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IFAC PapersOnLine 51-24 (2018) 74–81

Fault Detection and Diagnosis of the Homogenous Quadcopter Team in the Presence of Wind Disturbance

Zhang Hepeng * **Zhao Zhiyao** ** **Quan Quan** ***

* *School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, P.R.China (e-mail: zhanghepeng@buaa.edu.cn).*

** *School of Computer and Information Engineering, Beijing Technology and Business University, No. 11/33 Fucheng Road, Haidian District, Beijing 100048, P.R.China (e-mail: zhaozy@btbu.edu.cn)*

*** *School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, P.R.China, (e-mail: qq_buaa@buaa.edu.cn)*

Abstract: In order to ensure the normal operation and increase the safety, reliability and mission dependability of quadcopters, the fault detection and diagnosis is very important. In this paper, the problem of fault detection and diagnosis is investigated for the homogenous quadcopter team subject to non-negligible wind disturbance. However, it is difficult to accurately diagnose the fault, because it is coupled with the wind disturbance, whose effect is difficult to eliminate. For this problem, first, the observability is analyzed for the sum effect of the fault and the wind disturbance of one quadcopter, for the fault of one quadcopter, and for the fault of the homogenous quadcopter team, respectively. By analysis, the fault in the situation of the homogenous quadcopter team is observable under some reasonable assumptions. Based on this, a procedure is further proposed to detect and diagnose the fault for the homogenous quadcopter team, eliminating the effect of the wind disturbance. Finally, the simulation demonstrates the effectiveness of the proposed procedure.

Keywords: Fault detection and diagnosis, homogenous quadcopter team, extended Kalman filter, observability.

1. INTRODUCTION

Quadcopters have been more and more popular in both civilian and military domains, due to the simplicity of their construction and maintenance, their ability to hover, and their vertical take-off and landing capability [Quan (2017)]. They can undertake some special tasks in which risks to pilots are high, beyond normal human endurance is required, or human presence is not necessary.

However, the actuators, namely the motor-propeller systems, are prone to faults due to component degradation or damage to the motors, propellers, and so on. The occurrence of such actuator faults could cause undesirable effect on the stability of the closed-loop control system and then the tracking performance. Once quadcopter actuators fail or a quadcopter suffers some structural airframe damage, the flying performance of the quadcopter deteriorates, and the consequences of even a minor fault can be catastrophic [Zhang and Jiang (2008)]. Therefore, in order to ensure the normal operation and increase the safety, reliability and mission dependability of quadcopters, the fault detection and diagnosis (FDD) is very important.

Towards this, many methods have been developed for fault detection and diagnosis in the literature. The problem of designing and developing a hybrid fault detection and isolation (FDI) scheme was investigated in [Meskin et al. (2010)] for a network of unmanned vehicles subject to large environmental disturbances, where the proposed FDI algorithm was a hybrid architecture composed of a bank of continuous-time residual generators and a discrete-event fault diagnoser. The problems of actuator fault diagnosis and control were addressed for a realistic nonlinear six degree-of-freedom quadcopter model [Candido et al. (2014)], based on interacting multiple model filter and a switching multi-model predictive controller. In this paper, a new performance index was proposed in order to reduce the false alarms caused by universal residuals. In [He et al. (2013)] and [Liu et al. (2014)], the fault was treated as an augmented state and estimated via Kalman filter (KF) or extended Kalman filter (EKF). In [Aguilar-Sierra et al. (2014)], a polynomial observer was utilized to estimate the fault. Concretely, the fault was expressed by using the available measurement and known inputs of the system, as well as their differentials. In [Jiang et al. (2006)], an adaptive estimator was constructed for the simultaneous estimation of the system states and process faults, based

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.09.531

on which the fault tolerant control was carried out. To sum up, two approaches have been developed in the literature for the FDD problem, namely the residual generation and the direct fault estimation. In the former approach, several residuals are established, which are only sensitive to certain faults while insensitive to the others. The residuals should be close to zero in fault-free conditions. Otherwise, the residuals will deviate from zero, usually beyond a given threshold, after the occurrence of faults to which they are sensitive. While the latter is to estimate the fault directly, designing an observer or utilizing other mathematical methods. Therefore, the faulty situation could be detected upon the fault occurrence, through the online parameter estimation. However, by using no matter which approach, it is difficult to eliminate the effect of the wind disturbance, which is coupled with that of the fault. In fact, for one quadcopter, the fault is unobservable in the presence of wind disturbance (see *Corollary 2*).

In this paper, an FDD procedure is proposed, which could eliminate the effect of the wind disturbance by a comparison among the homogenous quadcopters in a team. First, the sum of fault and wind disturbance is considered as an additional state and is then estimated through EKF. As is mentioned above, they are actually coupled. Therefore, the fault is unobservable. Since the homogenous quadcopter team is considered, it is possible to distinguish the fault from the wind disturbance by imposing several reasonable assumptions. The feasibility is proved (see *Corollary 4*). Simulation results are further presented to demonstrate the effectiveness of the proposed method. To summarize, the main contributions of the work are as follows: (i) a simple method to diagnose the fault, in which the effect of wind disturbance has been eliminated taking advantage of the homogeneity of the quadcopter team; (ii) observability analysis to show observability and unobservability for different scenarios.

The remainder of the paper is organized as follows. The modeling and problem formulation are described in Section II. The observability analysis proving the feasibility of the proposed assumptions in Section II is given in Section III. In Section IV, the concrete solution to the FDD problem is detailed. Some simulation results are shown in Section V, and Section VI gives the conclusion.

2. MODELING AND PROBLEM FORMULATION

2.1 Quadcopter Model

The nonlinear dynamic model of the ith ($i = 1, 2, \ldots, n$) quadcopter is [Liu et al. (2014)]:

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{G}(\mathbf{x}_i) + \mathbf{B}(\mathbf{x}_i)\mathbf{u}_i \\ \mathbf{y}_i = \mathbf{x}_i \end{cases}. \tag{1}$$

Here $\mathbf{x}_i = [\phi_i\ \theta_i\ \psi_i\ z_i\ p_i\ q_i\ r_i\ v_{i,z}]^{\mathrm{T}} \in \mathbb{R}^8$ are the roll angle, the pitch angle, the yaw angle, the height, the roll angle rate, the pitch angle rate, the yaw angle rate and the vertical velocity, respectively; $\mathbf{y}_i \in \mathbb{R}^8$ denotes the system output, and $\mathbf{u}_i \in \mathbb{R}^4$ is the control input defined as

$$\mathbf{u}_i = [\,T_i\ \tau_{i,x}\ \tau_{i,y}\ \tau_{i,z}\,]^{\mathrm{T}} \tag{2}$$

where $T_i \in \mathbb{R}$ is the sum of thrusts of all rotors, $\tau_{i,x}$, $\tau_{i,y}$, $\tau_{i,z} \in \mathbb{R}$ are the roll moment, pitch moment and yaw moment, respectively. Furthermore,

$$\mathbf{G}(\mathbf{x}_i) = \begin{bmatrix} p_i + \tan\theta_i(r_i\cos\phi_i + q_i\sin\phi_i) \\ q_i\cos\phi_i - r_i\sin\phi_i \\ \sec\theta_i(r_i\cos\phi_i + q_i\sin\phi_i) \\ v_{i,z} \\ \dfrac{(J_{yy}-J_{zz})r_iq_i}{J_{xx}} \\ \dfrac{(J_{zz}-J_{xx})p_ir_i}{J_{yy}} \\ \dfrac{(J_{xx}-J_{yy})p_iq_i}{J_{zz}} \\ g \end{bmatrix} \tag{3}$$

,

$$\mathbf{B}(\mathbf{x}_i) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{1}{J_{xx}} & 0 & 0 & 0 \\ 0 & \dfrac{1}{J_{yy}} & 0 & 0 \\ 0 & 0 & \dfrac{1}{J_{zz}} & 0 \\ 0 & 0 & 0 & -\dfrac{\cos\phi_i\cos\theta_i}{m} \end{bmatrix} \tag{4}$$

where J_{xx}, J_{yy}, $J_{zz} \in \mathbb{R}$ represent the moments of inertia, $m \in \mathbb{R}$ denotes the mass of each quadcopter, and g denotes the acceleration due to gravity. The control input $\mathbf{u}_i$ is converted from the thrust $\mathbf{T}_i$ produced by the four rotors as

$$\mathbf{u}_i = \mathbf{B}_T\mathbf{T}_i. \tag{5}$$

Here

$$\mathbf{B}_T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & d & 0 & -d \\ d & 0 & -d & 0 \\ k_\mu & k_\mu & k_\mu & k_\mu \end{bmatrix}, \mathbf{T}_i = \begin{bmatrix} T_{i,1} \\ T_{i,2} \\ T_{i,3} \\ T_{i,4} \end{bmatrix} \tag{6}$$

where $d \in \mathbb{R}$ represents the distance from each rotor to the center of mass of the quadcopter; $k_\mu \in \mathbb{R}$ is the mapping coefficient from lift to torque. The n quadcopters have the same model and therefore are said to be *homogenous*.

2.2 Fault Model for a Single Quadcopter

During flight, the fault and the wind disturbance may occur, decreasing the lift supplied by the rotors, probably causing an actual output less than expected. In this case, in order to describe the output of rotors, a residual matrix $\mathbf{H}_i$ is introduced as

$$\mathbf{T}_i = \mathbf{H}_i\mathbf{T}_{c,i} \tag{7}$$

where $\mathbf{T}_{c,i} = [\,T_{c,i,1}\ T_{c,i,2}\ T_{c,i,3}\ T_{c,i,4}\,]^{\mathrm{T}} \in \mathbb{R}^4$ is the lift output of the controller, and $\mathbf{H}_i = \mathrm{diag}(\eta_{i,1}, \eta_{i,2}, \eta_{i,3}, \eta_{i,4})$, $\eta_{i,j} \in [0, 1]$, $j = 1, 2, 3, 4$. Here, $\eta_{i,j} = T_{i,j}/T_{c,i,j}$ indicates that the ratio of the remaining output of the jth rotor under the effect of both fault and wind disturbance is $100\eta_{i,j}\%$. In particular, $\eta_{i,j} = 1$ means that the rotor is absolutely *healthy*. On the other hand, the closer $\eta_{i,j}$ is to 0, the less the ratio is, and the less lift would be supplied. The residual matrix is caused by two aspects, namely the fault and the wind disturbance. They have the following relationship

$$\mathbf{H}_i + \mathbf{H}_{f,i} + \mathbf{H}_{w,i} = \mathbf{I}_4 \tag{8}$$

where $\mathbf{H}_{f,i}, \mathbf{H}_{w,i} \in \mathbb{R}^{4\times4}$ are the losses caused by fault and wind disturbance, respectively. Based on (8), the equation (5) can be rewritten as

$$\begin{aligned}\mathbf{u}_i &= \mathbf{B}_T\mathbf{H}_i\mathbf{T}_{c,i}\\ &= \mathbf{B}_T\left(\mathbf{I}_4 - \mathbf{H}_{f,i} - \mathbf{H}_{w,i}\right)\mathbf{T}_{c,i}\\ &= \mathbf{u}_{c,i} - \mathbf{f}_i - \mathbf{w}_i \end{aligned} \tag{9}$$

where

$$\mathbf{u}_{c,i} = \mathbf{B}_T\mathbf{T}_{c,i}, \tag{10}$$
$$\mathbf{f}_i = \mathbf{B}_T\mathbf{H}_{f,i}\mathbf{T}_{c,i}, \tag{11}$$
$$\mathbf{w}_i = \mathbf{B}_T\mathbf{H}_{w,i}\mathbf{T}_{c,i}. \tag{12}$$

Here, $\mathbf{u}_{c,i} \in \mathbb{R}^4, \mathbf{f}_i = \left[\, f_{i,1}\ f_{i,2}\ f_{i,3}\ f_{i,4}\,\right]^{\mathrm{T}} \in \mathbb{R}^4, \mathbf{w}_i \in \mathbb{R}^4$ are the controller command, the fault, and the wind disturbance, respectively.

Notice that

$$\mathbf{f}_i + \mathbf{w}_i = \mathbf{B}_T(\mathbf{I}_4 - \mathbf{H}_i)\mathbf{T}_{c,i} \tag{13}$$

which would be utilized to calculate the residual matrix $\mathbf{H}_i$.

Then, the model (1) becomes

$$\begin{cases}\dot{\mathbf{x}}_i = \mathbf{G}(\mathbf{x}_i) + \mathbf{B}(\mathbf{x}_i)\mathbf{u}_{c,i} - \mathbf{B}(\mathbf{x}_i)(\mathbf{f}_i + \mathbf{w}_i)\\ \mathbf{y}_i = \mathbf{x}_i\end{cases}. \tag{14}$$

Obviously, the fault $\mathbf{f}_i$ and the wind disturbance $\mathbf{w}_i$ are actually coupled.

2.3 Problem Formulation

The following assumptions are proposed to simplify the analysis.

Assumption 1. The fault $\mathbf{f}_i$ and wind disturbance $\mathbf{w}_i$ are constant, namely

$$\begin{cases}\dot{\mathbf{f}}_i = \mathbf{0}_{4\times 1}\\ \dot{\mathbf{w}}_i = \mathbf{0}_{4\times 1}\end{cases}$$

where $i = 1, 2, \ldots, n$.

Assumption 2. The homogenous quadcopter team is in the same wind field, i.e.,

$$\mathbf{w}_1 = \cdots = \mathbf{w}_n = \mathbf{w}.$$

Assumption 3. The homogenous quadcopter team has the same loss of effectiveness caused by the wind disturbance, i.e.,

$$\mathbf{H}_{w,1} = \cdots = \mathbf{H}_{w,n} = \mathbf{H}_w.$$

Assumption 4. No more than one fault occurs on the jth rotor of the n quadcopters, namely,

$$\forall i_1, i_2 = 1, 2, \cdots, n, f_{i_1,j} \cdot f_{i_2,j} = 0, j = 1, 2, 3, 4.$$

Based on *Assumption 1*, the model (14) is augmented as

$$\begin{cases}\begin{bmatrix}\dot{\mathbf{x}}_i\\ \dot{\mathbf{f}}_i + \dot{\mathbf{w}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{G}(\mathbf{x}_i) - \mathbf{B}(\mathbf{x}_i)(\mathbf{f}_i + \mathbf{w}_i)\\ \mathbf{0}_{4\times 1}\end{bmatrix} + \begin{bmatrix}\mathbf{B}(\mathbf{x}_i)\\ \mathbf{0}_{4\times 1}\end{bmatrix}\mathbf{u}_{c,i}\\ \mathbf{y}_i = \mathbf{x}_i\end{cases} \tag{15}$$

or

$$\begin{cases}\begin{bmatrix}\dot{\mathbf{x}}_i\\ \dot{\mathbf{f}}_i\\ \dot{\mathbf{w}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{G}(\mathbf{x}_i) - \mathbf{B}(\mathbf{x}_i)(\mathbf{f}_i + \mathbf{w}_i)\\ \mathbf{0}_{4\times 1}\\ \mathbf{0}_{4\times 1}\end{bmatrix} + \begin{bmatrix}\mathbf{B}(\mathbf{x}_i)\\ \mathbf{0}_{4\times 1}\\ \mathbf{0}_{4\times 1}\end{bmatrix}\mathbf{u}_{c,i}\\ \mathbf{y}_i = \mathbf{x}_i\end{cases} \tag{16}$$

where $i = 1, 2, \ldots, n$. Under *Assumptions 1-4*, based on (15) and (16), we have the *FDD problem* stated in the following.

FDD Problem. A homogenous team consisted of n quadcopters is modeled in (15) or (16). We will solve the following four problems under (15) or (16):

(i) is $\mathbf{f}_i + \mathbf{w}_i$ observable for one quadcopter?

(ii) is $\mathbf{f}_i$ observable for one quadcopter?

(iii) is $\mathbf{f}_i$ observable for the quadcopter team?

(iv) how could $\mathbf{f}_i$ be estimated for the quadcopter team if the problem (iii) is true?

Remark 1 (on *Assumptions 1-3*). In *Assumption 1*, the fault and wind disturbance are supposed to be constant or slow varying. Of course, in reality, the wind disturbance may not be constant. While in that situation, all that we need to do is to make a slight change in the model (15) and (16). This change has influence neither in the FDD procedure to be presented in Section IV nor in the observability analysis to be presented in Section III and the *Appendix*. *Assumption 1* is just for the simplification of the analysis and the simulation, and the algorithm presented here also applies to a time-varying wind disturbance.

For the sake of safety, quadcopters always operate with a distance that is remote enough. Therefore, the interaction among them could be ignored. The homogeneity of the quadcopters and the same environment they are located in lead to *Assumption 2*. Considering that the homogenous quadcopters carry out the same task and have nearly the same control command $\mathbf{T}_{c,i}$, (12) and *Assumption 2* imply that *Assumption 3* is reasonable.

Remark 2 (on *Assumption 4*). In fact, *Assumption 4* cannot hold absolutely, but it can hold in the sense of probability. For clarification, it could be assumed that the probability that each independent rotor encounters a fault per hour is $\epsilon > 0$, which is generally rather low. Consider the jth rotor of the n quadcopters. First, some events are defined. Denote S the sample space. Define a series of events

$$\bigcup_{1\leq k\leq n} F_k = S$$

where F_k, $k = 1, 2, \cdots, n$ indicates that explicit k faults occur on the jth rotor of the n quadcopters. Define four mutually exclusive random events S_k, $k = 1, 2, 3, 4$. Let S_1 be

$$S_1 = \{\forall i_1, i_2 = 1, 2, \cdots, n, f_{i_1,j} \cdot f_{i_2,j} = 0\}$$

which indicates that no more than one fault has occurred on the jth rotor of the n quadcopters. Let S_2 be

$$S_2 = \{\forall i = 1, 2, \cdots, n, |f_{i,j}| = f > 0\}$$

which indicates that the jth rotor of all n quadcopters has encountered the same fault. Let S_3 be

$$S_3 = \{\exists i_0 = 1, 2, \cdots, n, \forall i \neq i_0, |f_{i_0,j}| > |f_{i,j}| = f > 0\}$$

which indicates that the jth rotor of all n quadcopters has encountered a fault, while one of them is greater than the other equivalent ones. Let S_4 be

$$S_4 = S - S_1 - S_2 - S_3$$

which contains all other cases.

With these definitions above, *Assumption 4* could be described in the form of probability as $P(S_1) = 100\%$. The FDD procedure of this paper is based on the occurrence of S_1, but it would not happen definitely, which leads to the following four cases:

- If and only if S_1 happens, the fault could be detected and diagnosed correctly. By a simple calculation,
$$\begin{aligned}P(S_1) &= P(F_0) + P(F_1)\\ &= (1-\epsilon)^n + n(1-\epsilon)^{n-1}\epsilon.\end{aligned}$$
A necessary condition to apply the FDD procedure proposed in this paper is that $P(S_1)$ must be high enough, at least greater than 95%.
- If and only if S_2 happens, the FDD procedure fails to detect the fault.
- If and only if S_3 happens, the fault could be detected correctly but diagnosed falsely. S_2 and S_3 impose quantitative restrictions on F_n, so $S_2 \cup S_3 \subset F_n$, $P(S_2) + P(S_3) = P(S_2 \cup S_3) < P(F_n) = \epsilon^n$. The probability is low enough and tends to zero.
- If and only if S_4 happens, the occurrence of fault could be concluded, while no diagnosis conclusion could be drawn (see *Separable Condition*). $P(S_4) = 1 - P(S_1) - P(S_2) - P(S_3)$.

For example, suppose that $n = 4$, $\epsilon = 0.05(/h)$, then $P(S_1) \approx 0.986(/h)$, $P(S_2) + P(S_3) \approx 6.25 \times 10^{-6}(/h)$, $P(S_4) \approx 0.014(/h)$. On the one hand, the probability that *Assumption 4* is true is near 100%. On the other hand, we have a probability near 100% to correctly detect and diagnose the fault under *Assumption 4*. Therefore, *Assumption 4* is reasonable in the sense of probability.

3. OBSERVABILITY ANALYSIS

In this section, the feasibility of the proposed assumptions is further illustrated via observability analysis. For this purpose, first, some definitions and a lemma of nonlinear system observability are introduced, based on which the observability analysis is further carried out.

3.1 Preliminaries

Consider a general nonlinear system
$$\Sigma : \begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \\ \mathbf{y} = \mathbf{h}(\mathbf{x}) \end{cases} \tag{17}$$
where $\mathbf{x} \in \mathcal{M}$, an open subset of $\mathbb{R}^n$, $\mathbf{f}(\mathbf{x}) = [\, f_1 \ \cdots \ f_n \,]^{\mathrm{T}} \in \mathbb{R}^n$, $\mathbf{g}(\mathbf{x}) = [\, \mathbf{g}_1 \cdots \mathbf{g}_p \,] \in \mathbb{R}^{n\times p}$, $\mathbf{u} \in \mathbb{R}^p$, $\mathbf{h}(\mathbf{x}) = [\, h_1 \ \cdots \ h_m \,]^{\mathrm{T}} \in \mathbb{R}^m$. Let $\mathbf{g}_0 = \mathbf{f}(\mathbf{x})$.

For observability analysis, first, some definitions are given [Hermann and Krener (1977)].

Definition 1 ($\mathcal{U}$-Indistinguishability): A pair of state points $\mathbf{x}_0$ and $\mathbf{x}_1$ are $\mathcal{U}$*-indistinguishable* (denoted $\mathbf{x}_0 I_{\mathcal{U}} \mathbf{x}_1$) if, for any admissible input $\mathbf{u}(t)$ that ensures the trajectories $\mathbf{x}_0(t)$ and $\mathbf{x}_1(t)$ with initial states $\mathbf{x}_0$ and $\mathbf{x}_1$ lie in an open subset $\mathcal{U} \subset \mathcal{M}$, they realise always the same output, i.e., $\mathbf{x}_0(t), \mathbf{x}_1(t) \in \mathcal{U}$, $\mathbf{h}(\mathbf{x}_0(t)) = \mathbf{h}(\mathbf{x}_1(t))$. Denote $I_{\mathcal{U}}(\mathbf{x}_0) = \{\mathbf{x}|\mathbf{x} I_{\mathcal{U}} \mathbf{x}_0\}$ the set of all $\mathcal{U}$-indistinguishable states of $\mathbf{x}_0$.

The notation $I_{\mathcal{M}}$ is simplified to I.

Definition 2 (Observability): The system Σ is said to be *observable* at $\mathbf{x}_0$ if $I(\mathbf{x}_0) = \{\mathbf{x}_0\}$ and is further said to be *observable* if $\forall \mathbf{x} \in \mathcal{M}$, $I(\mathbf{x}) = \{\mathbf{x}\}$.

Notice that in the global concept of observability, it may be necessary to travel a long distance to distinguish the state points of $\mathcal{M}$. Therefore, a local while stronger concept is led in.

Definition 3 (Local Observability): The system Σ is said to be *locally observable* at $\mathbf{x}_0$ if for every neighborhood $\mathcal{U}$ of $\mathbf{x}_0$, $I_{\mathcal{U}}(\mathbf{x}_0) = \{\mathbf{x}_0\}$ and is said to be *locally observable* if it is so at every $\mathbf{x} \in \mathcal{M}$.

Let $\mathcal{U} = \mathcal{M}$, and clearly, local observability implies observability [Hermann and Krener (1977)].

However, in practice, it may suffice to be able to distinguish $\mathbf{x}_0$ from its neighbors, which leads to a weaker concept of observability.

Definition 4 (Weak Observability): The system Σ is said to be *weakly observable* at $\mathbf{x}_0$ if there exists a neighborhood $\mathcal{V}$ of $\mathbf{x}_0$ such that $I(\mathbf{x}_0) \cap \mathcal{V} = \{\mathbf{x}_0\}$ and Σ is *weakly observable* if it is so at every $\mathbf{x} \in \mathcal{M}$.

Notice again that it may be necessary to travel a long distance to distinguish the state points of $\mathcal{M}$. Therefore, a relatively stronger concept is led in.

Definition 5 (Local Weak Observability): The system Σ is said to be *locally weakly observable* at $\mathbf{x}_0$ if there exists a neighborhood $\mathcal{V}$ of $\mathbf{x}_0$ such that for every neighborhood $\mathcal{U}$ of $\mathbf{x}_0$, $I_{\mathcal{U}}(\mathbf{x}_0) \cap \mathcal{V} = \{\mathbf{x}_0\}$, and Σ is *locally weakly observable* if it is so at every $\mathbf{x} \in \mathcal{M}$.

The relationships of the four forms of observability is shown as below:
$$\begin{array}{ccc} \text{Local Observability} & \Rightarrow & \text{Observability} \\ \Downarrow & & \Downarrow \\ \text{Local Weak Observability} & \Rightarrow & \text{Weak Observability} \end{array}$$

In general, there are no other implications, but for autonomous linear systems, all the four concepts are equivalent. Generally speaking, the *local weak observability* is taken into consideration because it could be determined by a simple algebraic test.

Definition 6 (Unobservability): The system Σ is said to be *unobservable* if it does not satisfy any of the four concepts of observability. Considering the implications among them, Σ is *unobservable* if and only if it is not *weakly observable*.

Recall that $C^\infty(\mathcal{M})$ is the vector space of all smooth functions defined on $\mathcal{M}$, and $L_{\mathbf{g}}(h) = \nabla h \cdot \mathbf{g}$ represents the Lie derivation. Then the observation space could be defined.

Definition 7 (Observation Space): Denote $\mathcal{G}$ the *observation space*
$$\mathcal{G} = \mathrm{span}_{\mathbb{R}} \left\{ L_{\mathbf{g}_{s_1}} L_{\mathbf{g}_{s_2}} \dots L_{\mathbf{g}_{s_r}}(h_s), r \geq 0, s_r = 0, 1, \dots, p, \; s = 1, 2, \dots, m \right\}$$
which is the smallest subspace of $C^\infty(\mathcal{M})$ which contains the functions $h_1, \cdots, h_m$ and is closed under differentiation along the vector fields $\mathbf{f}, \mathbf{g}_1, \cdots, \mathbf{g}_p$. Denote $\nabla\mathcal{G}$ the gradient of $\mathcal{G}$
$$\nabla\mathcal{G} = \mathrm{span}_{\mathbb{R}_{\mathbf{x}}} \{\nabla\phi, \phi \in \mathcal{G}\}$$
where $\mathbb{R}_{\mathbf{x}}$ is the field consisted of meromorphic functions on $\mathcal{M}$.

The following lemma gives a necessary and sufficient condition of the local weak observability [Anguelova (2004)].

Lemma 1. The system Σ is locally weakly observable if and only if $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G}) = n$.

Remark 3. The dimension $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G})$ is constant on $\mathcal{M}$ except at certain singular points where the dimension is smaller. Therefore, $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G})$ is defined to be the generic or maximal dimension on $\mathcal{M}$, i.e. $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G}) = \max\limits_{\mathbf{x}\in\mathcal{M}}(\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G}(\mathbf{x})))$.

3.2 Observability Analysis of Quadcopters

The following four corollaries are given based on the definitions above and *Lemma 1*.

Solution to Problem (i)-(ii)

Corollary 1 (for Problem i). For the ith quadcopter, under *Assumption 1,* system Σ_i in (15) is *locally weakly observable* with the states being $\mathbf{x}_i$ and $\mathbf{f}_i + \mathbf{w}_i$, $i = 1, 2, \ldots, n$.

Corollary 2 (for Problem ii). For the ith quadcopter, under *Assumption 1,* system Σ_i in (16) is *unobservable* with the states being $\mathbf{x}_i, \mathbf{f}_i, \mathbf{w}_i$, $i = 1, 2, \ldots, n$.

It is shown in *Corollary 2* that, the fault $\mathbf{f}_i$ could not be detected and diagnosed only through a single quadcopter. As a consequence, the homogenous quadcopter team should be taken into consideration.

Solution to Problem (iii)

Corollary 3 (for Problem iii). For 1st,$\cdots$,nth quadcopters, under *Assumption 1,* the multiple system $(\Sigma_1, \cdots, \Sigma_n)$ is *unobservable* with the states being $\mathbf{x}_i, \mathbf{f}_i, \mathbf{w}_i$, $i = 1, 2, \ldots, n$.

Corollary 4 (for Problem iii). For 1st,$\cdots$,nth quadcopters, under *Assumptions 1-4*, the multiple system $(\Sigma_1, \cdots, \Sigma_n)$ is *locally weakly observable* with the states being $\mathbf{x}_i, \mathbf{f}_i, \mathbf{w}_i$, $i = 1, 2, \ldots, n$.

The proofs are given in the *Appendix*.

Remark 4. The four corollaries illustrate the feasibility of the proposed assumptions. Obviously, the fault $\mathbf{f}_i$ could not be diagnosed directly through one quadcopter. Since the model in *Corollary 4* considering a homogenous quadcopter team is relatively complex, an alternative is to estimate firstly the sum of the fault and wind disturbance $\mathbf{f}_i + \mathbf{w}_i$ and then to separate them utilizing *Assumptions 2-4*.

4. FDD PROCEDURE

While the first three problems in the *FDD Problem* have been solved in the previous section via observability analysis, this section aims to answer the fourth problem, i.e., how could the fault $\mathbf{f}_i$ be estimated to accomplish the FDD mission. First, the EKF is adopted to estimate the sum of fault and wind disturbance. After that, a theorem is given to diagnose the fault, by separating it from the wind disturbance.

In order to carry out EKF, the system should be discretized. Through the observability analysis in the previous section, the model in (15) is utilized. The EKF process is omitted here. For details, please refer to Chapter 8 in [Quan (2017)].

Applying EKF, the sum of fault and wind disturbance could be estimated as $\hat{\mathbf{f}}_{i,k} + \hat{\mathbf{w}}_{i,k}$, further the residual matrix $\hat{\mathbf{H}}_{f,i} + \hat{\mathbf{H}}_{w,i}$, through (11) and (12). However, they could not be distinguished, according to *Corollary 3*. Therefore, in this section, *Theorem 1* gives a solution to the FDD problem based on *Assumptions 3-4*. The following *Diagnosable Condition* is an extension of the *Assumption 4* to the executive step. Intuitively, the wind disturbance causes the same loss of effectiveness $\eta_{w,j}$. No more than one fault occurs, causing a loss of effectiveness $\eta_{f,i,j}$. Therefore, no more than one remaining ratio $\eta_{i,j} = 1 - \eta_{f,i,j} - \eta_{w,j}$ should be less than others. So does the estimation $\hat{\eta}_{i,j}$.

Diagnosable Condition: The *Diagnosable Condition* is satisfied for the jth$(j = 1, 2, 3, 4)$ rotor if

$$\forall i_1, i_2 = 1, 2, \cdots, n, (\max_i(\hat{\eta}_{i,j}) - \hat{\eta}_{i_1,j}) \cdot (\max_i(\hat{\eta}_{i,j}) - \hat{\eta}_{i_2,j}) = 0$$

which implies that no more than one $\hat{\eta}_{i,j}$ is less than others.

With it, we have the following theorem.

Theorem 1. Suppose that the *Diagnosable Condition* is satisfied. Then

$$\begin{cases} \hat{\eta}_{w,j} = 1 - \max\limits_i(\hat{\eta}_{i,j}) & \forall j = 1, 2, 3, 4 \\ \hat{\eta}_{f,i,j} = 1 - \hat{\eta}_{w,j} - \hat{\eta}_{i,j} & \forall i = 1, 2, \ldots, n \end{cases}.$$

Proof. The *Diagnosable Condition* ensures than no more than one $\hat{\eta}_{i,j}$ is less than others. Therefore, two cases may occur: they are all equal, i.e., $\hat{\eta}_{i,j} = \eta, \forall i = 1, 2, \cdots, n$, or one among them is less than other equivalent ones, i.e., $\exists i_0 = 1, 2, \cdots, n, \forall i \neq i_0, \hat{\eta}_{i,j} = \eta > \hat{\eta}_{i_0,j}$.

For the first case $\hat{\eta}_{i,j} = \eta, \forall i = 1, 2, \cdots, n$. Recall that $\hat{\eta}_{f,i,j} + \hat{\eta}_{w,j} + \hat{\eta}_{i,j} = 1$ according to (8), and the wind disturbance has the same influence $\hat{\eta}_{w,j}, j = 1, 2, 3, 4$ on each quadcopter according to *Assumption 3*. Therefore, $\hat{\eta}_{f,i,j}$ $(i = 1, 2, \cdots, n)$ coincide. Furthermore, *Assumption 4* with the restriction of no more than one fault concludes $\hat{\eta}_{f,i,j} = 0, \forall i = 1, 2, \cdots, n$.

For the second case $\exists i_0 = 1, 2, \cdots, n, \forall i \neq i_0, \hat{\eta}_{i,j} = \eta > \hat{\eta}_{i_0,j}$. Similarly the conclusion that $\forall i \neq i_0, \hat{\eta}_{f,i,j} = 0$ is drawn, which indicates that $\hat{\eta}_{f,i_0,j} = \eta - \hat{\eta}_{i_0,j} > 0$.

In both two cases, $\hat{\eta}_{w,j} = 1 - \eta = 1 - \max\limits_i(\hat{\eta}_{i,j})$. □

Remark 5 (on *Theorem 1***)**. *Remark 5* extends *Remark 2* to give a probabilistic illustration to the reliability of *Theorem 1*. In the first case where $\hat{\eta}_{i,j} = \eta, \forall i = 1, 2, \cdots, n$, we conclude that $\hat{\eta}_{f,i,j} = 0, \forall i = 1, 2, \cdots, n$ which we define an event E_0. Of course, there is a possibility that $\hat{\eta}_{f,i,j} = \eta_0 > 0, \forall i = 1, 2, \cdots, n$ which we define another event E_1. According to *Remark 2*, $P(E_1) \approx 0$. Therefore, the probability that the estimation is correct is $P(E_0|E_0 \cup E_1) = \frac{P(E_0)}{P(E_0)+P(E_1)} \approx 100\%$. Similarly the second case. In the case where the *Diagnosable Condition* is not satisfied, only the conclusion that the fault has occurred could be drawn. Thus, *Theorem 1* supplies a reliable algorithm.

The estimate of fault $\hat{\mathbf{f}}_i$ could be evaluated based on the loss caused by fault $\hat{\mathbf{H}}_{f,i} = \mathrm{diag}(\hat{\eta}_{f,i,1}, \hat{\eta}_{f,i,2}, \hat{\eta}_{f,i,3}, \hat{\eta}_{f,i,4})$ through (11). If the *Diagnosable Condition* is not satisfied, the occurrence of fault is assured, while no diagnosis conclusion could be drawn. However, its probability is rather low. Up to now, the procedure to solve the FDD problem could be summarized in Table 1.

Table 1. FDD Procedure

Step 1	State estimation of $\hat{\mathbf{x}}_i, i = 1, 2, \cdots, n$, including the sum of fault and wind disturbance $\hat{\mathbf{f}}_i + \hat{\mathbf{w}}_i$, utilizing the model (15) through EKF;
Step 2	Calculation of the residual matrix $\hat{\mathbf{H}}_i$, based on the sum of fault and wind disturbance $\hat{\mathbf{f}}_i + \hat{\mathbf{w}}_i$, $i = 1, 2, \cdots, n$, through (13);
Step 3	Verification of whether the *Diagnosable Condition* is satisfied for the jth rotor, $j = 1, 2, 3, 4$. If yes, goto Step 4; Else the algorithm ends with merely the conclusion that the fault has occurred on the jth rotor of at least one quadcopter;
Step 4	Separation of the loss of effectiveness caused by the fault $\hat{\eta}_{f,i,j}$ from that of the wind disturbance $\hat{\eta}_{w,j}$, by a comparison among the homogenous quadcopter team (*Theorem 1*);
Step 5	Estimation of the fault $\hat{\mathbf{f}}_i$ and wind disturbance $\hat{\mathbf{w}}$, based on the estimation $\hat{\mathbf{H}}_{f,i}$ and $\hat{\mathbf{H}}_w$, through (11) and (12), respectively.

5. SIMULATION

Four quadcopters are adopted here for the simulation, hovering at $10m$ under the control of a PD controller, and the parameters are given in Table 2.

Table 2. Simulation Parameters

Sampling time T_{step}	$0.01s$
Distance between rotors and center of mass d	$0.28m$
Coefficient from lift to torque k_μ	$1m$
Moment of inertia J_{xx}	$0.0411kg \cdot m^2$
Moment of inertia J_{yy}	$0.0478kg \cdot m^2$
Moment of inertia J_{zz}	$0.0599kg \cdot m^2$
Lift supplied by rotors $T_{ij} \in [a, b]N$	$T_{ij} \in [0, 6]\ N$
Mass m	$1.535kg$

They are placed under the same wind field, and their residual matrix are, $\mathbf{H}_1$ =diag$(0.9, 0.65, 0.9, 0.9)$, and $\mathbf{H}_2 = \mathbf{H}_3 = \mathbf{H}_4$ =diag$(0.9, 0.9, 0.9, 0.9)$. Only the simulation results of #1 rotor and #2 rotor are shown in that the situations of rotors #2, #3 and #4 are nearly the same. The simulation result of #1 rotor is given in Fig.1. It is shown that $\hat{\eta}_{1,2} = \hat{\eta}_{2,2} = \hat{\eta}_{3,2} = \hat{\eta}_{4,2} = 1 - 0.1 = 0.9$. It is inferred by *Theorem 1* that $\hat{\eta}_{w,2} = 0.1; \hat{\eta}_{f,1,2} = \hat{\eta}_{f,2,2} = \hat{\eta}_{f,3,2} = \hat{\eta}_{f,4,2} = 0$. That is to say: (i) The wind disturbance causes 10% loss of effectiveness for #1 rotor; (ii) For #1 rotor of all four quadcopters, the fault does not occur. The simulation result of #2 rotor is given in Fig.2. It is shown that $\hat{\eta}_{1,2} = 1 - 0.35 = 0.65$, $\hat{\eta}_{2,2} = \hat{\eta}_{3,2} = \hat{\eta}_{4,2} = 1 - 0.1 = 0.9$. It is inferred by *Theorem 1* that $\hat{\eta}_{w,2} = 0.1$, $\hat{\eta}_{f,1,2} = 0.25$, $\hat{\eta}_{f,2,2} = \hat{\eta}_{f,3,2} = \hat{\eta}_{f,4,2} = 0$. That is to say (i) The wind disturbance causes 10% loss of effectiveness for #2 rotor; (ii) For #2 rotor of #1 quadcopter, the fault causes 25% loss of effectiveness; (iii) For #2 rotor of the others, the fault does not occur.

Consider another case where their residual matrix are, $\mathbf{H}_1 = \mathbf{H}_2$ =diag$(0.65, 0.9, 0.9, 0.9)$, and $\mathbf{H}_3 = \mathbf{H}_4$ =diag$(0.9, 0.9, 0.9, 0.9)$. The simulation result of #1 rotor is given in

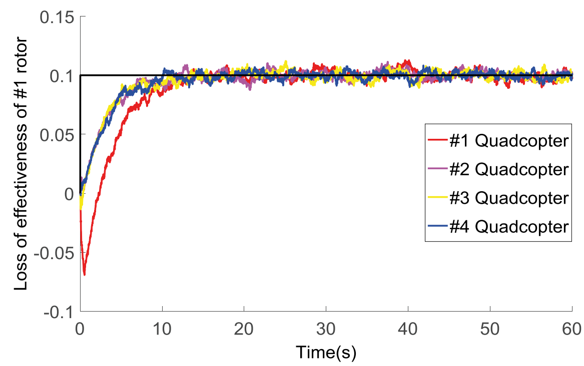

Fig. 1. Simulation result of rotor #1

Fig.3. It is shown that $\hat{\eta}_{1,2} = \hat{\eta}_{2,2} = 1 - 0.35 = 0.65, \hat{\eta}_{3,2} = \hat{\eta}_{4,2} = 1 - 0.1 = 0.9$. In this situation, the *Diagnosable Condition* is not satisfied for #1 rotor, and merely the occurrence of fault on the #2 rotor could be concluded, with no diagnosis conclusion. While, the probability is rather low, as emphasized in *Remark 2*. The simulation results of #2-4 rotors are like that of #1 rotor in the previous case, and the same conclusion could be drawn.

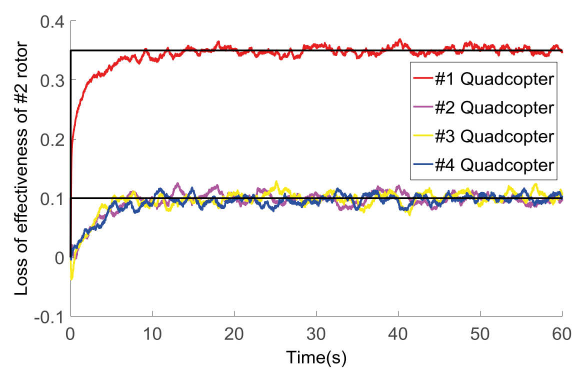

Fig. 2. Simulation result of rotor #2

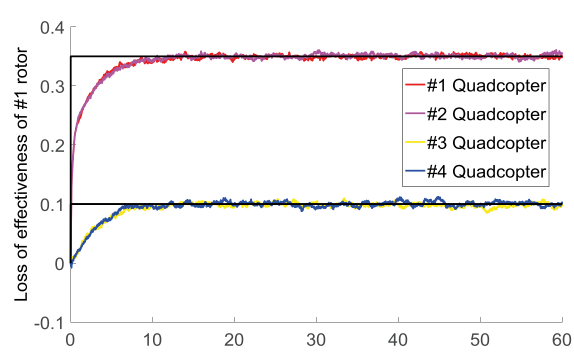

Fig. 3. Simulation result of rotor #1

6. CONCLUSIONS

This paper proposes an FDD method which could estimate the fault of a homogenous quadcopter team while eliminating the effect of the wind disturbance. Its main idea is to evaluate the sum effect of fault and the wind disturbance, regarding them as additional states and utilizing EKF. Then, by a comparison among the homogenous quadcopters in a team under several reasonable assumptions, the effect of fault is distinguished from that of the wind disturbance. Simulation results demonstrate that it is a simple but useful method. For simplicity, the wind disturbance is supposed constant, which is usually not true. Therefore, the next goal is to study the case under a time-varying wind field. Furthermore, merely the FDD is not adequate. As a consequence, the future study includes the reliability analysis of a quadcopter team.

ACKNOWLEDGEMENTS

The financial supports from the National Natural Science Foundation of China (61473012) for this work are greatly acknowledged.

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Appendix A. PROOFS OF COROLLARIES

In the following, $\mathbf{x}_i$ is the traditional state defined in (1); $\mathbf{G}(\mathbf{x}_i)$ and $\mathbf{B}(\mathbf{x}_i)$ are the matrices defined in (3) and (4), respectively; $\mathbf{f}_i$ and $\mathbf{w}_i$ are the fault and wind disturbance defined in (11) and (12).

A.1 Proof of Corollary 1

The system (15) could be written in the form of (17) with

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_i \\ \mathbf{f}_i + \mathbf{w}_i \end{bmatrix}, \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{G}(\mathbf{x}_i) - \mathbf{B}(\mathbf{x}_i)(\mathbf{f}_i + \mathbf{w}_i) \\ \mathbf{0}_{4\times 1} \end{bmatrix}$$
$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \mathbf{B}(\mathbf{x}_i) \\ \mathbf{0}_{4\times 1} \end{bmatrix}, \mathbf{h}(\mathbf{x}) = \mathbf{x}_i.$$

The proof is to determine the dimension $\dim_{\mathbb{R}_\mathbf{x}}(\nabla\mathcal{G})$ of the gradient $\nabla\mathcal{G}$ of the observation space $\mathcal{G} = \text{span}_{\mathbb{R}_\mathbf{x}} \{L_{\mathbf{g}_{s_1}} L_{\mathbf{g}_{s_2}} \ldots L_{\mathbf{g}_{s_r}}(h_s), r \geq 0, s_r = 0,1,2,3,4, s = 1, 2, \ldots, 8\}$, namely the maximum number of linearly independent vectors in $\nabla\mathcal{G}$ regardless of the singular points.

First, when $r = 0$, $\mathbf{v}_s = \nabla h_s, s = 1, 2, \cdots, 8$ supply 8 linearly independent vectors:

$$\begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_8 \end{bmatrix} = \begin{bmatrix} \nabla h_1 \\ \vdots \\ \nabla h_8 \end{bmatrix} = [\, \mathbf{I}_8 \;\; \mathbf{0}_{8\times 4} \,].$$

Second, when $r = 1$, we can obtain the 4 following vectors:

$$L_\mathbf{f}(h_5) = \frac{(J_{yy} - J_{zz})r_i q_i - (f_{i,1} + w_{i,1})}{J_{xx}},$$
$$L_\mathbf{f}(h_6) = \frac{(J_{zz} - J_{xx})p_i r_i - (f_{i,2} + w_{i,2})}{J_{yy}},$$
$$L_\mathbf{f}(h_7) = \frac{(J_{xx} - J_{yy})p_i q_i - (f_{i,3} + w_{i,3})}{J_{zz}},$$
$$L_\mathbf{f}(h_8) = g + \frac{(f_{i,4} + w_{i,4})\cos\phi_i \cos\theta_i}{m} \Rightarrow$$
$$\mathbf{v}_9 = \nabla L_\mathbf{f}(h_5) = \begin{bmatrix} *_{1\times 8} & -\frac{1}{J_{xx}} & 0 & 0 & 0 \end{bmatrix},$$
$$\mathbf{v}_{10} = \nabla L_\mathbf{f}(h_6) = \begin{bmatrix} *_{1\times 8} & 0 & -\frac{1}{J_{yy}} & 0 & 0 \end{bmatrix},$$
$$\mathbf{v}_{11} = \nabla L_\mathbf{f}(h_7) = \begin{bmatrix} *_{1\times 8} & 0 & 0 & -\frac{1}{J_{zz}} & 0 \end{bmatrix},$$
$$\mathbf{v}_{12} = \nabla L_\mathbf{f}(h_8) = \begin{bmatrix} *_{1\times 8} & 0 & 0 & 0 & \frac{\cos\phi_i \cos\theta_i}{m} \end{bmatrix}.$$

We have obtained 12 linearly independent vectors in $\nabla\mathcal{G}$:

$$\begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{12} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_8 & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} \\ *_{1\times 8} & -\frac{1}{J_{xx}} & 0 & 0 & 0 \\ *_{1\times 8} & 0 & -\frac{1}{J_{yy}} & 0 & 0 \\ *_{1\times 8} & 0 & 0 & -\frac{1}{J_{zz}} & 0 \\ *_{1\times 8} & 0 & 0 & 0 & \frac{\cos\phi_i \cos\theta_i}{m} \end{bmatrix}.$$

Therefore, $\dim_{\mathbb{R}_\mathbf{x}}(\nabla\mathcal{G}) = 12$. According to *Lemma 1*, the 12-dimension system Σ_i is locally weakly observable.

A.2 Proof of Corollary 2

The system (16) could be written in the form of (17) with

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_i \\ \mathbf{f}_i \\ \mathbf{w}_i \end{bmatrix}, \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{G}(\mathbf{x}_i) - \mathbf{B}(\mathbf{x}_i)(\mathbf{f}_i + \mathbf{w}_i) \\ \mathbf{0}_{8\times 1} \end{bmatrix}$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \mathbf{B}(\mathbf{x}_i) \\ \mathbf{0}_{8\times 1} \end{bmatrix}, \mathbf{h}(\mathbf{x}) = \mathbf{x}_i.$$

We analyse directly the definitions of the four forms of observability. It is easy to find that, for every initial state $\mathbf{x}_{ini,0} \in \mathcal{M}$

$$\mathbf{x}_{ini,0} = [\, \mathbf{x}_{ini} \ f_1 \ f_2 \ f_3 \ f_4 \ w_1 \ w_2 \ w_3 \ w_4 \,]^{\mathrm{T}}$$

where all these parameters are constant, there exists another initial state $\mathbf{x}_{ini,1}$ for example

$$\mathbf{x}_{ini,1} = [\, \mathbf{x}_{ini} \ f_1 + \epsilon \ f_2 \ f_3 \ f_4 \ w_1 - \epsilon \ w_2 \ w_3 \ w_4 \,]^{\mathrm{T}}$$

where $\epsilon > 0$ which could be infinitely small, such that $\mathbf{x}_{ini,0}$ and $\mathbf{x}_{ini,1}$ realise always the same output

$$\mathbf{h}(\mathbf{x}_{ini,0}(t)) = \mathbf{h}(\mathbf{x}_{ini,1}(t))$$

which implies that for every neighborhood $\mathcal{V}$ of $\mathbf{x}_{ini,0}$, $I(\mathbf{x}_{ini,0}) \cap \mathcal{V} \supsetneq \{\mathbf{x}_{ini,0}\}$. Σ_i does not even meet the definition of weak observability, which is the weakest among the four concepts. Thus, Σ_i does not meet any of the four concepts. It is unobservable.

A.3 Proof of Corollary 3

The system Σ consisted of n homogenous systems in (16) could be written in the form of (17) with

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{f}_1 \\ \mathbf{w}_1 \\ \vdots \\ \mathbf{x}_n \\ \mathbf{f}_n \\ \mathbf{w}_n \end{bmatrix}, \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{G}(\mathbf{x}_1) - \mathbf{B}(\mathbf{x}_1)(\mathbf{f}_1 + \mathbf{w}_1) \\ \mathbf{0}_{8\times 1} \\ \vdots \\ \mathbf{G}(\mathbf{x}_n) - \mathbf{B}(\mathbf{x}_n)(\mathbf{f}_n + \mathbf{w}_n) \\ \mathbf{0}_{8\times 1} \end{bmatrix}$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \mathbf{B}(\mathbf{x}_1) \\ \mathbf{0}_{8\times 1} \\ \vdots \\ \mathbf{B}(\mathbf{x}_n) \\ \mathbf{0}_{8\times 1} \end{bmatrix}, \mathbf{h}(\mathbf{x}) = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix}. \tag{A.1}$$

Similar with the previous one, it could be proved with ease that Σ does not meet any of the four definitions and hence is unobservable.

A.4 Proof of Corollary 4

Reconsider the system Σ in (A.1). *Assumption 2* simplifies the wind disturbance $\mathbf{w}_i, i = 1, 2, \cdots, n$ to $\mathbf{w}$. Based on *Assumption 2*, combining *Corollary 1* which gives the observability basis of (15) to evaluate the sum of fault and wind disturbance and *Assumption 4* that assures no more than one fault, the fault could be located, which would be summarized as *Theorem 1*. Therefore, the fault $\mathbf{f}_i, i = 1, 2, \cdots, n$ could be simplified to $\mathbf{f}_s = [\, f_{i_1,1} \ f_{i_2,2} \ f_{i_3,3} \ f_{i_4,4} \,]^{\mathrm{T}}$, where $f_{i_j,j}$ represents the fault of the jth rotor of a certain (i_jth) quadcopter, and *Assumption 4* ensures that all other faults are zero. The reconsidered model could be rewritten in the form of (17) with

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \\ \mathbf{f}_s \\ \mathbf{w} \end{bmatrix}, \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{G}(\mathbf{x}_1) - \mathbf{B}(\mathbf{x}_1)(\mathbf{f}_1 + \mathbf{w}) \\ \vdots \\ \mathbf{G}(\mathbf{x}_n) - \mathbf{B}(\mathbf{x}_n)(\mathbf{f}_n + \mathbf{w}) \\ \mathbf{0}_{8\times 1} \end{bmatrix}$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \mathbf{B}(\mathbf{x}_1) \\ \vdots \\ \mathbf{B}(\mathbf{x}_n) \\ \mathbf{0}_{8\times 1} \end{bmatrix}, \mathbf{h}(\mathbf{x}) = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix}.$$

The proof is also to determine the dimension $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G})$, namely the maximum number of linearly independent vectors in $\nabla\mathcal{G}$ regardless of the singular points. First, when $r = 0$, $8n$ trivial linearly independent vectors are obtained as

$$\begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{8n} \end{bmatrix} = \begin{bmatrix} \nabla h_1 \\ \vdots \\ \nabla h_{8n} \end{bmatrix} = [\mathbf{I}_{8n} \ \mathbf{0}_{8n\times 8}].$$

Second, when $r = 1$: through the calculation of the rotors without fault, we can obtain 4 linearly independent vectors as in the proof of *Corollary 1* that

$$\begin{bmatrix} \mathbf{v}_{8n+1} \\ \vdots \\ \mathbf{v}_{8n+4} \end{bmatrix} = \begin{bmatrix} \nabla L_{\mathbf{f}}(h_*) \\ \vdots \\ \nabla L_{\mathbf{f}}(h_*) \end{bmatrix} = [*_{4\times 8n} \ \mathbf{0}_{4\times 4} \ \mathbf{A}_*]$$

where

$$\mathbf{A}_* = \begin{bmatrix} -\frac{1}{J_{xx}} & 0 & 0 & 0 \\ 0 & -\frac{1}{J_{yy}} & 0 & 0 \\ 0 & 0 & -\frac{1}{J_{zz}} & 0 \\ 0 & 0 & 0 & \frac{\cos\phi_*\cos\theta_*}{m} \end{bmatrix}.$$

Through the calculation of the rotors with fault, we can obtain the linearly independent vectors as in the proof of *Corollary 2* that

$$\begin{bmatrix} \mathbf{v}_{8n+5} \\ \vdots \\ \mathbf{v}_{8n+8} \end{bmatrix} = \begin{bmatrix} \nabla L_{\mathbf{f}}(h_*) \\ \vdots \\ \nabla L_{\mathbf{f}}(h_*) \end{bmatrix} = [\, *_{4\times 8n} \ \mathbf{A}_* \ \mathbf{A}_* \,].$$

We have obtained $(8n+8)$ linearly independent vectors in $\nabla\mathcal{G}$

$$\begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_{8n+8} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{8n} & \mathbf{0}_{4\times 4} & \mathbf{0}_{4\times 4} \\ *_{4\times 8n} & \mathbf{0}_{4\times 4} & \mathbf{A}_* \\ *_{4\times 8n} & \mathbf{A}_* & \mathbf{A}_* \end{bmatrix}.$$

Therefore, $\dim_{\mathbb{R}_{\mathbf{x}}}(\nabla\mathcal{G}) = 8n + 8$. According to *Lemma 1*, the $(8n+8)$-dimension system Σ is locally weakly observable.