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ABSTRACT

The robust sampled-data repetitive control (RC, or repetitive controller, also designated RC) problem for non-minimum phase nonlinear systems is both challenging and practical. This paper proposes a sampled-data output-feedback RC design for a class of non-minimum phase systems with measurable nonlinearities to improve the robustness against the period variation. The design relies on additive-state decomposition, by which the output-feedback RC problem is decomposed into an output-feedback RC problem for a linear time-invariant component and a state-feedback stabilisation problem for a nonlinear component. Thanks to the decomposition, existing controller design methods in both the frequency domain and the time domain are employed to make the robustness and discretisation for a continuous-time nonlinear system tractable. In order to demonstrate the effectiveness, an illustrative example is given.

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Repetitive control; sampled-data systems; non-minimum phase; nonlinear systems; robustness; additive state decomposition

1. Introduction

In nature, numerous examples of periodic phenomena are found and observed, ranging from the orbital motion of heavenly bodies to the rhythm of hearts. In practice, many control tasks are of periodic nature as well. When executing operations of picking, placing or painting, industrial manipulators are often required to track or reject periodic exogenous signals (Fateh et al., 2013). Besides these, special applications further include magnetic spacecraft attitude control (Pittelkau, 1993; Silani & Lovera, 2005), active control of vibrations in helicopters (Arcara et al., 1997; Bittanti & Cuzzola, 2002), autonomous vertical landing on an oscillating platform (Isidori et al., 2003; Marconi et al., 2002), harmonics elimination in aircraft power supplies (Escobar et al., 2006), satellite formation (Hu et al., 2014), LED light tracking (Scalzi et al., 2015), control of hydraulic servomechanisms (Yao et al., 2015), and control of lower limb exoskeleton (Yang et al., 2016). Repetitive Control (RC, or repetitive controller, also designated RC) is a control method used specifically in tracking or rejecting periodic signals (Hara et al., 1988; Longman, 2010; Quan & Cai, 2010).

- One of the major drawbacks of RC is that the control accuracy is sensitive to the period variation of the external signals. It has been shown in Steinbuch (2002) that, with a period variation as small as 1.5% for an LTI system, the gain of the internal model part of the RC drops from ∞ to 10. As a result, the tracking accuracy may be far from satisfactory, especially for high-precision control. For such a purpose, higher-order RCs composed of several delay blocks in series were proposed to improve the robustness of the control accuracy against period variation (Kurniawan et al., 2014a; Kurniawan & Cao, 2014b; Pipeleers et al., 2008; Steinbuch, 2002; Steinbuch et al., 2007). However, these methods are inapplicable to nonlinear systems directly as they are all based on transfer functions and frequency-domain analysis. *The primary motivation* is to design an RC for nonlinear systems to improve the robustness against the period variation.
- Although modern control systems are often implemented via digital processors, RC schemes are often designed for nonlinear continuous-time systems. Unlike LTI systems, the zero-order hold equivalent of a nonlinear continuous-time system cannot be

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represented as explicit, exact sampled-data models. Only approximate controller design methods can be applied by taking the discretisation error as an external disturbance. This often requires the resulting closed-loop system to be input-to-state stable (ISS, or input-to-state stability, also designated ISS) with respect to the discretisation error (Nesić & Teel, 2001). However, a linear continuous-time RC system is a neutral-type system in a critical case.[1] The characteristic equation of the neutral-type system has an infinite sequence of poles with negative real parts approaching zero. Consequently, only non-exponential stability can be guaranteed in the critical case (Quan et al., 2010). This further implies that the ISS property cannot be obtained. Therefore, in theory, a sampled-data RC cannot be derived by discretising a continuous-time RC directly for nonlinear systems. *The second motivation* is to design a sampled-data RC for continuous-time nonlinear systems.

For clarity, a sampled-data *output-feedback* robust RC problem for a class of systems with measurable nonlinearities is considered in this paper. It is not easy to convert the considered system into a linear system directly with a periodic disturbance term (see Appendix A.1). The first reason is that the output is only available from measurements. This makes feedback linerization difficult. The other reason is the unknown periodic disturbance. This makes state estimation difficult because a general periodic disturbance cannot be modelled as a finite-dimensional autonomous exosystem. Even if it can be modelled as a finite-dimensional autonomous exosystem, the subsequent design and proof will be inconvenient. In addition, to say the least, if the considered system can be reduced to a linear system with a periodic disturbance term, the continuous-time nonlinear control term (compensate for nonlinearity) and continuous-time linear RC term (tracking and rejection) will be coupled together. It is still problematic how to guarantee the stability under discretisation.

Based on these reasons mentioned above, the sampled-data output-feedback robust RC problem is solved under a recently developed tracking framework, named the *additive-state-decomposition-based tracking control framework* (Quan et al., 2015, 2014). The key idea is to decompose the output-feedback RC problem into two well-solved control problems by additive-state decomposition: an output-feedback RC for an LTI component and a state-feedback stabilising control for a nonlinear component. Since the RC problem is only limited to the LTI component, existing robust higher-order RC methods can be applied directly. Moreover, according to the properties of the two control problems, two different methods are adopted to design sampled-data controllers, i.e. the sampled-data model design for the LTI component and the emulation design for the nonlinear component.[2] Finally, one can combine the sampled-data output-feedback robust RC with the sampled-data state-feedback stabilising controller to achieve the original control goal. The design process here is similar to Quan et al. (2015), namely the additive-state decomposition, observer design, controllers design for the primary and secondary systems, and controller integration. But, this paper focuses itself only on the discrete-time domain and robust RC problem rather than the continuous-time domain and general tracking problem. For example, the relationship between the sampled-data nonlinear RC system and the semiglobal practical stability property is established. The contributions of this paper are (i) the robust sampled-data RC problem is solved for a class of non-minimum phase nonlinear systems *for the first time* (covering the sampled-data output-feedback robust RC problem for a class of nonlinear continuous-time systems); (ii) more importantly, an intermediate step is built between existing RC design methods for LTI systems and a class of nonlinear systems so that more RC problems for nonlinear systems become tractable once this step is taken, such as rejection of nonperiodic disturbances (Kurniawan et al., 2014a; Kurniawan & Cao, 2014b) and the transient behaviour improvement (Chen & Tomizuka, 2014).

The following notations are used. $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}_+$ is the set of positive real numbers. $\mathbb{N}$ denotes the set of nonnegative integers. $\|\cdot\|$ denotes the Euclidean vector norm or induced matrix norm. The symbol $f \in L_\infty$ implies that $\|f\|_\infty \triangleq \sup_{t\in[0,\infty)} \|f(t)\| < \infty$. $\mathcal{L}$ and $\mathcal{L}^{-1}$ denote the Laplace transform and the inverse Laplace transform, respectively. $\mathcal{Z}$ and $\mathcal{Z}^{-1}$ denote the Z-transform and the inverse Z-transform, respectively. The following definitions can also be found in Khalil (2002). A continuous function $\alpha : [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. Furthermore, it belongs to class $\mathcal{K}_\infty$ if

$a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

2. Problem formulation

2.1. System description

Consider the class of single–input–single–output (SISO) nonlinear systems (Ding, 2003; Lee & Tsao, 2004; Marino & Tomei, 1995):

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) + \phi\left(y(t)\right) + d(t), x(0) = x_0 \\ y(t) &= c^{\mathrm{T}} x(t) \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is a constant matrix, $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ are constant vectors, $\phi : \mathbb{R} \to \mathbb{R}^n$ is a nonlinear function with $\phi(0) = 0$, $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the control, and $d(t) \in \mathbb{R}^n$ is a periodic bounded disturbance with a period $T > 0$. The reference $r(t) \in \mathbb{R}$ is sufficiently smooth with a period T. In the following, for convenience, we will omit the variable t except when necessary. Two assumptions on the nonlinear system (1) are made as follows:

Assumption 2.1: The pair (A, c^{T}) is observable and the matrix $A \in \mathbb{R}^{n\times n}$ is stable.

Assumption 2.2: Only $y(t)$ is available from measurements.

Remark 2.1: If A is unstable, then, by the observability of (A, c^{T}) in Assumption 2.1, there always exists a vector $p \in \mathbb{R}^n$ such that $A + pc^{\mathrm{T}}$ is stable, whose eigenvalues can be assigned freely (Kautsky & Nichols, 1985). Then, (1) can be rewritten as $\dot{x} = (A + pc^{\mathrm{T}})x + bu + (\phi(y) - py) + d$. Therefore, without loss of generality, A is assumed to be stable. There is no constraint on ϕ in contrast to a more specific class usually assumed in other literature. If $\phi(0) = a \neq 0$, then (1) can be rewritten as $\dot{x}(t) = Ax(t) + bu(t) + \phi'(y(t)) + (d(t) + a)$, where $\phi'(y(t)) = \phi(y(t)) - a$ with $\phi'(0) = 0$. Furthermore, if (1) is subject to model uncertainties like $\dot{x} = Ax + bu + \phi(y) + \Delta\phi(x) + d$ and the model uncertainty $\Delta\phi(x)$ is not too large, then the controller can also be designed according to (1) by taking $\Delta\phi(x) + d$ as a new disturbance. The robustness analysis is further carried out based on the small gain theorem (Wei et al., 2016). Here, for simplicity, $\Delta\phi(x)$ is ignored.

In the following, the non-minimum or minimum phase property of the nonlinear system (1) is discussed.

Proposition 2.1: *If and only if the following linear system*

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= c^{\mathrm{T}} x. \end{aligned} \tag{2}$$

is non-minimum or minimum phase, then system (1) without external signals is non-minimum or minimum phase, respectively.

Proof: See Appendix 6.2. ■

For system (2), the transfer function from u to y is

$$c^{\mathrm{T}}\left(sI_n - A\right)^{-1} b = \frac{N(s)}{D(s)}.$$

According to Proposition 2.1, system (1) without external signals is non-minimum or minimum phase, if $N(s)$ has zeros on the right-half s-plane or open left-half s-plane, respectively.

2.2. Objective

Let $T > 0$ be the period of the disturbance and the reference in practice. The continuous-time system (1) is controlled by using a sampled-data RC with a sampling period $T_s > 0$, where NT_s instead of T is taken as the period in the sampled-data controller design, $N \in \mathbb{N}$. More precisely, u in (1) is constant during the sampling interval, so that $u(t) = u(kT_s)$, $t \in [kT_s, (k+1)T_s)$, $k \in \mathbb{N}$. In practice, the disturbance period of T is not known exactly or is varying, namely period T is uncertain. This will cause the period variation, namely $T - NT_s$. On the other hand, since NT_s instead of T is used in the controller and $NT_s \neq T$ in general, T can be also considered as a variation of NT_s. Let $T = NT_s + \Delta$ be the true period, where Δ is the perturbation. By using NT_s in the design, $y{-}r$ is uniformly ultimately bounded with the ultimate bound $d_{e_\Delta} > 0$.

Under Assumptions 2.1 and 2.2, for a given desired output r, the objective is to design a sampled-data output-feedback RC for the nonlinear system (1) such that $y-r$ is uniformly ultimately bounded with the ultimate bound robust against the period variation. Here, robustness can be roughly understood that d_{e_Δ} is not sensitive to Δ or say d_{e_Δ}/Δ is small.

Remark 2.2: Here, for simplicity, the higher-order RC is used to handle the period variation caused by multiple sources. If only the period variation caused by sampling needs to be compensated for, the interpolator design or low-pass filter modification is suggested to use (Longman, 2010, pp. 477–480).

3. Sampled-data output-feedback robust RC by additive state decomposition

Based on additive-state decomposition, the considered system (1) is decomposed into two subsystems: an LTI system including all external signals as the primary system, together with a secondary nonlinear system whose equilibrium point is zero, as shown in Figure 1. In the following, the decomposition process and benefits from decomposition are introduced.

3.1. Additive state decomposition

3.1.1. Decomposition process

Consider the system (1) as the original system. The primary system is chosen as follows:

$$\begin{aligned} \dot{x}_p &= Ax_p + bu_p + \phi(r) + d \\ y_p &= c^{\mathrm{T}} x_p, x_p(0) = x_0. \end{aligned} \tag{3}$$

Then the secondary system is determined by subtracting the primary system (3) from the original system (1) as

$$\begin{aligned} \dot{x} - \dot{x}_p &= Ax + bu + \phi(y) + d \\ &\quad - \left(Ax_p + bu_p + \phi(r) + d\right) \\ y - y_p &= c^{\mathrm{T}} x - c^{\mathrm{T}} x_p, x(0) - x_p(0) = 0. \end{aligned} \tag{4}$$

Let

$$x_s \triangleq x - x_p, y_s \triangleq y - y_p, u_s \triangleq u - u_p. \tag{5}$$

Then the secondary system (4) is further written as

$$\begin{aligned} \dot{x}_s &= Ax_s + bu_s + \phi\left(r + y_s + e_p\right) - \phi(r) \\ y_s &= c^{\mathrm{T}} x_s, x_s(0) = 0 \end{aligned} \tag{6}$$

where $e_p \triangleq y_p - r$. According to the definitions,

$$y \equiv r + y_s + e_p.$$

If $e_p \equiv 0$, then

$$\begin{aligned} \dot{x}_s &= Ax_s + bu_s + \phi\left(r + c^{\mathrm{T}} x_s\right) - \phi(r) \\ y_s &= c^{\mathrm{T}} x_s, x_s(0) = 0. \end{aligned} \tag{7}$$

Thus $(x_s(t), u_s(t)) \equiv 0$ is an equilibrium point of (7) because it can make the left and right sides equal no matter what the reference r is. According to (5), it holds that

$$x = x_p + x_s, y = y_p + y_s, u = u_p + u_s. \tag{8}$$

Controller design for the decomposed systems (3) and (4) will use the output y_p and state x_s as feedback. For such a purpose, an observer is proposed.

Lemma 3.1 (Quan et al., 2015): *Suppose that an observer is designed to estimate y_p and x_s in (3), (6) as follows:*

$$\hat{y}_p = y - c^{\mathrm{T}} \hat{x}_s \tag{9}$$

$$\dot{\hat{x}}_s = A\hat{x}_s + bu_s + \phi(y) - \phi(r), \hat{x}_s(0) = 0. \tag{10}$$

Then $\hat{y}_p \equiv y_p$ and $\hat{x}_s \equiv x_s$.

Proof: Subtracting (10) from (6) results in $\dot{\tilde{x}}_s = A\tilde{x}_s$, $\tilde{x}_s(0) = 0$, where $\tilde{x}_s = x_s - \hat{x}_s$. Then $\tilde{x}_s \equiv 0$. This implies that $\hat{x}_s \equiv x_s$. Consequently, by (8), we have $\hat{y}_p \equiv y - c^T \hat{x}_s \equiv y_p$. ■

Remark 3.1: The observer proposed in Lemma 3.1 is different from classical observers because the state estimate of x is still unknown. Also, the proposed observer does not estimate the unknown disturbance or equivalent-input disturbance (She et al., 2008; Zhou & Li, 2018). A further explanation about the result of Lemma 3.1 is given in the following. Since (6) and (10) are only the models existing in the design, the initial values $x_s(0), \hat{x}_s(0)$ are both assigned by the designer and are all determinate. With this, $\hat{x}_s \equiv x_s$. Consequently, $\hat{y}_p \equiv y_p$. This will simplify the following stability analysis. The measurement y may be inaccurate in practice. In this case, it is expected that small uncertainties still maintain $\hat{x}_s$ close to x_s eventually. Accordingly, the matrix A is required to be stable (see Assumption 2.1) in the relationship $\dot{\tilde{x}}_s = A\tilde{x}_s$ in the proof above.

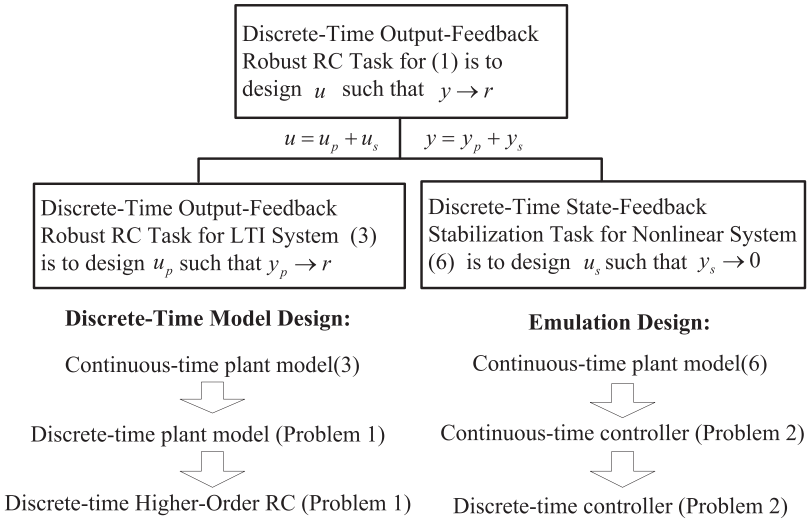

Figure 1. Additive-state decomposition and different discrete-time controller design for the primary system (3) and the secondary system (4).

3.1.2. Benefits from decomposition

Additive-state decomposition brings in two benefits.

- For a given desired output r, the objective is to design a sampled-data output-feedback RC for the nonlinear system (1) such that $y{-}r$ is uniformly ultimately bounded. First, since the output of the primary system and the state of the secondary system can be observed,[3] the original tracking problem for the system (1) is correspondingly decomposed into two problems: an *output-feedback* tracking problem (because only $\hat{y}_p$ is available) for an LTI 'primary' system ($y_p - r \to \mathcal{B}(\delta_1)$,[4] $\delta_1 \in \mathbb{R}_+$) and a *state-feedback* stabilisation problem (because $\hat{x}_s$ is available) for the complementary 'secondary' system ($y_s \to \mathcal{B}(\delta_2)$, $\delta_2 \in \mathbb{R}_+$). This is shown in Figure 1. As a result, $y - r \to \mathcal{B}(\delta_1 + \delta_2)$ according to (8). So, the objective is achieved. Since the tracking task is only assigned to the LTI component, it is therefore much easier than that for the nonlinear system (1). The state-feedback stabilisation is also easier than the output-feedback stabilisation.
- Secondly, for the two decomposed components, different discrete-time controller design methods can be employed (shown in Figure 1). It is appropriate to follow the discrete-time model design for the discrete-time RC design of the linear primary system. On the other hand, a state-feedback stabilisation problem for the secondary system is independent of RC (The ISS property cannot be obtained for a traditional RC system). The resultant closed-loop system can be rendered ISS. Then, the emulation design will be adopted for the discrete-time controller design of the nonlinear secondary system.

3.2. Controller design for primary system and secondary system

Thus far, the original tracking problem for the system (1) is correspondingly decomposed into two problems: an *output-feedback* tracking problem for an LTI primary system (3) and a *state-feedback* stabilisation problem for the complementary secondary system (6). In the following, we will solve them one by one.

3.2.1. Problem 3.1 on primary system (3)

Since (3) is an LTI system with an exogenous additive perturbation term given by $\phi(r) + d$, by using the sample-and-hold on the input and output with the sampling period T_s, (3) can be written as

$$y_p(z) = P(z)\, u_p(z) + d_r(z) \tag{11}$$

where $P(z) = c^{\mathrm{T}}(zI - F)^{-1}Hb$, $F = e^{AT_s}$, $H = \int_0^{T_s} e^{As}\mathrm{d}s$, and $d_r(z)$ represents the contribution of $\phi(r) + d$ to the output. Since A is stable by Assumption 2.1, namely $\mathrm{Re}(\lambda(A)) < 0$, then $|\lambda(F)| < 1$ no matter what $T_s > 0$ is. This implies that $P(z)$ is stable. Similarly to Steinbuch et al. (2007) and Pipeleers et al. (2008), a sampled-data output-feedback RC can be designed for (11). The corresponding problem is stated in the following.

problem 3.1: For (11) (or (3)), design a sampled-data output-feedback RC as

$$u_p(z) = C(z)\, e_p(z) \tag{12}$$

such that, in time domain, $e_p(kT_s) = r(kT_s) - y_p(kT_s) \to \mathcal{B}(\delta)$ as $k \to \infty$, where

$$C(z) = 1 + L(z)\frac{Q(z)\,W(z)\,z^{-N}}{1 - Q(z)\,W(z)\,z^{-N}},$$

and $\delta > 0$ is expected as small as possible.

The stability analysis of the closed-loop system corresponding to (11) and (12) is given by Proposition 3.1.

Proposition 3.1: *Let u_p in* (11) *be designed as in* (12). *Suppose* (i) $P_c(z), L(z), Q(z)$ *are stable,* (ii)

$$\left|Q(z)\,W(z)\,z^{-N}(1 - T(z)\,L(z))\right| < 1, \forall\, |z| = 1 \tag{13}$$

where $P_c(z) = 1/(1 + P(z))$ and $T(z) = P(z)P_c(z)$. Then the tracking error e_p is uniformly ultimately bounded. Furthermore, if

$$\mathcal{Z}^{-1}\left(\left(1 - Q(z)\,W(z)\,z^{-N}\right)(r(z) - d_r(z))\right) \to 0,$$

then $e_p(kT_s) = r(kT_s) - y_p(kT_s) \to 0$ as $k \to \infty$.

Proof: By substituting (12) into (11), the tracking error of the primary system is written as

$$e_p(z) = P_c(z)\,K(z)\left[\left(1 - Q(z)\,W(z)\,z^{-N}\right)(r(z) - d_r(z))\right] \tag{14}$$

where $K(z) = 1/(1 - Q(z)W(z)z^{-N}(1 - T(z)L(z)))$. A sufficient criterion for stability of the closed-loop system now becomes that $P_c(z)$ and $K(z)$ are both stable. The transfer function $P_c(z)$ is stable by condition (i). For stability of $K(z)$, to apply the small gain theorem (Green & Limebeer, 1994, pp. 97-98), $Q(z)W(z)z^{-N}(1 - T(z)L(z))$ is required to be stable first. This requires that $P_c(z)$, $P(z)$ and $Q(z)W(z)z^{-N}$ be stable, which are satisfied by given conditions. Therefore, if (13) holds, then $K(z)$ is stable by the small gain theorem. Then the tracking error e_p is uniformly ultimately bounded. Furthermore, taking $(1 - Q(z)W(z)z^{-N})$ $(r(z) - d_r(z))$ as a new input in (14), since $P_c(z)K(z)$ is stable, $e_p(k) = r(k) - y_p(k) \to 0$ if $\mathcal{Z}^{-1}(1 - Q(z)z^{-N}(r(z) - d_r(z))) \to 0$ as $k \to \infty$. ∎

From Proposition 3.1, one can see that the stability depends on three main elements of the controller (12): $L(z)$, $Q(z)$ and $W(z)$. The ideal design is

$$1 - T(z)\,L(z) = 0, Q(z) = 1. \tag{15}$$

If so, then the condition (13) is satisfied and (14) becomes

$$e_p(z) = P_c(z)\left(1 - W(z)\,z^{-N}\right)(r(z) - d_r(z)). \tag{16}$$

Since $r(z) - d_r(z)$ is periodic, $\mathcal{Z}^{-1}((1 - W(z)z^{-N})$ $(r(z) - d_r(z))) \to 0$. As a result, $e_p(kT_s) = r(kT_s) - y_p(kT_s) \to 0$ based on (16). However, (15) is not often satisfied. The following remarks comment this.

Remark 3.2 (design of $L(z)$): In practice, the transfer function $T(z)$ may be *non-minimum-phase*. So, a stable $L(z)$ cannot satisfy $T(z)L(z) = 1$ exactly. Here, Taylor expansions of the transfer function inverse is used to design $L(z)$. When there are zeros outside the unit circle for $T(z)$, one can rewrite $T(z)$ in the following form

$$T(z) = k_T\frac{T_n(z)}{T_d(z)} = k_T\frac{T_n^+(z)\,T_n^-(z)}{T_d(z)}$$

where $T_n^+(z)$ is the cancelable part containing only the stable zeros, $T_n^-(z)$ is the noncancelable part containing only the unstable zeros, and k_T is the gain. Based on the decomposition, the filter $L(z)$ is designed as

$$L(z) = \frac{1}{k_T}\frac{T_d(z)}{T_n^+(z)}\hat{T}_{n,inv}^-(z) \tag{17}$$

where $\hat{T}_{n,inv}^-(z)$ is the Taylor expansions of $1/T_n^-(z)$. Although the designed $L(z)$ is noncausal, (12) can be realised, thanks to the one-period delay term z^{-N}. Such a design can often assure $|1 - T(e^{i\omega T_s})L(e^{i\omega T_s})| \approx 0$ at least for low frequency band, or for all frequencies $\omega \in [0, \pi/T_s]$. The detailed design above and other related designs are presented in Longman (2010, pp. 468–470).

Remark 3.3 (design of $Q(z)$): The design of $L(z)$ has assured that $T(z)L(z) \approx 1$ in low frequency band so that the stability criterion (13) holds in low frequency band. However, the stability criterion may be violated in high-frequency band. Based on the choice of $L(z)$, the filter $Q(z)$ is chosen to be a zero-phase low-pass filter (Longman, 2010; Smith, 2007, pp. 473–475)

$$Q(z) = \sum_{k=-n_q}^{k=n_q} a_k z^k$$

which aims to attenuate the term $|Q(z)W(z)z^{-N}(1 - T(z)L(z))|$ in high-frequency band. Similarly, although $Q(z)$ is noncausal, (12) can be realised because of the one-period delay term z^{-N}. On the other hand, by (14), the term $1 - Q(z)W(z)z^{-N}$ will determine the tracking performance directly. Especially, if $|1 - T(e^{i\omega T_s})L(e^{i\omega T_s})| \approx 0$ for all frequencies $\omega \in [0, \pi / T_s]$, then the ideal design is $Q(z) = 1$.

Remark 3.4 (design of $W(z)$): $W(z)$ is the gain adjusting or the higher-order RC function, given by

$$W(z) = \sum_{i=1}^{p} w_i z^{-(i-1)N} \tag{18}$$

with $\sum_{i=1}^{p} w_i = 1$. For a traditional RC, $W(z) = 1$. With the redundant freedom, one can design appropriate weighting coefficients $w_1, w_2, \ldots w_p$ to improve the robustness of the tracking accuracy with respect to the period variation of $r - d_r$ (Steinbuch, 2002; Steinbuch et al., 2007). An approach was further proposed in Pipeleers et al. (2008) to design higher-order RCs that yield an optimal trade-off between the robustness for period-time uncertainty and the sensitivity for non-periodic inputs.

Remark 3.5: Remarks 3.2–3.4 introduce the design of $L(z), Q(z), W(z)$ separately. Unlike it, Kurniawan et al. (2014a) and Kurniawan and Cao (2014b) proposed a comprehensive design and analysis method. Only the parameters of $L(z)$ is optimised by taking the RC problem with time-varying sampling periods as a robust control problem for an uncertain system.

3.2.2. Problem 3.2 on secondary system (6)

So far, Problem 3.1 has been solved. In the following, the design of a sampled-data controller for the nonlinear system (6) is discussed. Before proceeding further, a lemma is introduced.

Definition 3.1 (Nesić & Teel, 2001): The system

$$\dot{x} = f(x, u(x), d_c) \tag{19}$$

is ISS with respect to d_c if there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that the solutions of the system satisfy $\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma(\|d_c\|_\infty)$, $\forall x(0)$, $d_c \in \mathcal{L}_\infty$, $\forall t \geq 0$.

Suppose that the feedback is implemented by a sample-and-hold as

$$u(t) = u(x(kT_s)), t \in [kT_s, (k+1)T_s), k \in \mathbb{N}. \tag{20}$$

Lemma 3.2 (Nesić & Teel, 2001): *If the continuous-time system (19) is ISS, then there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that given any triple of strictly positive numbers $(\Delta_x, \Delta_{d_c}, \nu)$, there exists $T^* > 0$ such that for all $T_s \in (0, T^*)$, $\|x(0)\| \leq \Delta_x$, $\|d_c\|_\infty \leq \Delta_{d_c}$, the solutions of the sampled-data system $\dot{x} = f(x, u(x(k)), d_c)$ satisfy:*

$$\|x(k)\| \leq \beta(\|x(0)\|, kT_s) + \gamma(\|d_c\|_\infty) + \nu, k \in \mathbb{N}. \tag{21}$$

Lemma 3.2 states that if the continuous-time closed-loop system is ISS, then the sampled-data system with the emulated controller will be semiglobally practically ISS with a sufficiently small T_s. With Lemma 3.2 in hand, Problem 3.2 will be introduced.

problem 3.2: For (6), design a controller

$$u_s(t) = \kappa(x_s(kT_s)), \quad t \in [kT_s, (k+1)T_s) \tag{22}$$

such that the closed-loop system is ISS with respect to the input e_p, namely

$$\|x_s(t)\| \leq \beta(\|x_s(0)\|, t) + \gamma\left(\sup_{t\geq 0}\|e_p(t)\|\right) + \nu, \tag{23}$$

where $t \geq 0$, γ is a class $\mathcal{K}$ function, β is a class $\mathcal{KL}$ function, $\nu > 0$ can be made small by reducing the sampling period T_s.

According to the emulation design, a continuous-time controller $u_s(t)$ is first designed based on the continuous-time plant model (6). Then, the obtained continuous-time controller is discretised according to the sampling period. For the secondary system (6), a locally Lipschitz static state feedback is designed as

$$u_s(t) = \kappa(x_s(t)) \tag{24}$$

whose discretisation form is (22). Then, substituting (24) into (6) yields

$$\dot{x}_s = f(t, x_s, e_p) \tag{25}$$

where $f(t, x_s, e_p) = Ax_s + b\kappa(x_s) + \phi(r + c^{\mathrm{T}}x_s + e_p) - \phi(r)$.

With respect to the ISS problem for (25), one has the following result.

Proposition 3.2: *For* (25), *suppose that there exists a continuously differentiable function* $V_1 : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}$ *such that*

$$\alpha_1(\|x_s\|) \le V_1(t, x_s) \le \alpha_2(\|x_s\|)$$
$$\frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial x_s} f\left(t, x_s, e_p\right) \le -V_2(x_s), \forall \|x_s\| \ge \rho\left(\|e_p\|\right) > 0$$

$\forall (t, x_s, e_p) \in [0,\infty) \times \mathbb{R}^n \times \mathbb{R}$, *where* α_1, α_2 *are class* $\mathcal{K}_\infty$ *functions,* ρ *is a class* $\mathcal{K}$ *function, and* $V_2(x)$ *is a continuous positive definite function on* $\mathbb{R}^n$*. Then, given any triple of strictly positive numbers* $(\Delta_{x_s}, \Delta_{e_p}, \nu)$*, there exist a class* $\mathcal{K}$ *function* γ*, a class* $\mathcal{KL}$ *function* β *and* $T^* > 0$ *such that for all* $T_s \in (0, T^*)$, $\|x_s(0)\| \le \Delta_{x_s}$, $\sup_{t\ge 0}\|e_p(t)\| \le \Delta_{e_p}$, *the solutions of the sampled-data system formed by* (6) *and* (22) *satisfy* (23), *where* $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

Proof: One can imitate the proof of Khalil (2002, Theorem 4.19, p. 176) to show that the continuous-time system (25) is ISS with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$. Then, based on Lemma 3.2, it can be concluded that the solutions of the sampled-data system formed by (6) and (22) are semiglobally practically ISS. ■

Remark 3.6: Since the structure of nonlinear term ϕ is not specified, then it will be complex or conservative to establish an explicit Lyapunov function for Problem 3.2 through a constructive method. To avoid discussion case-by-case, we turn to conditions on Lyapunov functions to establish a general solvability condition.

3.3. Controller integration

With the two designed controllers (12) and (22) for the two subsystems, one can combine them together to solve the original problem. The result is stated in Theorem 3.1.

Theorem 3.1: *Suppose* (i) *Problems* 3.1 *and* 3.2 *are solved;* (ii) *the observer-controller for system* (1) *is designed as:*

$$\hat{y}_p(kT_s) = y(kT_s) - c^{\mathrm{T}}\hat{x}_s(kT_s)$$
$$\hat{x}_s((k+1)T_s) = F\hat{x}_s(kT_s) + Hbu_s(kT_s) + \int_0^{T_s} e^{As}\left(y(kT_s+s) - r(kT_s+s)\right) \times \mathrm{d}s, \hat{x}_s(0) = 0 \tag{26}$$

and the controller for system (1) *is designed as:*

$$\begin{aligned} u_p(t) &= u_p(kT_s) = \mathcal{Z}^{-1}\left(C(z)\left(r(z) - \hat{y}_p(z)\right)\right) \\ u_s(t) &= \kappa(\hat{x}_s(kT_s)) \\ u(t) &= u_p(t) + u_s(t) \end{aligned} \tag{27}$$

for $t \in [kT_s, (k+1)T_s), k \in \mathbb{N}$. *Then the output of system* (1) *satisfies that* $y(kT_s) - r(kT_s) \to \mathcal{B}(\delta + \|c\|\gamma(\delta) + \|c\|\nu)$ *as* $k \to \infty$, *where* δ *is defined in Problem* 3.1, *and* ν, γ *are defined in Problem* 3.2. *Furthermore, if* $\|\dot{\phi}(y_d(t))\| \le l_{\dot{\phi}}$ *and* $\|\dot{d}(t)\| \le l_{\dot{d}}, \forall t \ge 0$, *then* $y(t) - r(t) \to \mathcal{B}(\delta + \|c\|\gamma(\delta) + \|c\|\nu + \|c\|\nu')$ *as* $t \to \infty$, *where* $\nu' = \int_0^{T_s}\|e^{A(T_s-s)}\|(l_{\dot{\phi}} + l_{\dot{d}})dsT_s$.

Proof: See Appendix 6.3. ■

Remark 3.7: According to Theorem 3.1, the ultimate bound $d_{e_\Delta} \le \delta + \|c\|\gamma(\delta) + \|c\|\nu + \|c\|\nu'$, which is determined by the property of the original system, the reference/disturbance, the controller, and also the sampling period. The robust RC is designed for the primary system in order to reduce δ.

4. An illustrative example

4.1. Problem formulation

In this paper, a single-link robot arm with a revolute elastic joint rotating in a vertical plane is served as an application (Marino & Tomei, 1995):

$$\begin{aligned} \dot{x} &= A_0 x + bu + \phi_0(y) + d, x(0) = x_0 \\ y &= c^{\mathrm{T}}x. \end{aligned} \tag{28}$$

Here

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{J_l} & -\frac{F_l}{J_l} & \frac{K}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{J_m} & 0 & -\frac{K}{J_m} & -\frac{F_m}{J_m} \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$c=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\phi_0\left(y\right)=\begin{bmatrix}0\\-\frac{Mgl\sin y}{J_l}\\0\\0\end{bmatrix},d=\begin{bmatrix}0\\d_1\\0\\d_2\end{bmatrix} \quad (29)$$

where $x=[x_1\ x_2\ x_3\ x_4]^{\mathrm{T}}$ are the link displacement (rad), link velocity, rotor displacement and rotor velocity, respectively; d_1 and d_2 are unknown disturbances. The parameters in (29) are $J_l=2$, $J_m=0.5$, $k=0.05$, $M=0.5$, $g=9.8$, $l=0.5$, $F_l=F_m=0.2$. The control u is the torque delivered by the motor. It is found that A_0 is unstable. Choose $p=[-2.10\ -1.295\ -9.36\ 3.044]^{\mathrm{T}}$. Then the system (28) can be formulated as (1) with $A=A_0+pc^{\mathrm{T}}$ and $\phi(y)=\phi_0(y)-py$, where A is stable. Assume that the desired trajectory is $r(t)=0.05+0.1\sin(2\pi t/T)$, while the periodic disturbances are $d_1(t)=0.04\sin(2\pi t/T)$ and $d_2(t)=0.02\cos(2\pi t/T)\sin(2\pi t/T)$, where $T=20\pi/3$ s. Let the sampling period be $T_s=0.1$ s. Then $N=209$.

4.2. Controller design

4.2.1. Controller design for primary system

Under the sampling period $T_s=0.1$ s, the discrete-time transfer function $T(z)$ is

$$T\left(z\right)=\left(z+9.399\right)T_m\left(z\right)$$

where

$$T_m\left(z\right)=\frac{9.8895*10^{-8}(z+0.9493)(z+0.09589)}{(z^2-1.819z+0.8279)(z^2-1.929z+0.9314)}$$

and there exists an unstable zero -9.399 in $T(z)$. Therefore, $T(z)$ is non-minimum phase. According to (17), $L(z)$ is designed as

$$L\left(z\right)=\frac{\hat{T}^{-}_{n,inv}\left(z\right)}{T_m\left(z\right)} \quad (30)$$

where $\hat{T}^{-}_{n,inv}(z)$ is the Taylor expansions of $1/(z+9.399)$ and is designed as

$$\hat{T}^{-}_{n,inv}\left(z\right)=\frac{1}{9.399}-\frac{z}{9.399^2}+\frac{z^2}{9.399^3}\approx\frac{1}{z+9.399}.$$

It is easy to check that $|1-T(e^{i\omega T_s})L(e^{i\omega T_s})|\le 0.0013$ for all frequencies $\omega\in[0,\pi/T_s]$. Then choose $Q(z)=1$. According to Steinbuch et al. (2007), a simpler second-order RC function is chosen as

$$W\left(z\right)=1.85-0.85z^{-N} \quad (31)$$

to improve robustness against the period variation. The amplitude of the transfer function in (14) with both $W(z)=1$ and $W(z)=1.85-0.85z^{-N}$ are plotted in Figure 2. As shown, if a small variation around the given period occurs (corresponding to frequencies at $0.3, 0.6, 0.9, \ldots$), then the magnitude variation by the higher-order RC is less than that by the RC. Therefore, the higher-order RC is less sensitive to the period variation. So, the higher-order RC can improve the robustness of the tracking accuracy against the period variation. This will be further confirmed next.

4.2.2. Controller design for secondary system

For the system (4), by the backstepping technique (Khalil, 2002), design

$$u_s\left(x_s\right)=\mu_1+\frac{J_l}{K}\left(v+\mu_2\right) \quad (32)$$

where $v=-7.5x_{s,1}-19x_{s,2}-17\eta_3-7\eta_4$, $\mu_1=-\eta_3+(K/J_m)x_{s,1}-(K/J_m)x_{s,3}-(F_m/J_m)x_{s,4}$, $\mu_2=(F_l/J_l)\eta_4+(Mgl/J_l)(\eta_3+\ddot{r})\cos(x_{s,1}+r)-(Mgl/J_l)((x_{s,2}+\dot{r})^2\sin(x_{s,1}+r)+\ddot{r}\cos(r)-\dot{r}^2\sin(r))$, $\eta_3=-(F_l/J_l)x_{s,2}-(K/J_l)(x_{s,1}-x_{s,3})-(Mgl/J_l)(\sin(x_{s,1}+r)-\sin(r))$, $\eta_4=-(F_l/J_l)\eta_3-(K/J_l)(x_{s,2}-x_{s,4})-(Mgl/J_l)((x_{s,2}+\dot{r})\cos(x_{s,1}+r)-\dot{r}\cos(r))$, $x_s=[x_{s,1}x_{s,2}x_{s,3}x_{s,4}]^{\mathrm{T}}$. The controller (32) can solve Problem 3.2.

4.3. Controller integration and simulation

The final controller is given by (27), where $L(z)$, $Q(z)$ in u_p are chosen as in Section 4.2, while u_s is chosen as in (32). The variables y_p and x_s are estimated by the observer (26) with the sensor sampling rate $T_{ss}=0.01s$. In the controller combination, the variables y_p and x_s will be replaced by $\hat{y}_p$ and $\hat{x}_s$. To compare the robustness of the tracking accuracy against the period variation, both $W(z)=1$ and $W(z)=1.85-0.85z^{-N}$ are taken into consideration, and the true period is assumed to be $20\pi(1+\alpha)/3$, where α is the perturbation. First, the transient response and tracking performance are shown in Figure 3. The tracking error is uniformly ultimately bounded. At the beginning, the traditional RC and the second-order RC both have a big overshooting, because, at the first period,

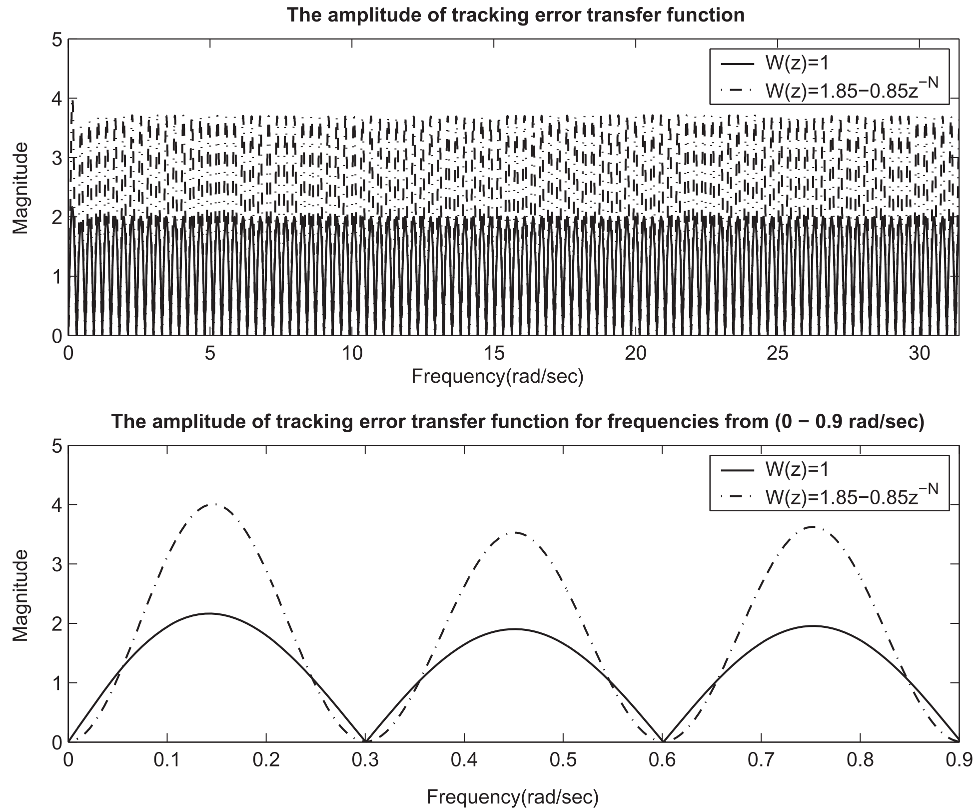

Figure 2. The amplitude of $(1/(1+P(z)))((1-Q(z)W(z)z^{-N})/(1-Q(z)W(z)z^{-N}(1-T(z)L(z))))$.

the RC does not work. Compared with the traditional RC, the second-order RC takes more time to adjust because it uses the previous two-period tracking error rather than one period by the traditional RC. However, after several periods, the output corresponding to each controller tracks the reference gradually with a satisfied steady-state tracking error. In the simulation, the convergence rate of the second-order RC is slower compared to that of the traditional RC. This is because the second-order RC is not optimised based on the system. Interested readers can refer to Kurniawan et al. (2014a) and Kurniawan and Cao (2014b) for optimisation methods. In order to examine the robustness against the period variation quantitatively, the ultimate bound is plotted as a function of the perturbation α. As shown in Figure 4, the ultimate bound is small if α is small. This implies that the proposed sampled-data RC can drive y to track r. More importantly, the ultimate bound of the steady-state tracking error produced by the proposed second-order RC is less sensitive to the perturbation α in comparison with that produced by the traditional RC. Therefore, our initial goal is achieved that a discrete-time output-feedback RC is designed for the nonlinear system (1) such that y can track r and the robustness against the period variation is improved.

4.4. Comparison

To demonstrate the effectiveness of the proposed method, a comparison is made. The considered system is (28) with parameters (29) except for

$$\phi_0(y) = \begin{bmatrix} 0 \\ -\dfrac{Mgl\left(\sin y + y^2\right)}{J_l} \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

where the new nonlinear term has the term y^2 additionally to make the comparison obvious. The other is the same as that in Section 4.1. The compared control is based on linearisation by Taylor's expansion. By linearisation, the system (28) becomes

$$\begin{aligned} \dot{x} &= Ax + bu + d, x(0) = x_0 \\ y &= c^{\mathrm{T}} x \end{aligned} \tag{34}$$

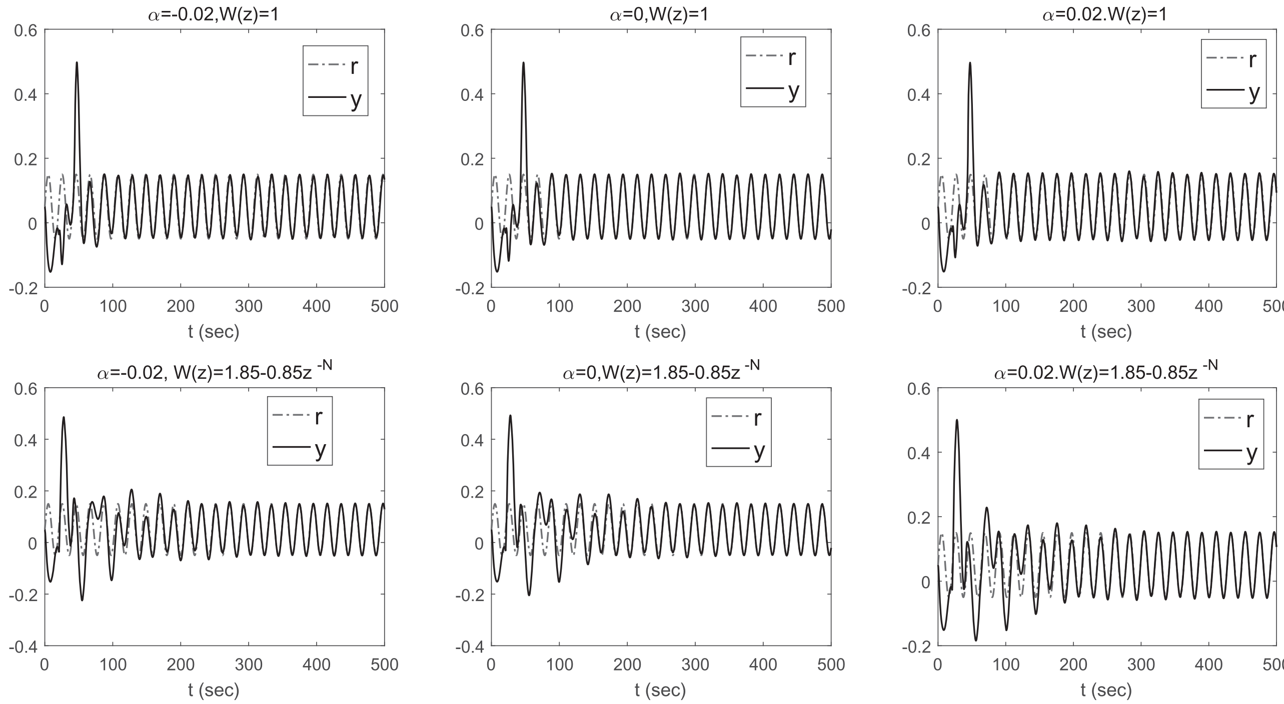

Figure 3. The transient response and tracking performance of the period-mismatch with traditional RC and second-order RC.

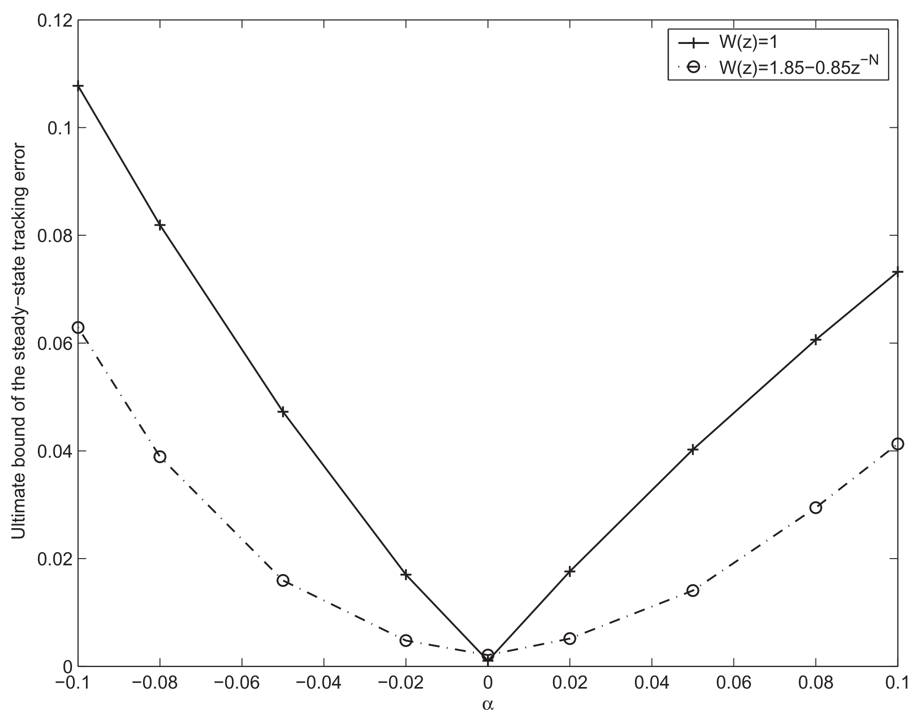

Figure 4. The ultimate bound as function of the period-mismatch with traditional RC and second-order RC.

where

$$A = A_0 + \left.\frac{\partial \phi_0\left(c^{\mathrm{T}} x\right)}{\partial x}\right|_{x=0}.$$

Since only y rather than x is available and A is stable for this case, the poles of A are not assigned here by feedback. Under the sampling period $T_s = 0.1$ s, the sampled-data RC is designed for comparison based on (34) following the procedure in Section 4.2.1. The proposed method is designed similar to Section 4.2, but the controller for the secondary system is designed based on (33). The transient response and tracking

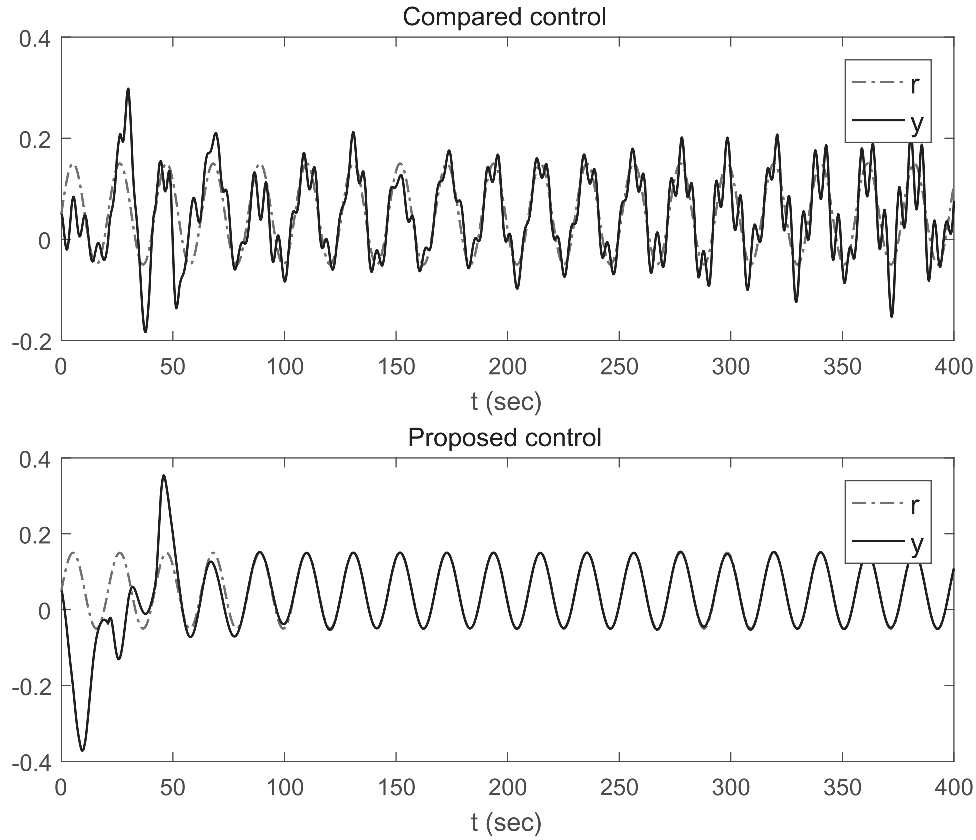

Figure 5. The transient response and tracking performance for comparison with traditional RC.

performance are shown in Figure 5. As shown, the compared control based on linearisation cannot track the period reference any more, but the proposed control method can achieve satisfying tracking performance. This demonstrates the effectiveness of the proposed method.

5. Conclusions

In this paper, a sampled-data output-feedback robust RC problem for a class of non-minimum phase systems with measurable nonlinearities was solved under the additive-state-decomposition-based tracking control framework. Existing controller design methods in both the frequency domain and the time domain were employed to make the robustness and discretisation for a continuous-time nonlinear system tractable. From the given illustrative example, our initial goal was achieved that, compared a discrete-time output-feedback RC using a simpler second-order filter with a classical RC, the output could track the reference and the robustness against the period variation was improved. Moreover, the proposed method outperformed the RC based on linearisation. Since the sampled-data control (sample-and-hold control) is a type of hybrid control, the sampled-data controller for the secondary nonlinear systems can be also designed under the framework of hybrid control (Goebel et al., 2012). Also, observer-based sampled-data control methods can also apply on the secondary nonlinear systems (Mao et al., 2018).

Notes

1. The neutral-type system in the critical case is in the form of $\dot{x}(t) - \dot{x}(t-T) = A_1 x(t) + A_2 x(t-T)$, where $x(t) \in \mathbb{R}^n$, $T > 0$, and $A_1, A_2 \in \mathbb{R}^{n\times n}$ (Hale & Verduyn, 1993; Quan et al., 2010).
2. A continuous-time controller is first designed based on a continuous-time plant model. The sampling is completely ignored at this step. Then, the obtained continuous-time controller is discretised and implemented using a sample-and-hold device (Nesić & Teel, 2001).
3. By (9) and (10), $\hat{y}_p$ and $\hat{x}_s$ are obtained instead of the true state x. So far, x or x_p is still unknown. In fact, we avoid solving x by using the additive-state decomposition.
4. Here, $\mathcal{B}(\delta) \triangleq \{\xi \in \mathbb{R} \,|\, \|\xi\| \leq \delta\}, \delta > 0$; the notation $x(t) \to \mathcal{B}(\delta)$ means $\min_{y\in\mathcal{B}(\delta)} |x(t) - y| \to 0$ as $t \to \infty$.

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Appendix

A.1 An example

An example is presented to show that output feedback makes feedback linearisation difficult. Consider the following uncertain non-minimum phase system

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 + \frac{x_2^2}{1+x_2^2} - 0.5d\\ \dot{x}_2 &= -5x_1 - 4x_2 + u - 0.5d\\ y &= x_2\end{aligned} \tag{A1}$$

where only $x_2 \in \mathbb{R}$ is available from measurement, and $d \in \mathbb{R}$ is unknown for controller designer. In the following, (A1) is transformed into a linear system subject to a periodic disturbance term. Let

$$z_2 = x_2 + \frac{x_2^2}{1+x_2^2} \tag{A2}$$

whose derivative is

$$\dot{z}_2 = \left(1 + \frac{2x_2}{\left(1+x_2^2\right)^2}\right)(-5x_1 - 4x_2 + u - 0.5d).$$

By designing

$$u = 5x_1 + 4x_2 + \left(1 + \frac{2x_2}{\left(1+x_2^2\right)^2}\right)^{-1} v, \tag{A3}$$

one has

$$\dot{z}_2 = v + \tilde{d} \tag{A4}$$

where $v \in \mathbb{R}$ is a virtual control input, and

$$\tilde{d} = -0.5\left(1 + \frac{2x_2}{\left(1+x_2^2\right)^2}\right) d \in \mathbb{R} \tag{A5}$$

is a new disturbance. On the other hand, substituting (A2) into (A1) results in

$$\dot{x}_1 = x_1 + z_2 - 0.5d \tag{A6}$$

Combining (A4) and (A6) yields

$$\begin{aligned}\dot{x}_1 &= x_1 + z_2 - 0.5d\\ \dot{z}_2 &= v + \tilde{d}.\end{aligned} \tag{A7}$$

In the process, there exist two problems: (i) In (A3), the state x_1 is used. However, only x_2 (output) is available from measurements. So, the feedback linearisation *cannot* be realised directly. (ii) In addition, the new disturbance $\tilde{d}$ in (A5) involves state

$2x_2/(1+x_2^2)^2$. So, (A7) is in fact *not* a linear system. Therefore, from this simple example, it is *not* easy to transfer (1) into a linear system with a periodic disturbance term.

A.2 Proof of Proposition 2.1

According to Khalil (2002, Theorem 13.1, p. 516), for (2), there exists a diffeomorphism transformation

$$z = \begin{bmatrix} \eta \\ \xi \end{bmatrix} = \mathcal{T}(x) = \begin{bmatrix} \mathcal{T}_1(x) \\ \mathcal{T}_2(x) \end{bmatrix}$$

and $\gamma : \mathbb{R}^n \to \mathbb{R}, \alpha : \mathbb{R}^n \to \mathbb{R}$ such that

$$\begin{bmatrix} \dot{\eta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} f_0(\eta, \xi) \\ A_c\xi + b_c\gamma(x)(u - \alpha(x)) \end{bmatrix}$$
$$y = c_c^{\mathrm{T}}\xi$$

where $\eta \in \mathbb{R}^\rho, \xi \in \mathbb{R}^{n-\rho}$, (A_c, b_c, c_c) is a canonical form representation of a chain of $\rho \in \mathbb{N}$ integrators;

$$f_0(\eta, \xi) = \left.\frac{\partial \mathcal{T}_1(x)}{\partial x}(Ax + bu)\right|_{x=\mathcal{T}^{-1}(z)}$$
$$A_c\xi + b_c\gamma(x)(u - \alpha(x)) = \left.\frac{\partial \mathcal{T}_2(x)}{\partial x}(Ax + bu)\right|_{x=\mathcal{T}^{-1}(z)}.$$

The system (2) is said to be *minimum phase*, if the zero dynamics $\dot{\eta} = f_0(\eta, 0)$ has an asymptotically stable equilibrium point. Otherwise, it is *non-minimum phase*. By using the same transformation $\mathcal{T}(x)$ for system (1) without external signals, we can get

$$\begin{bmatrix} \dot{\eta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial \mathcal{T}_1(x)}{\partial x}\left(Ax + bu + \phi\left(c^{\mathrm{T}}x\right)\right)\right|_{x=\mathcal{T}^{-1}(z)} \\ \left.\frac{\partial \mathcal{T}_2(x)}{\partial x}\left(Ax + bu + \phi\left(c^{\mathrm{T}}x\right)\right)\right|_{x=\mathcal{T}^{-1}(z)} \end{bmatrix}$$

$$= \begin{bmatrix} f_0(\eta, \xi) + \left.\frac{\partial \mathcal{T}_1(x)}{\partial x}\phi\left(c^{\mathrm{T}}x\right)\right|_{x=\mathcal{T}^{-1}(z)} \\ A_c\xi + b_c\gamma(x)(u - \alpha(x)) \\ + \left.\frac{\partial \mathcal{T}_2(x)}{\partial x}\left(\phi\left(c^{\mathrm{T}}x\right)\right)\right|_{x=\mathcal{T}^{-1}(z)} \end{bmatrix}$$
$$y = c_c^{\mathrm{T}}\xi$$

whose zero dynamics is

$$\dot{\eta} = f_0(\eta, 0)$$

as well because

$$\left.\frac{\partial \mathcal{T}_1(x)}{\partial x}\phi\left(c^{\mathrm{T}}x\right)\right|_{x=\mathcal{T}^{-1}(z),\xi=0}$$
$$= \left.\frac{\partial \mathcal{T}_1(x)}{\partial x}\phi\left(c_c^{\mathrm{T}}\xi\right)\right|_{x=\mathcal{T}^{-1}(z),\xi=0}$$
$$= \left.\frac{\partial \mathcal{T}_1(x)}{\partial x}\right|_{x=\mathcal{T}^{-1}(z),\xi=0}\phi(0)$$
$$= 0.$$

Based on the analysis above, the system (2) and system (1) without external signals have the same zero dynamics. According to the definition, if the linear system (2) being non-minimum or minimum phase is equivalent to the system (1) without external signals being non-minimum or minimum phase, respectively.

A.3 Proof of Theorem 3.1

By Lemma 3.1, the estimates in the observer (26) satisfy $\hat{x}_p \equiv x_p$ and $\hat{x}_s \equiv x_s$. Then the controller u_p in (27) can drive $e_p(kT_s) = y_p(kT_s) - r(kT_s) \to \mathcal{B}(\delta)$ as $k \to \infty$ thanks to Problem 3.1 being solved. On the other hand, suppose that Problem 3.2 is solved. According to (23), one has

$$\|x_s(kT_s)\| \le \beta'\left(\left\|x_s\left(k'T_s\right)\right\|, \left(k - k'\right)T_s\right) + \gamma\left(\sup_{t \ge k'T}\left\|e_p(t)\right\|\right) + \nu,$$

where β' *is a class* $\mathcal{KL}$ function and $k' \le k$. Then

$$\left\|y_s(kT_s)\right\| \le \|c\|\,\|x_s(kT_s)\|$$
$$\le \|c\|\,\beta'\left(\left\|x_s\left(k'T_s\right)\right\|, \left(k - k'\right)T_s\right)$$
$$+ \|c\|\,\gamma\left(\sup_{t \ge k'T}\left\|e_p(t)\right\|\right) + \|c\|\,\nu.$$

According to Problem 3.1, $e_p(kT_s) \to \mathcal{B}(\delta)$ as $k \to \infty$. This implies that, for a given $\varepsilon \in \mathbb{R}_+$, there exists a $N_0 \in \mathbb{N}$ such that $\sup_{t \ge k'T}\|e_p(t)\| \le \delta + \varepsilon$ when $k' \geqslant N_0$. Then $\|y_s(kT_s)\| \le \|c\|\beta'(\|x_s(N_0T_s)\|, (k - N_0)T_s) + \|c\|\gamma(\delta + \varepsilon) + \|c\|\nu, k \geqslant N_0$. Since $\|c\|\beta'(\|x_s(N_0T_s)\|, (k - N_0)T_s) \to 0$ as $k \to \infty$ and ε can be chosen arbitrarily small, it can be concluded that $y_s(kT_s) \to \mathcal{B}(\|c\|\gamma(\delta) + \|c\|\nu)$ as $k \to \infty$.

Furthermore, let us consider the behaviour of the sampled-data system during a sampling interval. By using (23), $y_s(t) \to \mathcal{B}(\|c\|\gamma(\delta) + \|c\|\nu)$ as $t \to \infty$. In the following, we further consider the primary system. Let

$$\tilde{x}_{p,k}(\Delta t) = x_p(kT_s + \Delta t) - x_p(kT_s), \tilde{y}_{p,k}(\Delta t) = c^{\mathrm{T}}\tilde{x}_p(\Delta t)$$

where $\Delta t \in [0, T_s]$. Then, by using Lagrange mean value theorem, one has

$$\dot{\tilde{x}}_{p,k}(\Delta t) = A\tilde{x}_{p,k}(\Delta t) + \dot{\phi}\left(y_d(\xi_1)\right)\Delta t + \dot{d}(\xi_2)\Delta t$$

where $kT_s \le \xi_1, \xi_2 \le kT_s + \Delta t$. Since $\|\dot{\phi}(y_d(t))\| \le l_{\dot{\phi}}$ and $\|\dot{d}(t)\| \le l_{\dot{d}}$, one has

$$\left\|\tilde{y}_{p,k}(\Delta t)\right\| \le \|c\|\,\left\|\tilde{x}_{p,k}\right\|$$
$$\le \|c\| \int_0^{\Delta t}\left\|e^{A(\Delta t - s)}\right\|\left(l_{\dot{\phi}} + l_{\dot{d}}\right)\mathrm{d}s\Delta t$$
$$\le \|c\|\,\nu'$$

for all $k \in \mathbb{N}$. Therefore, $y(t) - r(t) \to \mathcal{B}(\delta + \|c\|\gamma(\delta) + \|c\|\nu + \|c\|\nu')$.