajectories [16,17].

This paper studies the model of the probe-drogue aerial refueling system under aerodynamic disturbances, and proposes a docking control method based on TILC to compensate for the docking errors caused by aerodynamic disturbances. In the ATP-56(B) issued by NATO [18], chasing the drogue directly is identified as a dangerous operation which may cause the overcontrol of the receiver. Therefore, the proposed TILC controller is designed by imitating the docking operations of human pilots to predict the terminal position of the drogue with an offset to compensate for the docking errors caused by aerodynamic disturbances. The designed controller works as an additional unit for the trajectory generation of the original autopilot system. Simulations based on our previously published MATLAB/SIMULINK environment [5,6] show that the proposed control method has a fast learning speed to achieve a successful docking control under aerodynamic disturbances, including the bow wave effect.

Nomenclature

- $F_T$: tanker joint frame
- $F_{bow}$: disturbance force from bow wave effect
- $F_r, F_{led}$: disturbance forces on receiver and hose-drogue
- $p_{dr}(T), p_{pr}(T)$: terminal positions of drogue and probe
- $p_{dr}(t), p_{pr}(t)$: current positions of drogue and probe
- $p_{dr}$: drogue initial equilibrium position
- $\mathbb{R}^n, \mathbb{R}^+_n$: real number set and positive real number set
- $R_C$: threshold radius for a successful docking attempt
- $T$: terminal time of a docking attempt
- $\hat{u}_{pr}$: reference trajectory for autopilot
- $\Delta F_0$: bow wave disturbance force
- $\Delta p_{fr}$: drogue position offsets from equilibrium position
- $\Delta p_{fr/pr}$: position error between drogue and probe
- $\Delta R_{fr/pr}$: radial error between hose and drogue

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The paper is organized as follows. Section II gives comprehensive problem description and model analysis of a PDR system. Section III describes the details of the TILC controller design and the convergence analysis. Section IV gives simulations with the proposed TILC method. In the end, Sec. V presents the conclusions.

II. Problem Formulation

A. Frames and Notations

Because the tanker moves at a uniform speed in a straight and level line during the docking stage of AAR, a frame fixed to the tanker body can be treated as an inertial reference frame to describe the relative motion between the receiver and the drogue. As shown in Fig. 1, a tanker joint frame $F_T$ is defined with the origin $O_T$ fixed to the joint between the tanker body and the hose. $F_T$ is a right-handed coordinate system, whose $x_T$, horizontally points to the flight direction of the tanker, $z_T$, vertically points to the ground, and $y_T$ points to the right. For simplicity, the following rules are defined:

1) All position or state vectors are defined under the tanker joint frame $F_T$, unless explicitly stated.

2) The drogue position vector is expressed as $p_{dr} \triangleq [x_{dr} \ y_{dr} \ z_{dr}]^T$, and the probe position vector is $p_{pr} \triangleq [x_{pr} \ y_{pr} \ z_{pr}]^T$. In a similar way, the position error between the probe and the drogue is expressed as

$$\Delta p_{dr/pr}(t) \triangleq p_{dr}(t) - p_{pr}(t)$$

whose decomposition form is represented by $\Delta p_{dr/pr} = [\Delta x_{dr/pr} \ \Delta y_{dr/pr} \ \Delta z_{dr/pr}]^T$.

3) One docking attempt ends at the terminal time $T \in \mathbb{R}^+$, when the probe contacts with the central plane of the drogue ($\Delta x_{dr/pr} = 0$) for the first time, which is defined as

$$T = \min_t[\Delta x_{dr/pr}(t) = 0]$$

The value at time $t = T$ is called the terminal value. For example, $p_{dr}(T)$ is the terminal position of the drogue and $\Delta p_{dr/pr}(T)$ is the terminal position error.

4) The value in the $k$th docking attempt is marked by a right superscript. For example, $p_{dr}^{(k)}$ denotes the drogue position $p_{dr}$ in the $k$th docking attempt, $T^{(k)}$ denotes the $k$th terminal time, and $p_{dr}^{(k)}(T^{(k)})$ denotes the $k$th terminal position of the drogue.

![Fig. 1 Simplified schematic diagram of PDR systems.](Image)

B. System Overview

The overall structure of the AAR system proposed in this paper is shown in Fig. 2, where the whole AAR system is divided into two parts: the mathematical model and the control system. The AAR mathematical model contains three components: the aerodynamic disturbance model, the hose-drogue dynamic model, and the receiver dynamic model; the control system contains two components: the autopilot and the TILC controller. The autopilot focuses on stabilizing the aircraft attitude and tracking the given reference trajectory, and the TILC controller works as a human pilot that learns from historical experience and sends trajectory commands to the autopilot. This paper focuses on the design of the TILC controller.

C. Mathematical Model

1. Aerodynamic Disturbance Model

The aerodynamic disturbances will change the flow field around the receiver and the drogue, and then produce disturbance forces on them to affect their relative motions. There are mainly two sources of aerodynamic disturbances: one is from the atmospheric environment such as the tanker vortex, the wind gust, and the atmospheric turbulence [10]; the other is from the bow wave flow field of the receiver forebody. In an AAR system, the receiver mainly suffers the atmospheric disturbance force $F_r \in \mathbb{R}^3$, whereas the hose-drogue suffers both the atmospheric disturbance force $F_{hd} \in \mathbb{R}^3$ and the bow wave disturbance force $F_{bow} \in \mathbb{R}^3$.

The modeling and simulation methods for $F_r$ and $F_{bd}$ have been well studied in the existing literature, where the detailed mathematical expression for $F_r$ can be found in [10], and the detailed mathematical expression for $F_{bd}$ can be found in [5,10]. The bow wave disturbance force $F_{bow}$, according to [5], is determined by the position error between the drogue and the probe $\Delta p_{dr/pr}$, which can be expressed as

$$F_{bow} = f_{bow}(\Delta p_{dr/pr})$$

where $f_{bow}(\cdot)$ is the bow wave effect function whose expression can be obtained by the method proposed in [5].

Among these disturbances, $F_r$ and $F_{hd}$ are independent of the states of the AAR system, and the corresponding control methods are mature; $F_{bow}$ is strongly coupled with the system output $\Delta p_{dr/pr}$, the control strategy for which is challenging and still lacking. Therefore, this paper puts more effort on the control of the bow wave effect.

2. Hose-Drogue Model

The soft hose can be modeled by a finite number of cylinder-shaped rigid links based on the finite-element theory [19]. Then, the hose-drogue dynamic equation can be written as

$$\begin{cases} \dot{x}_{hd}(t) = f_{hd}(x_{hd}(t), F_{hd}(t), F_{bow}(t)) \\ \dot{p}_{dr}(t) = g_{hd}(x_{hd}(t)) \end{cases}$$

![Fig. 2 Overall structure of the AAR system.](Image)
where \( f_{\text{hd}}(\cdot) \) is a nonlinear vector function, \( x_{\text{hd}} \) is the hose-drogue state vector, and \( F_{\text{hd}}(t) \) and \( F_{\text{bow}}(t) \) are the disturbance forces acting on the drogue. The dimensions of \( x_{\text{hd}} \) and \( f_{\text{hd}}(\cdot) \) depend on the number of the links that the hose is divided into.

The most concerned value in the TILC method is the terminal position of the drogue. Therefore, it is necessary to study the terminal state of the hose-drogue system (4). According to [6], when there is no random disturbance, the drogue will eventually settle at an equilibrium position marked as \( p_{\text{d}}^0 \). Then, under the bow wave effect, the drogue will be pushed to a new terminal position \( p_{\text{d}}(T) \). The drogue position offset \( \Delta p_{\text{d}} \in \mathbb{R}^3 \) is defined as

\[
\Delta p_{\text{d}} = p_{\text{d}}(T) - p_{\text{d}}^0
\]

where \( \Delta p_{\text{d}} \) is further determined by the strength of terminal bow wave disturbance force \( F_{\text{bow}}(T) \) as

\[
\Delta p_{\text{d}} = f_{\text{d}}(F_{\text{bow}}(T))
\]

Then, substituting Eq. (3) into Eq. (6) yields

\[
\Delta p_{\text{d}} = f_{\text{d}}(F_{\text{bow}}(\Delta p_{\text{d}}/p_{\text{pr}}(T))) = \tilde{f}_{\text{d}}(\Delta p_{\text{d}}/p_{\text{pr}}(T))
\]

Noticing that \( \Delta p_{\text{d}}/p_{\text{pr}}(T) \to 0 \), the Taylor expansion can be applied to Eq. (7), which results in

\[
\Delta p_{\text{d}} \approx m_0 + M_1 \cdot \Delta p_{\text{d}}/p_{\text{pr}}(T)
\]

where

\[
m_0 \equiv \tilde{f}_{\text{d}}(0), \quad M_1 \equiv \frac{\partial \tilde{f}_{\text{d}}(x)}{\partial x} \bigg|_{x=0}
\]

In practice, the drogue is sensitive to the aerodynamic disturbances, and the actual terminal position of the drogue always oscillates around its stable position. Therefore, a bounded disturbance term \( v_{\text{d}} \in \mathbb{R}^3 \) should be added to Eq. (7) as

\[
\Delta p_{\text{d}} = m_0 + M_1 \cdot \Delta p_{\text{d}}/p_{\text{pr}}(T) + v_{\text{d}}
\]

where \( \|v_{\text{d}}\| \leq B_{\text{d}} \) represents the position fluctuation of the drogue due to random disturbances such as atmospheric turbulence. According to (10), there is a functional relationship between the terminal docking error \( \Delta p_{\text{d}}/p_{\text{pr}}(T) \) and the drogue bow wave offset \( \Delta p_{\text{d}} \). Therefore, it is possible to use TILC methods to compensate for the bow wave position offset \( \Delta p_{\text{d}} \) with the terminal docking error \( \Delta p_{\text{d}}/p_{\text{pr}}(T) \).

The detailed mathematical expression of \( \tilde{f}_{\text{d}}(\cdot) \) can be obtained through methods in [5], and then the Jacobian matrix \( M_1 \) can be obtained from Eq. (9). Because \( \tilde{f}_{\text{d}}(\cdot) \) is monotonically decreasing along each axial direction, for the receiver aircraft with symmetrical forebody layout, it is easy to verify that \( M_1 \) is a negative definite matrix.

3. Receiver Aircraft Model

As previously mentioned, in the docking stage, the tanker joint frame \( F_T \) can be simplified as an inertial frame. Under this condition, the commonly used aircraft modeling methods as presented in [20] can be applied to the receiver aircraft with the following form:

\[
\begin{align*}
\dot{x}_r(t) &= f_r(x_r(t), F_r(t), u_r(t)) \\
\dot{p}_{\text{pr}}(t) &= g_{\text{pr}}(x_r(t))
\end{align*}
\]

(11)

where \( f_r(\cdot) \) is a nonlinear function, \( x_r \) is the state of the receiver, and \( u_r \) is the control input of the receiver aircraft.

Because the nonlinear model (11) is too complex for controller design, a linearization method [20] is applied to Eq. (11) to simplify the receiver dynamic model. Assume that the receiver equilibrium state is \( x_{r0} \) and the trimming control is \( u_{r0} \), then the linear model can be expressed as

\[
\begin{align*}
\dot{x}_r(t) &= A_r \cdot \Delta x_r(t) + B_r \cdot \Delta u_r(t) + G_r \cdot F_r(t) \\
\dot{p}_{\text{pr}}(t) &= C_r \cdot \Delta x_r(t)
\end{align*}
\]

(12)

where \( \Delta x_r \equiv x_r - x_{r0} \) is the state vector of the linearized system, \( \Delta u_r \equiv u_r - u_{r0} \) is the linearized control input vector, and \( \Delta p_{\text{pr}} \equiv p_{\text{pr}} - p_{\text{pr0}} \) is the probe position offset from the initial probe position \( p_{\text{pr0}} \).

D. Control System

1. Autopilot

Based on the linear model (12), the autopilot can be simplified as a state feedback controller [11] in the form as

\[
\Delta u_r(t) = -K_r \cdot \Delta x_r(t) - K_i \cdot e_r(t)
\]

(13)

where \( \Delta u_r(t) \in \mathbb{R}^3 \) is the reference trajectory vector of the probe, and \( K_r \) and \( K_i \) are the gain matrices. Essentially, Eq. (13) is a PI controller, where \( -K_r \cdot \Delta x_r(t) \) is the state feedback control term for stabilizing the aircraft, and \( -K_i \cdot e_r(t) \) is the integral control term for tracking the given trajectory. Because it is very convenient to obtain \( K_r \) and \( K_i \) through LQR function in MATLAB, the procedures are omitted here. In practice, a saturation function is required for \( e_r(t) \) in Eq. (13) to slow down the response speed and resist integral saturation. For instance, the approaching speed should be constrained within a reasonable range about 0.5–1 m/s, because the probe should have enough closure speed to open the valve on the drogue safely [2].

As analyzed in [11,20], when the autopilot (13) is well designed and the disturbance force \( F_r(t) \equiv 0 \), the tracking error can converge to zero:

\[
\hat{u}_{\text{pr}}(t) - p_{\text{pr}}(t) \to 0, \quad \text{as} \quad t \to \infty
\]

(15)

However, in practice, the disturbance force \( F_r(t) \neq 0 \) and the terminal time \( T \ll \infty \), then the tracking error cannot reach zero at terminal time \( T \). Therefore, an error term should be added to Eq. (15) at \( T \) as

\[
\hat{u}_{\text{pr}}(T) - p_{\text{pr}}(T) = \nu_{\text{pr}}
\]

(16)

where \( \nu_{\text{pr}} \in \mathbb{R}^3 \) is a bounded random disturbance term with \( \|\nu_{\text{pr}}\| \leq B_{\text{pr}} \). The random disturbance \( \nu_{\text{pr}} \) may come from the unrepeatable disturbances such as atmospheric turbulence.

2. Objective of Docking Control

According to [2], in each docking attempt, the receiver should follow the drogue for seconds until the hose-drogue levels off. Then, the receiver starts to drive the probe to approach the drogue with a slow constant speed, until the probe hits the central plane of the drogue as shown in Fig. 3. The basic requirement for the AAR system is that the relative position between the probe and the drogue (represented by the docking error \( \Delta p_{\text{d}}/p_{\text{pr}} \)) can reach zero at the terminal time \( T \). In practice, the radial error \( \Delta R_{\text{d}}/p_{\text{pr}} \in \mathbb{R}^3 \) is an important evaluation index for the docking performance which defined in \( \sigma_{xy} \) plane as

Fig. 3 Success and failure criteria of a docking attempt [2].
\[ \Delta R_{n/p}(t) = \sqrt{\Delta^2_{\text{dr}}(t) + \Delta^2_{\text{pr}}(t)} \quad (17) \]

Because the docking error is inevitable due to disturbances, a threshold radius (criterion radius) \( R_C \in \mathbb{R}_+ \) should be defined as

\[ \Delta R_{n/p}(T) < R_C \quad (18) \]

If criterion (18) is satisfied, a success docking is declared for this docking attempt [2]. Otherwise, a failure or miss is declared. In fact, according to the previous definition, there is \( \Delta R_{n/p}(T) \equiv 0 \). Therefore, the terminal radial error \( \Delta R_{n/p}(T) \) always equals to the terminal docking error \( \Delta p_{n/p}(T) \).

### III. TILC Design

As shown in Fig. 2, the role of the TILC controller in AAR system is the same as the human pilot in manned refueling system. The inputs of the TILC controller are the terminal position errors, a simple and safe control strategy is to predict and track the terminal radial error \( \Delta R_{n/p}(T) \equiv 0 \) and the tracking error caused by the response lag of the autopilot should have the following form:

\[ \begin{align*}
\Delta R_{n/p}(T) &< R_C \\
\Delta p_{n/p}(T) &> 0
\end{align*} \]

Therefore, the terminal radial error

\[ \Delta p_{n/p}(T) \]

and the probe terminal tracking error caused by the response lag of the autopilot should have the following form:

\[ u_k = p_{\text{pr}}^{(0)}(t) + u_{\text{dr},k}^{(0)} + u_{\text{pr}}^{(k)} \quad (19) \]

where \( p_{\text{pr}}^{(0)}(t) \) is the original stable position of the drogue, \( u_{\text{dr},k}^{(0)} \) is an estimation term for the drogue position offset, and \( u_{\text{pr}}^{(k)} \) is an ILC term to compensate for the tracking error of the probe. Note that, since \( p_{\text{dr}}^{(0)} \) can be directly measured during the flight, it is treated as a known parameter. Then, \( u_{\text{dr},k}^{(0)} \) and \( u_{\text{pr}}^{(k)} \) should be updated in each iteration, and the updating laws are given below.

1) The updating law of \( u_{\text{dr},k}^{(0)} \) is given by

\[ u_{\text{dr},k}^{(k)} = K_s \cdot u_{\text{dr},k}^{(k-1)} + (I - K_n) \cdot \Delta p_{\text{dr}}^{(k-1)} \quad (20) \]

where \( K_n = \text{diag}(k_1, k_2, k_3) \), with \( k_1, k_2, k_3 \in (0, 1) \) is a constant diagonal matrix, and \( \Delta p_{\text{dr}}^{(k-1)} \) is the drogue terminal offset position as defined in Eq. (5) whose iterative feature can be written as

\[ \Delta p_{\text{dr}}^{(k)} = \Delta p_{\text{dr}}^{(k-1)}(T^{(k)}) - p_{\text{pr}}^{(0)} \quad (21) \]

2) The updating law of \( u_{\text{pr}}^{(k)} \) is given by

\[ u_{\text{pr}}^{(k)} = u_{\text{pr}}^{(k-1)} + K_p \cdot e_{\text{pr}}^{(k-1)} \quad (22) \]

where \( K_p = \text{diag}(k_1, k_2, k_3) \) is a constant diagonal matrices with \( k_1, k_2, k_3 \in (0, 1) \) and \( e_{\text{pr}}^{(k)} \) represents the probe terminal tracking error with the \( k \)th iterative feature defined as

\[ e_{\text{pr}}^{(k)} = p_{\text{pr}}^{(0)}(t) + u_{\text{dr},k}^{(0)} - p_{\text{pr}}^{(k)}(T^{(k)}) \quad (23) \]

### IV. Simulation and Verification

#### A. Simulation Configuration

A MATLAB/SIMULINK-based simulation environment has been developed to simulate the docking stage of the AAR. The detailed introduction of the modeling methods and the simulation parameters can be found in the authors’ previous work [5]. A video has also been released to introduce the AAR simulation environment and demonstrate the TILC simulation results. The URL of the video is [https://youtu.be/VoplIDA6D5I](https://youtu.be/VoplIDA6D5I).
B. TILC Simulation Results

1. Iterative Learning Process

To verify the effectiveness of the proposed TILC method, all the initial values in Eq. (19) are set zeroes as $u^{(0)}_{d,e} = 0$, $u^{(0)}_{d,r} = 0$, and the learning procedures are shown in Fig. 4.

In Fig. 4, there are four docking attempts performed in sequence (the four docking attempts start at time 50, 100, 150, and 200 s, respectively), where the first two docking attempts fail, and the following two attempts both succeed. In each attempt, the probe moves close to until contact with the drogue at $T^{(k)}$ (marked by the vertical dotted lines), then the probe returns to the standby position and gets ready for the next docking attempt.

In the first docking attempt as shown in Fig. 4, the receiver remains at the standby position (5 m behind the drogue, with simulation time from 50 to 60 s) to observe the drogue movement and estimate the equilibrium position of the drogue. Then, the receiver approaches the drogue to perform a docking attempt during the simulation time from 60 to 71 s in Fig. 4. The docking control ends at the terminal time $T^{(1)} = 71$ s, and this docking attempt is declared as a failure because the radial error $\Delta R^{(1)}_{d,r} = 0.5$ m is larger than the desired radial error threshold $R_C = 0.15$ m.

With more docking attempts (not presented in Fig. 4) are simulated, a docking success rate over 90% will be obtained under the given threshold $R_C = 0.15$ m. According to the Monte Carlo simulations, the success rate depends on many factors, including the docking error threshold $R_C$, the strength of the atmospheric turbulence, and other random disturbances. The simulation results are consistent with results in [2,4]. When the aerodynamic disturbances are strong, both the drogue position oscillation and the receiver tracking error will be significant, then the success rate will be low.

2. Aerodynamic Disturbance Simulations

Figure 5 presents the total aerodynamic disturbance force $F_{\text{total}} = [\Delta F_x, \Delta F_y, \Delta F_z]^T$ applied on the drogue during the first docking attempt (50–71 s) in Fig. 4. In this simulation, the tanker vortex disturbance comes from the model presented in [10], the wind gust and the atmospheric turbulence come from the MATLAB/SIMULINK Aerospace Blockset based on the mathematical representations from Military Specification MIL-F-8785C, and the bow wave effect disturbance comes from the authors’ previous work [6]. When the receiver remains at the standby position (50–60 s in Fig. 5), the drogue is far away from the receiver and the disturbance forces mainly come from the tanker vortex and the atmospheric turbulence as illustrated on the left half of Fig. 5. As the receiver moves closer to the drogue, the receiver bow wave starts to cause a
large disturbance force on the drogue as illustrated on the right half of Fig. 5.

A comprehensive simulation is performed to verify the performance of the proposed TILC method with the initial value from the previous learning results. In addition to the atmospheric turbulence and the bow wave disturbance as shown in Fig. 5, a wind gust (5 m/s in the lateral direction and vertical direction respectively) is added at 100 s to verify the control effect of the proposed method under aerodynamic disturbances. The simulation results are presented in Fig. 6.

It can be observed from Fig. 6 that, with a good initial value, the docking control succeeds at the first attempt. Then, the second docking attempt (115 s in Fig. 6) fails due to the addition of a strong wind gust at 100 s. In the next two docking attempts (165 and 215 s in Fig. 6), the controller can rapidly recover and achieve successful docking control without being much affected by the wind gust disturbance. The simulation results demonstrate that the proposed TILC method has a certain ability to resist the aerodynamic disturbances.

V. Conclusions

This paper studies the model of the probe-drogue aerial refueling system under aerodynamic disturbances, and proposes a docking control method based on terminal iterative learning control to compensate for the docking errors caused by aerodynamic disturbances. The designed controller works as an additional unit for the trajectory generation function of the original autopilot system. Simulations based on our previously published simulation environment show that the proposed control method has a fast learning speed to achieve a successful docking control under aerodynamic disturbances, including the bow wave effect.

Appendix: Proof of Theorem 1

First, define the \( p_{dr}^{(k)}(T^{(k)}) \) as the probe terminal position in the \( k \)th docking attempt. Then, according to Eq. (16), one has

\[
P_{dr}^{(k)}(T^{(k)}) = \tilde{u}_{dr}^{(k)} - v_{pr}^{(k)}
\]

(A1)

where \( \tilde{u}_{dr}^{(k)} \) can be further expressed by Eq. (19), which yields

\[
P_{dr}^{(k)} + \tilde{u}_{dr}^{(k)} - p_{dr}^{(k)}(T^{(k)}) = v_{pr}^{(k)} - u_{pr}^{(k)}
\]

(A2)

Meanwhile, according to the definition of \( e_{pr}^{(k)} \) in Eq. (23), one has

\[
e_{pr}^{(k)} = v_{pr}^{(k)} - u_{pr}^{(k)}
\]

(A3)

Thus, substituting Eq. (22) into Eq. (A3) gives

\[
e_{pr}^{(k)}(I - K_p) \cdot e_{pr}^{(k-1)} + \tilde{v}_{pr}^{(k-1)}
\]

(A4)

where

\[
\tilde{v}_{pr}^{(k-1)} \equiv \tilde{v}_{pr}^{(k)} - v_{pr}^{(k-1)}
\]

(A5)

Second, according to Eq. (5), the drogue terminal position \( p_{dr}^{(k)}(T^{(k)}) \) in the \( k \)th docking attempt is given by

\[
p_{dr}^{(k)}(T^{(k)}) = p_{dr}^{(0,(k))} + \Delta p_{dr}^{(k)}
\]

(A6)

where \( p_{dr}^{(0,(k))} \) is the drogue original equilibrium position, and \( \Delta p_{dr}^{(k)} \) is the terminal position offset. According to Eq. (10), \( \Delta p_{dr}^{(k)} \) comes from the bow wave effect and can be expressed

\[
\Delta p_{dr}^{(k)} = m_0 + M_1 \cdot \Delta p_{dr/pr}^{(k)}(T^{(k)}) + \tilde{v}_{dr}^{(k-1)}
\]

(A7)

Thus, the docking error along the iteration axis is given by

\[
\Delta p_{dr/pr}^{(k)}(T^{(k)}) = p_{dr}^{(k)}(T^{(k)}) - p_{dr}^{(0,(k))} + \tilde{v}_{dr}^{(k-1)}
\]

(A8)

Substituting Eqs. (A6–A8) into Eqs. (20) and (21) gives

\[
\Delta p_{dr/pr}^{(k)}(T^{(k)}) = A_1 \cdot \Delta p_{dr/pr}^{(k-1)}(T^{(k-1)}) + A_2 \cdot e^{(k-1)}_{pr} + \tilde{v}_{dr}^{(k-1)}
\]

(A9)

where

\[
A_1 \equiv (M_1 - I)^{-1}(M_1 - K_a) = I - (I - M_1)^{-1}(I - K_a)
\]

(A10)

\[
A_2 \equiv (M_1 - I)^{-1}(K_p + K_a - I)
\]

(A11)

\[
\Delta p_{dr/pr}^{(k-1)} \equiv (M_1 - I)^{-1}(v_{dr}^{(k-1)} - v_{dr}^{(k)})
\]

(A12)

For simplicity, an augmented system is defined as

\[
X^{(k)} = A \cdot X^{(k-1)} + v^{(k-1)}
\]

(A13)
where

\[ X^{(k)} = \begin{bmatrix} \Delta P_{\text{dr/pf}}^{(k)} (T^{(k)}) \end{bmatrix}, \quad v^{(k)} = \begin{bmatrix} v_{\text{dr}}^{(k)} \end{bmatrix} \]  

(A14)

\[ A \triangleq \begin{bmatrix} A_1 & A_2 \\ 0_{3 \times 3} & A_3 \end{bmatrix}, \quad A_3 \triangleq I - K_p \]  

(A15)

Furthermore, Eq. (A14) can be written in the following form:

\[ X^{(k)} = A^k \cdot X^{(0)} + \sum_{i=0}^{k-1} A^i v^{(k-i)} \]  

(A16)

Because \( M \) is a negative definite matrix, according to Eqs. (A10), (A10), and (A14), it is easy to verify that the spectral radius of \( A \) is smaller than 1 (\( \rho(A) < 1 \)) when the following constraint is satisfied:

\[ 0 \leq k_{\text{as}} < 1, \quad 0 < k_p \leq 1, \quad i = 1, 2, 3 \]  

(A17)

Moreover, since the disturbances \( v^{(i)} \) and \( u^{(i)} \) are both bounded with \( \|v^{(i)}\| \leq B_{\text{pr}} \) and \( \|u^{(i)}\| \leq B_{\text{dr}} \), it is easy to obtain from Eqs. (A5), (A10), and (A14) that \( v^{(i)} \) is also bounded with

\[ \|v^{(i)}\| \leq 2 \sqrt{B_{\text{pr}}^2 + B_{\text{dr}}^2} \leq \sqrt{2} \sqrt{B_{\text{dr}}^2 + B_{\text{dr}}^2} \]  

Then, substituting Eq. (A18) into Eq. (A16) gives

\[ \|X^{(k)}\| \leq \|A\|^k \cdot X^{(0)} + \sum_{i=0}^{k-1} \|A\|^i \cdot \|v^{(k-i)}\| \leq \|A\|^k \cdot X^{(0)} + 2 \left( B_{\text{pr}}^2 + B_{\text{dr}}^2 \right) \sum_{i=0}^{k-1} \|A\|^i \]  

\[ = \|A\|^k \cdot X^{(0)} + 2 \left( B_{\text{pr}}^2 + B_{\text{dr}}^2 \right) (1 - \|A\|^k) \]  

(A19)

When the constraint in Eq. (A17) is satisfied, one has

\[ \rho(A) < 1 \Rightarrow \lim_{k \to \infty} \|A\|^k = 0 \]  

(A20)

which yields from Eq. (A19) that

\[ \lim_{k \to \infty} \|X^{(k)}\| \leq 2 \sqrt{B_{\text{pr}}^2 + B_{\text{dr}}^2} \]  

(A21)

According to the definition of \( X^{(k)} \) in Eq. (A14), one has

\[ \|\Delta P_{\text{dr/pf}}^{(k)} (T^{(k)})\| \leq \|X^{(k)}\| \]  

(A22)

Combining Eqs. (A21) and (A22) gives

\[ \lim_{k \to \infty} \|\Delta P_{\text{dr/pf}}^{(k)} (T^{(k)})\| = \|\Delta P_{\text{dr/pf}} (T)\| \leq 2 \left( B_{\text{pr}}^2 + B_{\text{dr}}^2 \right) = B_{\text{dr/pr}} \]  

(A23)

Thus, the docking error \( \Delta P_{\text{dr/pf}}^{(k)} (T^{(k)}) \) will converge to a bound \( B_{\text{dr/pr}} \) as \( k \to \infty \). In particular, by substituting \( B_{\text{dr}} = 0, B_{\text{pr}} = 0 \) into Eq. (A23), one has

\[ \lim_{k \to \infty} \|\Delta P_{\text{dr/pf}}^{(k)} (T^{(k)})\| = 0 \]

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