Filtered Repetitive Control with Nonlinear Systems

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Foreword

Since its inception in 1981, repetitive control (RC) has become a major chapter of control theory, with applications as diverse as power supplies, robotic manipulators, and quadcopters. These may have in common the requirement that the system track a periodic reference signal or reject a periodic disturbance or do both.

This book, by two well-known control researchers at the Beijing University of Aeronautics and Astronautics, aims to provide state-of-the-art coverage of RC, with due attention to theoretical precision combined with a strong emphasis on engineering design. The basic design challenge is to achieve an appropriate trade-off between the mutually conflicting goals of steady-state tracking accuracy and robust internal stability.

As their starting point, the authors introduce the familiar internal model principle of linear regulation, but now for a generic, not necessarily continuous, periodic reference signal. This infinite-dimensional extension raises new issues of stabilizability resolved by filtered repetitive control (FRC). FRC lays the groundwork for an extensive treatment of alternative design approaches to both linear and nonlinear systems, including the technique (original with the authors) of "additive state decomposition".

The book is well suited to a course on engineering design for readers with some preparation in ordinary differential-delay equations and Lyapunov stability. I recommend it as a timely and significant contribution to the current literature on RC.

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Preface

Repetition is the mother of all learning
—A Latin Phrase

In nature, numerous examples of periodic phenomena are found and observed, ranging from the orbital motion of celestial bodies to heart rate. In practice, many control tasks are often of periodic nature as well. Industrial manipulators are often required to track or reject periodic exogenous signals when performing operations such as picking, dropping, and painting. Besides, special applications include magnetic spacecraft attitude control, helicopter vibration active control and vertical landing on oscillating platform, aircraft power harmonic elimination, satellite formation, LED light tracking, control of hydraulic servo mechanism, and lower limb exoskeleton control. For these periodic control tasks, repetitive control (RC, or repetitive controller, also specified RC) enables high-precision control performance. RC is derived from the internal model principle and contains a special structure with time-delay components which play the memory role. RC is, at the root, based on the compensation control or the predictive control that uses the additional memory. RC was originally developed on continuous single-input, single-output linear time-invariant (LTI) systems for high-precision tracking of periodic signals within a known period. Later, RC extended to multiple-input multiple-output LTI systems. Since then, RC has been propelled to the forefront of research and development in control theory. However, previous studies focused on theories and applications that use frequency-domain methods in relation to LTI systems, while RC for nonlinear systems received limited attention. What is more, RC often faces robustness problem, including stability robustness against uncertain parameters of systems and performance robustness against uncertain or time-varying period-time of external signals. For these problems, filter design with the frequency-domain analysis is the main tool, which then develops into filtered RCs. But it is difficult to apply them, if possible, to nonlinear systems. Therefore, we write this book for the utilization of filtered RC with nonlinear systems.

As an outcome of a course developed at Beihang University (Beijing University of Aeronautics and Astronautics, BUAA), this book aims at providing more methods and tools for the students and researchers in the field of RC to explore the potential of RC. In this book, commonly used methods like the feedback linearization method and adaptive-control-like method are summarized and further modified to be filtered RC. However, feedback linearization or error dynamics derived is often difficult to perform due to various reasons. To solve this problem, three new methods parallel to the two methods mentioned above are also proposed: the additive-state-decomposition-based method, the actuator-focused design method, and the contraction mapping method. To be specific, an introduction (Chap. 1) and preliminaries (Chaps. 2–4) are presented in the first four chapters, where the preliminaries consist of mathematics preliminary (Chap. 2), a brief introduction to RC for linear systems (Chap. 3), and the robustness problem of RC system (Chap. 4) that will serve to be an illustration of what RC is and why filtered RC must be used. After that, this book will give basic but new methods to solve RC problems for some special nonlinear systems: commonly used methods like linearization method (Chap. 5) and adaptive-control-like method (Chap. 6) will be summarized. They consist of both previous research findings and authors' contributions. In addition, three new methods parallel to the two methods mentioned above will be proposed: the additive-state-decomposition-based method in Chaps. 7–8 that will bridge the LTI systems and nonlinear systems so that the linear RC methods can be used in nonlinear systems; the actuator-focused design method in Chap. 9 derived from another viewpoint of the internal model principle proposed by the authors; and the contraction mapping method (Chap. 10) being another attempt of the authors to solve the RC problems for nonlinear systems without the need of corresponding Lyapunov functions.

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Symbols

$=$	Equality
$\triangleq$	Definition. $x \triangleq y$ means that x is defined to be another name for y, under certain assumption
$\equiv$	Congruence relation
$\in$	Belong to
$\times$	Cross product
$*$	The convolution of f and g is written $(f * g)(t) \triangleq \int_{-\infty}^{\infty} f(s)g(t-s)\mathrm{d}s$, where $f(t), g(t) \in \mathbb{R}$
$\mathscr{B}$	$\mathscr{B}(\mathbf{o}, \delta) \triangleq \{\boldsymbol{\xi} \in \mathbb{R}^n \mid \|\boldsymbol{\xi} - \mathbf{o}\| \leq \delta\}$, and the notation $\mathbf{x}(t) \to \mathscr{B}(\mathbf{o}, \delta)$ means $\min\limits_{\mathbf{y} \in \mathscr{B}(\mathbf{o}, \delta)} \|\mathbf{x}(t) - \mathbf{y}\| \to 0$
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n \times m}$	Set of real numbers, Euclidean space of dimension n, and Euclidean space of dimension $n \times m$
$\mathbb{R}_+$	Set of positive real numbers
$\mathbb{C}, \mathbb{C}^n, \mathbb{C}^{n \times m}$	Set of complex numbers, complex vector of dimension n, complex matrix of dimension $n \times m$
$\mathbb{Z}$	Integer
$\mathbb{Z}_+$	Positive integer
$\mathbb{N}$	Nonnegative integers
$\mathscr{L}_{2e}$	$\mathscr{L}_{2\mathrm{e}} \triangleq \{\mathbf{f} \in \mathscr{L}_2[0, T], \text{for all } T < \infty\}$
$\mathscr{L}_2(-\infty, \infty)$	$\mathscr{L}_2(-\infty, \infty) \triangleq \{\|\mathbf{f}\|_2 < \infty\}$
$\mathscr{L}_2[0, \infty)$	$\mathscr{L}_2[0, \infty) \triangleq \{\mathbf{f} \in \mathscr{S} : \mathbf{f}(t) = 0 \text{ for all } t < 0\} \cap \mathscr{L}_2(-\infty, \infty)$
$\mathscr{L}_\infty[a, b]$	$\mathscr{L}_\infty[a, b] \triangleq \left\{\mathbf{f} \mid \sup_{t \in [a,b]} \|\mathbf{f}(t)\| < \infty\right\}$
$\mathbf{I}_n$	Identify matrix of dimension $n \times n$
$\mathbf{0}_{n \times m}$	Zero matrix of dimension $n \times m$
x	Scale
$\mathbf{x}$	Vector, x_i represents the ith element of vector $\mathbf{x}$
$\mathbf{X}$	Matrix, x_{ij} represents the element of matrix $\mathbf{X}$ at the ith row and the jth column

$\dot{\mathbf{x}}, \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}$	The first derivative with respect to time t
$\hat{\mathbf{x}}$	An estimate of $\mathbf{x}$
$\tilde{\mathbf{x}}$	An estimate error of $\mathbf{x}$
$\mathbf{A}^{\mathrm{T}}$	Transpose of $\mathbf{A}$
$\mathbf{A}^{-\mathrm{T}}$	Transpose of inverse $\mathbf{A}$
$\det(\mathbf{A})$	Determinant of $\mathbf{A}$
$\mathrm{tr}(\mathbf{A})$	Trace of a square matrix $\mathbf{A}$, $\mathrm{tr}(\mathbf{A}) \triangleq \sum_{i=1}^{n} a_{ii},\ \mathbf{A} \in \mathbb{R}^{n\times n}$
$\sigma_{\max}(\mathbf{C}), \sigma_{\min}(\mathbf{C})$	The maximum singular value and the minimum singular of matrix $\mathbf{C} \in \mathbb{C}^{n\times n}$, respectively
$\sigma(\mathscr{A})$	is the spectrum and $r_{\mathscr{A}} = \sup_{z\in\sigma(\mathscr{A})}\lvert z\rvert$ the spectral radius, where $\mathscr{A}$ be a linear compact operator
$\lambda(\mathbf{A}), \lambda_{\max}(\mathbf{A}), \lambda_{\min}(\mathbf{A})$	The eigenvalue, maximum eigenvalue, and the minimum eigenvalue of a matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, respectively
sup	The least upper bound of a set
inf	The greatest lower bound of a set
$(\cdot)^{(k)}$	The kth derivative with respect to time t
∇	Gradient
$\partial a/\partial \mathbf{x}$	$\partial a/\partial \mathbf{x} \triangleq [\,\partial a/\partial x_1 \quad \partial a/\partial x_2 \quad \ldots \quad \partial a/\partial x_n\,] \in \mathbb{R}^{1\times n}$
$\partial \mathbf{a}/\partial \mathbf{x}$	$(\partial \mathbf{a}/\partial \mathbf{x})_{ij} \triangleq \partial a_i/\partial x_j \in \mathbb{R}^{m\times n}$, where $\mathbf{a} = [\,a_1 \quad \ldots \quad a_m\,]^{\mathrm{T}}$ and $\mathbf{x} = [\,x_1 \quad \ldots \quad x_n\,]^{\mathrm{T}}$
$\mathscr{C}(\mathbf{A},\mathbf{B})$	Controllability matrix of pairs $(\mathbf{A},\mathbf{B})$, $\mathscr{C}(\mathbf{A},\mathbf{B}) = \left[\mathbf{B}\ \mathbf{AB}\ldots\mathbf{A}^{n-1}\mathbf{B}\right]$
$L_{\mathbf{f}}h$	The Lie derivative of the function h with respect to the vector field $\mathbf{f}$
$L_{\mathbf{f}}^{i}h$	The ith-order derivative of $L_{\mathbf{f}}h$, $L_{\mathbf{f}}^{i}h = \nabla\left(L_{\mathbf{f}}^{i-1}h\right)\mathbf{f}$
$\mathscr{O}(\mathbf{A},\mathbf{C}^{\mathrm{T}})$	Observability matrix of pairs $(\mathbf{A},\mathbf{C})$, $\mathscr{O}\left(\mathbf{A},\mathbf{C}^{\mathrm{T}}\right) = \begin{bmatrix} \mathbf{C}^{\mathrm{T}} \\ \mathbf{C}^{\mathrm{T}}\mathbf{A} \\ \vdots \\ \mathbf{C}^{\mathrm{T}}\mathbf{A}^{n-1} \end{bmatrix}$
$\mathrm{Re}(s)$	Real part of the complex number s
$\mathscr{L}, \mathscr{L}^{-1}$	Laplace transform and inverse Laplace transform, respectively
$\mathscr{Z}, \mathscr{Z}^{-1}$	Z-transform and inverse Z-transform, respectively
$\lvert\cdot\rvert$	Absolute value
$\lvert s\rvert$	The modulus of a complex number s is defined as $\lvert s\rvert \triangleq \sqrt{a^2+b^2}$, where $s = a+ib,\ a,b \in \mathbb{R}$
$\lVert\cdot\rVert$	Euclidean norm, $\lVert\mathbf{x}\rVert \triangleq \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}}, \mathbf{x} \in \mathbb{R}^{n}$
$\lVert\mathbf{C}\rVert$	The norm of a complex matrix $\lVert\mathbf{C}\rVert \triangleq \sigma_{\max}(\mathbf{C}),\ \mathbf{C} \in \mathbb{C}^{n\times n}$
$\lVert\cdot\rVert_{\infty}$	Infinity norm, $\lVert\mathbf{x}\rVert_{\infty} = \max\{\lvert x_1\rvert,\ldots,\lvert x_n\rvert\}, \mathbf{x} \in \mathbb{R}^{n}$

$\|\mathbf{f}\|_{2,[0,T]}$, $\|\mathbf{f}\|_2$	$\|\mathbf{f}\|_{2,[0,T]} \triangleq \left\{\int_0^T \|\mathbf{f}(t)\|^2 \mathrm{d}t\right\}^{\frac{1}{2}}$, $\|\mathbf{f}\|_2 \triangleq \left\{\int_{-\infty}^{\infty} \|\mathbf{f}(t)\|^2 \mathrm{d}t\right\}^{\frac{1}{2}}$, $\mathbf{f}: \mathbb{R} \to \mathbb{R}^n$
$\|\mathbf{f}\|_\infty$	$\|\mathbf{f}\|_\infty \triangleq \sup_{t\in[0,\infty)} \|\mathbf{f}(t)\|$
$\|\mathbf{f}\|_{[a,b]}$	$\|\mathbf{f}\|_{[a,b]} \triangleq \sup_{t\in[a,b]} \|\mathbf{f}(t)\|$
$\mathbf{A} \geq \mathbf{0}, \mathbf{A} > \mathbf{0}$	Represent that $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a positive semidefinite or positive definite matrix, respectively
$\bar{s}$	The conjugate of the complex number $s = a + ib$ is $\bar{s} \triangleq a - ib$
$\mathbf{C}^*$	The conjugate transpose is formally defined by $(\mathbf{C}^*)_{ij} \triangleq (\bar{\mathbf{C}})_{ji}$, $\mathbf{C} \in \mathbb{C}^{n\times m}$
$\mathscr{C}([a,b], \mathbb{R}^n)$	The space of continuous n-dimension function vector on $[a,b]$
$\mathscr{C}^m_{PT}([0,\infty), \mathbb{R}^n)$	The space of mth-order continuously differentiable functions $\mathbf{f}: [0,\infty) \to \mathbb{R}^n$ which are T-periodic, i.e., $\mathbf{f}(t+T) = \mathbf{f}(t)$
$\|\mathbf{x}_t\|_{[a,b]}$	$\|\mathbf{x}_t\|_{[a,b]} \triangleq \sup_{\theta\in[a,b]} \|\mathbf{x}(t+\theta)\|$, where $\mathbf{x}_t \triangleq \mathbf{x}_t(\theta) = \mathbf{x}(t+\theta), \theta \in [a,b]$
$\mathscr{K}$	A continuous function $\alpha: [0,a) \to [0,\infty)$ is said to belong to class $\mathscr{K}$ if it is strictly increasing and $\alpha(0) = 0$
$\mathscr{K}_\infty$	A continuous function α is said to belong to class $\mathscr{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$
$\mathscr{K}\mathscr{L}$	A continuous function $\beta: [0,a) \times [0,\infty) \to [0,\infty)$ is said to belong to class $\mathscr{K}\mathscr{L}$ if, for each fixed s, the mapping $\beta(r,s)$ belongs to $\mathscr{K}$ with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect with s and $\beta(r,s) \to 0$ as $s \to \infty$
$\|\mathbf{f}\|_a$	$\|\mathbf{f}\|_a \triangleq \limsup_{t\to\infty} \|\mathbf{f}(t)\|$, where $\mathbf{f} \in \mathscr{L}_\infty[0,\infty)$
$\mathscr{D}^+\mathbf{x}$	The upper right Dini derivative of a function and is defined by $\mathscr{D}^+\mathbf{x}(t_0) \triangleq \left(\lim_{t\to t_0+0} \sup \frac{x_1(t)-x_1(t_0)}{t-t_0}, \ldots, \lim_{t\to t_0+0} \sup \frac{x_n(t)-x_n(t_0)}{t-t_0}\right)^{\mathrm{T}}$
$\lvert\cdot\rvert$	Modulus of a complex number
$O(\mathbf{x})^n$	A function $\boldsymbol{\delta}(\mathbf{x})$ is said to be $O(\mathbf{x})^n$ if $\lim_{\|\mathbf{x}\|\to 0} \frac{\|\boldsymbol{\delta}(\mathbf{x})\|}{\|\mathbf{x}\|^n}$ exists and is $\neq 0$. In particular, a function $\boldsymbol{\delta}(\mathbf{x})$, said to be $O(\mathbf{x})^0$ or $O(1)$, implies $\lim_{\|\mathbf{x}\|\to 0} \|\boldsymbol{\delta}(\mathbf{x})\|$ exists and is $\neq 0$.
$\circ$	Function composition, $\mathbf{f}_1 \circ \mathbf{f}_2 : \mathbb{R}^n \to \mathbb{R}^m$ implies that $(\mathbf{f}_1 \circ \mathbf{f}_2)(\mathbf{x}) = \mathbf{f}_1(\mathbf{f}_2(\mathbf{x})), \forall \mathbf{x} \in \mathbb{R}^n$, where $\mathbf{f}_1 : \mathbb{R}^p \to \mathbb{R}^m, \mathbf{f}_2 : \mathbb{R}^n \to \mathbb{R}^p$