

Additive-state-decomposition-based station-keeping control for autonomous aerial refueling

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Dear editor,

Aerial refueling (AR) is an effective method of increasing the endurance and range of aircraft by refueling them in flight [1, 2]. Station-keeping control is the basis of autonomous aerial refueling (AAR). The station-keeping control for AAR is a difficult task for two main reasons. First, the receiver is disturbed by the atmospheric turbulence and the wake vortex of the tanker. Secondly, owing to the fuel injection in the refueling process, the mass and the center of mass of the receiver will change.

Many researches have focused on developing station-keeping control methods for AAR, for example, the linear quadratic regulator (LQR) method [3], the L1 adaptive control method [4], the proportional-integral-derivative (PID) control method, and active disturbance rejection control (ADRC) technique [5]. Among these existing methods, most station-keeping controllers are designed by using some control methods for linear systems after linearizing the nonlinear receiver system directly. However, abandoning the nonlinear term directly may limit the control effect and make the final closed-loop system fragile to system perturbation and external disturbances. If the nonlinearity information of the nonlinear receiver system can be considered properly, better control effect would be expected. Thus, in this study, an additive-state-decomposition-based (ASD-based) [6] station-keeping control method is proposed for the probe-and-drogue AAR, which is a typical representative of AAR.

Problem formulation. Because of fuel transfer in refueling, the receiver aircraft is a system of varying mass and moments of inertia. For station-keeping control, taking the variable mass receiver model into account is a big difference from other conventional control. Readers can refer to [7] for the variable mass receiver model. In the tanker frame, the variable mass receiver model can be represented by a compact form:

$$\dot{\mathbf{x}}_r = \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r, \mathbf{d}), \quad (1)$$

where $\mathbf{x}_r = [x_r \ y_r \ h_r \ \phi \ \theta \ \psi \ u_r \ v_r \ w_r \ p \ q \ r]^T$ is the state

vector, $\mathbf{u}_r = [\delta_t \ \delta_e \ \delta_a \ \delta_r]^T$ is the input vector, and \mathbf{d} denotes various aerodynamic disturbances including the atmospheric turbulence and the tanker vortex.

Suppose that, in the level and forward flight, $u_r = v_r = w_r = p = q = r = 0$. Under this trim condition, the trimmed state and trimmed input are \mathbf{x}_r^* and \mathbf{u}_r^* . By defining the disturbed state and disturbed input as $\tilde{\mathbf{x}}_r = \mathbf{x}_r - \mathbf{x}_r^*$, $\tilde{\mathbf{u}}_r = \mathbf{u}_r - \mathbf{u}_r^*$, the disturbed system can be written as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_r &= \mathbf{A}\tilde{\mathbf{x}}_r + \mathbf{B}\tilde{\mathbf{u}}_r + \mathbf{g}(\tilde{\mathbf{x}}_r) + \mathbf{d}(\tilde{\mathbf{x}}_r, \tilde{\mathbf{u}}_r), \\ \tilde{\mathbf{y}}_r &= \mathbf{C}\tilde{\mathbf{x}}_r, \quad \tilde{\mathbf{x}}_r(0) = \tilde{\mathbf{x}}_{r0}, \end{aligned} \quad (2)$$

where $\mathbf{A} \triangleq \frac{\partial \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r)}{\partial \mathbf{x}_r} \Big|_{\mathbf{x}_r = \mathbf{x}_r^*, \mathbf{u}_r = \mathbf{u}_r^*}$, $\mathbf{g}(\tilde{\mathbf{x}}_r)$ denotes nonlinear terms, $\mathbf{B} \triangleq \frac{\partial \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r)}{\partial \mathbf{u}_r} \Big|_{\mathbf{x}_r = \mathbf{x}_r^*, \mathbf{u}_r = \mathbf{u}_r^*}$, and $\mathbf{d}(\tilde{\mathbf{x}}_r, \tilde{\mathbf{u}}_r)$ includes unmodelled dynamics and disturbances. The output matrix $\mathbf{C} \in \mathbb{R}^{3 \times 12}$, and $\tilde{\mathbf{y}}_r = \mathbf{p}_r - \mathbf{p}_r^*$ with \mathbf{p}_r^* for the trimmed position. $\tilde{\mathbf{x}}_{r0} \in \mathbb{R}^{12}$ is the initial state value.

Control objective. Design a station-keeping controller \mathbf{u}_r for the receiver system (1) such that $\mathbf{p}_r(t) - \mathbf{p}_r^d(t) \rightarrow \mathbf{0}$ or $\mathbf{p}_r(t) - \mathbf{p}_r^d(t) \rightarrow \mathcal{B}(\mathbf{0}_{3 \times 1}, \delta)$ as $t \rightarrow \infty$, $\delta \in \mathbb{R}_+ \cup \{0\}$. Equivalently, design a tracking controller \mathbf{u}_r for the system (2) such that $\tilde{\mathbf{y}}_r(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathbf{0}$ or $\tilde{\mathbf{y}}_r(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathcal{B}(\mathbf{0}_{3 \times 1}, \delta)$ as $t \rightarrow \infty$ when there exist disturbances, where \mathbf{p}_r^d is the reference trajectory, and $\tilde{\mathbf{y}}_r^d = \mathbf{p}_r^d - \mathbf{p}_r^*$ is the disturbed reference trajectory.

ASD-based station-keeping controller design. ASD [6] is a decomposition method for nonlinear systems just like superposition principle for linear systems. In the following, ASD is introduced to decompose the aforementioned receiver model into two subsystems.

Consider system (2) as the original system. By applying ASD, the primary system is chosen as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{r,p} &= \mathbf{A}\tilde{\mathbf{x}}_{r,p} + \mathbf{B}\tilde{\mathbf{u}}_{r,p} + \mathbf{d}(\tilde{\mathbf{x}}_{r,p}, \tilde{\mathbf{u}}_{r,p}), \\ \tilde{\mathbf{y}}_{r,p} &= \mathbf{C}\tilde{\mathbf{x}}_{r,p}, \quad \tilde{\mathbf{x}}_{r,p}(0) = \tilde{\mathbf{x}}_{r0}. \end{aligned} \quad (3)$$

Then, subtracting the primary system (3) from the original

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1) $\mathcal{B}(\mathbf{o}, \delta) \triangleq \{\boldsymbol{\xi} \in \mathbb{R}^3 \mid \|\boldsymbol{\xi} - \mathbf{o}\| \leq \delta\}$, and the notation $\mathbf{x}(t) \rightarrow \mathcal{B}(\mathbf{o}, \delta)$ means $\min_{\mathbf{y} \in \mathcal{B}(\mathbf{o}, \delta)} \|\mathbf{x}(t) - \mathbf{y}\| \rightarrow 0$.

system (2) gives

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_r - \dot{\tilde{\mathbf{x}}}_{r,p} &= \mathbf{A}(\tilde{\mathbf{x}}_r - \tilde{\mathbf{x}}_{r,p}) + \mathbf{B}(\tilde{\mathbf{u}}_r - \tilde{\mathbf{u}}_{r,p}) + \mathbf{g}(\tilde{\mathbf{x}}_r), \\ \tilde{\mathbf{y}}_r - \tilde{\mathbf{y}}_{r,p} &= \mathbf{C}(\tilde{\mathbf{x}}_r - \tilde{\mathbf{x}}_{r,p}), \quad \tilde{\mathbf{x}}_r(0) - \tilde{\mathbf{x}}_{r,p}(0) = \mathbf{0}. \end{aligned} \quad (4)$$

Next, by defining

$$\tilde{\mathbf{x}}_{r,s} = \tilde{\mathbf{x}}_r - \tilde{\mathbf{x}}_{r,p}, \quad \tilde{\mathbf{y}}_{r,s} = \tilde{\mathbf{y}}_r - \tilde{\mathbf{y}}_{r,p}, \quad \tilde{\mathbf{u}}_{r,s} = \tilde{\mathbf{u}}_r - \tilde{\mathbf{u}}_{r,p},$$

systems (3) and (4) become

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{r,p} &= \mathbf{A}\tilde{\mathbf{x}}_{r,p} + \mathbf{B}\tilde{\mathbf{u}}_{r,p} + \mathbf{d}(\tilde{\mathbf{x}}_{r,s} + \tilde{\mathbf{x}}_{r,p}, \tilde{\mathbf{u}}_{r,p} + \tilde{\mathbf{u}}_{r,s}), \\ \tilde{\mathbf{y}}_{r,p} &= \mathbf{C}\tilde{\mathbf{x}}_{r,p}, \quad \tilde{\mathbf{x}}_{r,p}(0) = \tilde{\mathbf{x}}_{r0}, \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{r,s} &= \mathbf{A}\tilde{\mathbf{x}}_{r,s} + \mathbf{B}\tilde{\mathbf{u}}_{r,s} + \mathbf{g}(\tilde{\mathbf{x}}_{r,s} + \tilde{\mathbf{x}}_{r,p}), \\ \tilde{\mathbf{y}}_{r,s} &= \mathbf{C}\tilde{\mathbf{x}}_{r,s}, \quad \tilde{\mathbf{x}}_{r,s}(0) = \mathbf{0}. \end{aligned} \quad (6)$$

Conversely, the original system (2) can be replaced by putting the primary system (5) and the secondary system (6) together.

It is clear that if the controller $\tilde{\mathbf{u}}_{r,p}$ drives $\tilde{\mathbf{y}}_{r,p}(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathbf{0}$ or $\tilde{\mathbf{y}}_{r,p}(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathcal{B}(\mathbf{0}_{3 \times 1}, \delta)$ as $t \rightarrow \infty$ and the controller $\tilde{\mathbf{u}}_{r,s}$ drives $\tilde{\mathbf{x}}_{r,s} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$, then $\tilde{\mathbf{y}}_r(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathbf{0}$ or $\tilde{\mathbf{y}}_r(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathcal{B}(\mathbf{0}_{3 \times 1}, \delta)$ as $t \rightarrow \infty$. The strategy here is to assign the tracking subtask to the primary system (5) and the stabilization subtask to the secondary system (6). Because system (5) is a linear time-invariant (LTI) system including all disturbances, standard design methods in either frequency domain or time domain can be used. On the other hand, because system (6) is a deterministic nonlinear system, many nonlinear stabilizing control methods can be adopted.

Next, the controller design is investigated in the form of two problems with respect to two subtasks, respectively. Because the system dimension of the receiver is high, controllers are often designed for the decoupled longitudinal channel and lateral channel respectively. In the following, the controller design for the longitudinal channel is taken into consideration for an illustration (a subscript 'lon' is added to every variable in the following).

Problem 1 (Tracking problem). For (5), design a proportional-integral (PI) tracking controller such that $\mathbf{e}_{r,lon,p} = \tilde{\mathbf{y}}_{r,lon,p}(t) - \tilde{\mathbf{y}}_{r,lon}^d(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$, meanwhile keeping $\tilde{\mathbf{x}}_{r,lon,p}$ bounded.

Intuitively, to remove the tracking error, an integral action must be employed in the controller

$$\mathbf{q}_{r,lon,p} = \int_0^t \mathbf{e}_{r,lon,p}(s) ds. \quad (7)$$

According to [8], a state feedback controller can be designed as

$$\tilde{\mathbf{u}}_{r,lon,p} = -\mathbf{K}_{\mathbf{x}lon}\tilde{\mathbf{x}}_{r,lon,p} - \mathbf{K}_{\mathbf{e}lon}\mathbf{q}_{r,lon,p}, \quad (8)$$

where $\mathbf{K}_{\mathbf{x}lon} \in \mathbb{R}^{2 \times 6}$, $\mathbf{K}_{\mathbf{e}lon} \in \mathbb{R}^{2 \times 2}$. The LQR method is utilized to determine the feedback matrices $\mathbf{K}_{\mathbf{x}lon}$ and $\mathbf{K}_{\mathbf{e}lon}$.

Theorem 1. For system (5), if the controller is designed as

$$\tilde{\mathbf{u}}_{r,lon,p} = -\mathbf{K}_{\mathbf{x}lon}\tilde{\mathbf{x}}_{r,lon,p} - \mathbf{K}_{\mathbf{e}lon}\mathbf{q}_{r,lon,p}, \quad (9)$$

then $\tilde{\mathbf{y}}_{r,lon,p}(t) - \tilde{\mathbf{y}}_{r,lon}^d(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$, and $\tilde{\mathbf{x}}_{r,lon,p}$ is bounded.

Proof. See [8].

Problem 2 (Stabilization problem). For (6), design a stabilizing controller such that $\tilde{\mathbf{x}}_{r,lon,s} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$.

In the following, a feedback linearization controller will be designed. In order to make the controller design easier, a virtual output variable is defined as

$$\tilde{\mathbf{y}}_{r,lon,s} = \mathbf{C}_{slon}\tilde{\mathbf{x}}_{r,lon,s}, \quad (10)$$

where $\mathbf{C}_{slon} \in \mathbb{R}^{2 \times 6}$. If the new output matrix \mathbf{C}_{slon} makes the system from $\tilde{\mathbf{u}}_{r,lon,s}$ to $\tilde{\mathbf{y}}_{r,lon,s}$ be a minimum-phase system, then $\tilde{\mathbf{y}}_{r,lon,s} \rightarrow \mathbf{0}$ implies $\tilde{\mathbf{x}}_{r,lon,s} \rightarrow \mathbf{0}$. A method for determining the output matrix \mathbf{C}_{slon} is given in [9]. Differentiating (10), one has

$$\begin{aligned} \dot{\tilde{\mathbf{y}}}_{r,lon,s} &= \mathbf{C}_{slon}\mathbf{A}_{lon}\tilde{\mathbf{x}}_{r,lon,s} + \mathbf{C}_{slon}\mathbf{B}_{lon}\tilde{\mathbf{u}}_{r,lon,s} \\ &\quad + \mathbf{C}_{slon}\mathbf{g}(\tilde{\mathbf{x}}_{r,lon,s} + \tilde{\mathbf{x}}_{r,lon,p}). \end{aligned}$$

A control input can be designed as

$$\tilde{\mathbf{u}}_{r,lon,s} = (\mathbf{C}_{slon}\mathbf{B}_{lon})^{-1}(\mathbf{v}_{r,lon,s} - \mathbf{h}), \quad (11)$$

where $\mathbf{v}_{r,lon,s}$ is a virtual input, and $\mathbf{h} = \mathbf{C}_{slon}\mathbf{A}_{lon}\tilde{\mathbf{x}}_{r,lon,s} - \mathbf{C}_{slon}\mathbf{g}(\tilde{\mathbf{x}}_{r,lon,s} + \tilde{\mathbf{x}}_{r,lon,p})$. The choice of \mathbf{C}_{slon} needs to make $\mathbf{C}_{slon}\mathbf{B}_{lon}$ invertible. Then, it can be obtained that

$$\dot{\tilde{\mathbf{y}}}_{r,lon,s} = \mathbf{v}_{r,lon,s}.$$

Design

$$\mathbf{v}_{r,lon,s} = -\mathbf{K}_{r,lon}\tilde{\mathbf{y}}_{r,lon,s},$$

where $\mathbf{K}_{r,lon} \in \mathbb{R}^{2 \times 2}$ is the controller parameter. Then, one has

$$\dot{\tilde{\mathbf{y}}}_{r,lon,s} = -\mathbf{K}_{r,lon}\tilde{\mathbf{y}}_{r,lon,s},$$

which can guarantee that $\tilde{\mathbf{y}}_{r,lon,s} \rightarrow \mathbf{0}$ exponentially, and further can guarantee that $\tilde{\mathbf{x}}_{r,lon,s} \rightarrow \mathbf{0}$ exponentially.

Theorem 2. For system (6), if there exists a control input:

$$\tilde{\mathbf{u}}_{r,lon,s} = (\mathbf{C}_{slon}\mathbf{B}_{lon})^{-1}(-\mathbf{K}_{r,lon}\tilde{\mathbf{y}}_{r,lon,s} - \mathbf{h}), \quad (12)$$

where $\mathbf{K}_{r,lon} \in \mathbb{R}^{2 \times 2}$ and $\mathbf{C}_{slon} = \mathbf{P}\mathbf{B}_{lon}$, such that

$$\dot{\tilde{\mathbf{y}}}_{r,lon,s} = -\mathbf{K}_{r,lon}\tilde{\mathbf{y}}_{r,lon,s} \quad (13)$$

is stable, then $\tilde{\mathbf{x}}_{r,lon,s} \rightarrow \mathbf{0}$ as $t \rightarrow \infty$.

Proof. See [9].

Controller design for the decomposed systems (5) and (6) requires their states and outputs as feedback variables. However, they are virtual and unknown. For such a purpose, an observer is designed.

Theorem 3. Suppose that an observer is designed to estimate $\tilde{\mathbf{x}}_{r,lon,p}$, $\tilde{\mathbf{x}}_{r,lon,s}$ and $\tilde{\mathbf{y}}_{r,lon,p}$ in (5) and (6) as

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_{r,lon,s} &= \mathbf{A}_{lon}\hat{\mathbf{x}}_{r,lon,s} + \mathbf{B}_{lon}\tilde{\mathbf{u}}_{r,lon,s} + \mathbf{g}(\hat{\mathbf{x}}_{r,lon}), \\ \hat{\mathbf{x}}_{r,lon,p} &= \tilde{\mathbf{x}}_{r,lon} - \hat{\mathbf{x}}_{r,lon,s}, \\ \hat{\mathbf{y}}_{r,lon,p} &= \mathbf{C}_{lon}\hat{\mathbf{x}}_{r,lon,p}, \quad \hat{\mathbf{x}}_{r,lon,s} = \mathbf{0}. \end{aligned} \quad (14)$$

Then $\hat{\mathbf{x}}_{r,lon,p} \equiv \tilde{\mathbf{x}}_{r,lon,p}$, $\hat{\mathbf{x}}_{r,lon,s} \equiv \tilde{\mathbf{x}}_{r,lon,s}$, and $\hat{\mathbf{y}}_{r,lon,p} \equiv \tilde{\mathbf{y}}_{r,lon,p}$.

Proof. See [6].

With the solutions to the two problems in hand, one is ready to claim Theorem 4.

Theorem 4. Suppose (i) Problems 1 and 2 are solved; (ii) the controller for system (2) is designed as

$$\tilde{\mathbf{u}}_r = \begin{bmatrix} \tilde{\mathbf{u}}_{r\text{lon}} \\ \tilde{\mathbf{u}}_{r\text{lat}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{u}}_{r\text{lon,p}} + \tilde{\mathbf{u}}_{r\text{lon,s}} \\ \tilde{\mathbf{u}}_{r\text{lat,p}} + \tilde{\mathbf{u}}_{r\text{lat,s}} \end{bmatrix}. \quad (15)$$

Then, the output of system (2) satisfies $\tilde{\mathbf{y}}_r(t) - \tilde{\mathbf{y}}_r^d(t) \rightarrow \mathbf{0}$ as $t \rightarrow \infty$.

Proof. See Appendix A in the supplementary file.

Simulation results are shown in Appendix B in the supplementary file.

Conclusion. In this study, the station-keeping problem for AAR has been addressed by an ASD-based control method. Based on ASD, the original receiver system is decomposed into a primary system and a secondary system. Through designing a PI controller and a feedback linearization controller for these two decomposed systems respectively, the final control input can be obtained by combining these two controllers. While PI control and feedback linearization control are not new, the salient feature of the proposed control method lies in the fusion of them by using ASD to solve a challenging nonlinear tracking problem.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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