A stochastic approximation method for probability prediction of docking success for aerial refueling

Ying Liu, Zhiyao Zhao, Haibiao Ma, Quan Quan

Air Force Command College, Beijing, China
School of Artificial Intelligence, Beijing Technology and Business University, Beijing, China
School of Automation Science and Electrical Engineering, Beihang University, Beijing, China


1. Introduction

of docking phase with quantitative methods. Ref. [9] evaluated the docking risk with the probability of the drogue center located in a specific area. It computed the docking success probability in a two-dimensional space without considering the longitudinal distance between the receiver and tanker aircrafts. On this basis, Refs. [10,11] took the longitudinal dynamics into consideration during docking reachability analysis with Hamilton–Jacobi equation. Although these studies promote the safety analysis in the aerial refueling study and make a great contribution, a big problem in them is that the wind perturbations are not considered. During the aerial refueling, the wind perturbations bring randomness and uncertainties into the docking phase, making the docking success nondeterministic even if the flight information of the tanker and receiver aircrafts is accurately known.

In this case, the docking success problem is formulated into a probabilistic reachability problem by taking randomness and uncertainties in the docking phase into account, and the docking success risk evaluation turns to computing the docking success probability. An apparent approach to solve this problem is deriving an analytical solution of the docking success probability. However, the docking process is complex and nonlinear, which contains a large number of process variables. Thus, an exact analytical solution of the probabilistic reachable set is difficult to be obtained [12]. As an alternative, various numerical approaches have been proposed to solve the probabilistic reachability problem, which can be classified into three categories [13]: (1) Model abstraction: obtaining an abstract system that approximates the original complex process [14]. The gridding technique, a typical method of model abstraction, uses grids to discretize the state space, and generates a Markov chain as an approximation of the original process, and then calculates the probability of the system remaining in a permitted grid [15]. (2) Over-approximation: obtaining an efficient over-approximation of reachable sets. Polyhedral [16], hyperrectangles [17], ellipsoids [18] and zonotopes [19] are usually used to approximate the reachable set in a compact form. (3) Simulation: e.g., Monte Carlo simulation. The probability of a system remaining in a permitted area is obtained by randomly simulating many trajectories based on system dynamics and counting the number of trajectories that enter the permitted area [20]. The above three approaches have their own advantages and limitations. For deriving a solution of the docking success probability in the aerial refueling scenario, the over-approximation approach may get an overly conservative solution, and the simulation approach requires long computation cost to get an accurate result. In addition, since it is difficult to obtain a large amount of flight experimental data (e.g., flight information of the tanker and receiver aircrafts, magnitudes of wind perturbations, etc.), artificial intelligence-based data-driven methods are also limited to effectively estimate the docking success probability.

Considering the above deficiencies, this paper proposes a stochastic approximation method for the probability prediction of docking success, which belongs to the model abstraction approach. First, a stochastic differential equation is used to model the relative motion between the receiver and tanker aircrafts, where the receiver aircraft flies along a predefined flight path with influence of additive wind perturbations. The correlation between the magnitude of wind perturbations and the distance between the receiver and tanker aircrafts is considered. Then, based on the established relative motion model, the probability that the receiver aircraft enters the target set during a given time interval is predicted by the Markov chain stochastic approximation method. The docking success probability is computed by propagating the transition probabilities of the Markov chain backwards starting from the target set during the time interval. Finally, some comparative studies are provided to demonstrate the effectiveness and efficiency of the proposed method.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_\theta$</td>
<td>latitudinal correlation coefficient</td>
</tr>
<tr>
<td>$c_\phi$</td>
<td>longitudinal correlation coefficient</td>
</tr>
<tr>
<td>$N$</td>
<td>number of grid points</td>
</tr>
<tr>
<td>$p_{ij}^k(q)$</td>
<td>transition probability from the grid point $q$ to itself at the $k$th step</td>
</tr>
<tr>
<td>$p_{ij}^k(q)$</td>
<td>transition probability from the grid point $q$ to the grid point $q'$ at the $k$th step</td>
</tr>
<tr>
<td>$p^k(q)$</td>
<td>docking success probability of the grid point $q$ at the $k$th step</td>
</tr>
<tr>
<td>$t_f$</td>
<td>end time of docking phase</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time sampling interval</td>
</tr>
<tr>
<td>$V_t$</td>
<td>tanker speed</td>
</tr>
<tr>
<td>$V_r$</td>
<td>receiver speed</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>longitudinal distance between the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>latitudinal distance between the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>vertical distance between the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\Delta \gamma$</td>
<td>flight path angle of the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\Delta \varphi$</td>
<td>azimuth angle of the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>flight path angle of the receiver aircraft</td>
</tr>
<tr>
<td>$\varphi_r$</td>
<td>azimuth angle of the receiver aircraft</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>element of the diagonal matrix $\Gamma$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>maximum value among $\sigma_i (i = 1, 2, 3)$</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>variation of latitudinal stochastic perturbation</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>variation of longitudinal stochastic perturbation</td>
</tr>
<tr>
<td>$\delta$</td>
<td>grid size</td>
</tr>
<tr>
<td>$\mathcal{X}$</td>
<td>target set</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>state constraint set</td>
</tr>
<tr>
<td>$\mathcal{Q}$</td>
<td>state space</td>
</tr>
<tr>
<td>$\mathcal{Q}_0$</td>
<td>set of grids</td>
</tr>
<tr>
<td>$\partial \mathcal{Q}_\mathcal{L}$</td>
<td>boundary grids of state space</td>
</tr>
<tr>
<td>$\partial \mathcal{Q}_\mathcal{X}$</td>
<td>boundary grids of state constraint set</td>
</tr>
<tr>
<td>$\partial \mathcal{Q}_T$</td>
<td>boundary grids of target set</td>
</tr>
<tr>
<td>$\mathcal{N}_q$</td>
<td>adjacent grids of specific grid point $q$</td>
</tr>
<tr>
<td>$\mathbf{x}_t$</td>
<td>relative position of the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$\alpha (\mathbf{x}_t)$</td>
<td>drift term</td>
</tr>
<tr>
<td>$\beta (\mathbf{x}_t)$</td>
<td>diffusion term</td>
</tr>
<tr>
<td>$a_1(t)$</td>
<td>kinematic function of the probe</td>
</tr>
<tr>
<td>$a_2(t)$</td>
<td>kinematic function of the drogue</td>
</tr>
<tr>
<td>$f (\mathbf{x}, t)$</td>
<td>affine transformation of $\mathbf{x}$ in wind perturbations</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>dynamics of the probe tip with respect to the drogue center</td>
</tr>
<tr>
<td>$z(t)$</td>
<td>Gaussian process</td>
</tr>
<tr>
<td>$\rho (\mathbf{x})$</td>
<td>spatial correlation function</td>
</tr>
<tr>
<td>$B (\mathbf{x}_0, t)$</td>
<td>standard Brownian motion for a specific position $\mathbf{x}_0$</td>
</tr>
<tr>
<td>$B (\mathbf{x}, t)$</td>
<td>stochastic perturbation of current state $\mathbf{x}$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>three-dimensional identity matrix</td>
</tr>
<tr>
<td>$W(t)$</td>
<td>standard 3D Brownian motion</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>variance of the stochastic perturbation</td>
</tr>
</tbody>
</table>
The main contribution of this paper lies in three aspects: First, in order to consider the uncertainties and randomness brought by the wind perturbations in the aerial refueling process, probabilistic reachability is introduced to solve the docking risk evaluation problem by computing the docking success probability, which is an innovative idea different from previous studies. Second, a Markov chain stochastic approximation method is used to predict the docking success probability based on a stochastic differential equation of relative motion, which shows more advantages compared with other numerical approaches and artificial intelligence-based data-driven methods for aerial refueling process. Third, the correlation between the magnitude of the wind perturbations and the distance between the receiver and tanker aircrafts is considered in the modeling of wind perturbations, which is a more realistic and precise manner than modeling with a Gaussian white noise in previous studies.

The remainder of this paper is organized as follows. In Section 2, the relative motion between the receiver and tanker aircrafts with influence of additive wind perturbations is modeled, based on which the problem formulation is presented. In Section 3, the Markov chain stochastic approximation method is proposed, and the computation procedure of docking success probability is presented in detail. Section 4 provides a comprehensive simulation case and presents comparative studies to demonstrate the effectiveness and efficiency of the proposed method. Section 5 gives the conclusions and indicates future research.

2. Modeling and problem formulation

Fig. 1 shows a diagram of the relative motion between the receiver and tanker aircrafts in the docking phase of aerial refueling process, which contains two coordinate systems, namely the earth-fixed coordinate system Oxyz and the relative coordinate system Oxyh. As shown in Fig. 1, Vr and Vs represent the speeds of the tanker and the receiver aircrafts, respectively; the symbols Δy and Δφ refer to the relative flight path angle and the relative azimuth angle between the probe tip and the drogue center, respectively; the symbols Δx, Δy, Δh denote the longitudinal distance, latitudinal distance, and altitude of the probe tip with respect to the drogue center. The origin of the relative coordinate system is the drogue center, and the x-axis is aligned with the airspeed vector Vr of the tanker. The region in front of the drogue in Fig. 1 represents the target set D [21], which is defined as the successful docking region. If the probe enters this region, it means that the docking maneuver is successful.

Assumption 1. The flight path angle and the azimuth angle of the tanker are 0° in the docking phase, denoted as γt = 0°, φt = 0°.

Assumption 2. The flight path angle and the azimuth angle of the probe tip are equal to those of the receiver aircraft.

Remark 1. In the docking phase of aerial refueling, the attitude of the tanker aircraft needs to be stable. Hence, Assumption 1 is reasonable. Furthermore, for the probe of the receiver aircraft, there is no relative movement between the probe and the receiver aircraft during the docking phase. Thus, Assumption 2 is also reasonable.

Based on Assumptions 1 and 2, the relative flight path angle and the relative azimuth angle between the probe tip and the drogue center are Δy = γr - γt = γs, Δφ = φs - φt = ϕ, where γr and φr represent the flight path angle and the azimuth angle of the receiver aircraft, respectively.

In the docking phase, wind perturbations are the main source of uncertainties on the relative motion between the probe tip and the drogue center. Thus, the motions of the probe tip and the drogue can be described with the following stochastic differential equations, respectively [22]:

\[ \text{dx}_r(t) = a_1(t) dt + f(x_r, t) dt + \Gamma' dW(x_r, t) \]  
(1)

\[ \text{dx}_t(t) = a_2(t) dt + f(x_t, t) dt + \Gamma dW(x_t, t) , \]  
(2)

where the elements in state vector \( x = [x_r, y_r, h_r]^T \) represent the longitudinal distance, latitudinal distance and altitude of the probe tip in the earth-fixed coordinate system, and those in \( \text{x}_t = [x_t, y_t, h_t]^T \) represent the corresponding values of the drogue center. Functions \( a_1 : [0, +\infty) \to \mathbb{R}^3 \) and \( a_2 : [0, +\infty) \to \mathbb{R}^3 \) represent kinematic functions of the probe tip and the drogue center in the earth-fixed coordinate system, satisfying

\[ a_1(t) = \begin{bmatrix} V_r(t) \cos \gamma_r(t) \cos \phi_r(t) \\ V_r(t) \cos \gamma_r(t) \sin \phi_r(t) \\ V_r(t) \sin \gamma_r(t) \end{bmatrix}, \\ a_2(t) = \begin{bmatrix} V_t \\ 0 \\ 0 \end{bmatrix} . \]

The value of \( a_2 \) means that the tanker aircraft flies straightly at a constant speed \( V_t \). In Eqs. (1) and (2), the function \( f: \mathbb{R}^3 \times [0, \infty) \to \mathbb{R}^3 \) represents the wind perturbations, written as \( f(x, t) = \mathbf{r}(t)x + \mathbf{f}(t) \), where \( \mathbf{r}: [0, \infty) \to \mathbb{R}^3 \), \( \mathbf{f}(0, \infty) \to \mathbb{R}^3 \). The function \( B: \mathbb{R}^3 \times [0, \infty) \to \mathbb{R}^3 \) denotes stochastic perturbations, which is not a standard Brownian motion, but a function related to the current state \( x \). However, \( B(x_0, t) \) is a standard Brownian motion for a specified position \( x_0 \in \mathbb{R}^3 \). The covariance of \( B(\cdot) \) can be represented as [23]:

\[ E \left[ \left( \mathbf{B}(x_1, t_2) - \mathbf{B}(x_1, t_1) \right) \left( \mathbf{B}(y_1, t_2) - \mathbf{B}(y_1, t_1) \right)^T \right] = \rho \left( x_1 - y_1 \right) \left( t_2 - t_1 \right) I_1, \quad x_1, y_1 \in \mathbb{R}^3, \quad t_1 < t_2, \]

where \( x_1 \) and \( y_1 \) refer to the positions of the probe tip and the drogue center, respectively, \( I_1 \) represents the three-dimensional identity matrix and \( \rho: \mathbb{R}^3 \to \mathbb{R} \) represents the spatial correlation function. Supposing that the drogue center and the probe tip are in the same position, namely \( x_1 - y_1 = 0 \), then \( \rho(x_1 - y_1) = 1 \). If the distance is infinite, then \( \rho(x_1 - y_1) = 0 \). This means that the similarity between wind perturbations at the probe tip and those at the drogue center is negatively correlated to their relative position. The matrix \( \Gamma \) is used to characterize the variance of the stochastic perturbations. For simplicity, \( \Gamma \) is assumed to be a constant diagonal matrix, denoted as \( \Gamma = \text{diag} (\sigma_1, \sigma_2, \sigma_3) \).

Subtracting Eq. (2) from Eq. (1), the relative motion of the probe tip and the drogue center is written as

\[ \text{d}x_r(t) = \left( u(t) + r(t) \hat{x}_r(t) \right) dt + \Gamma' dW(t) . \]  
(4)

where \( \hat{x}_r = x_r - x_s = [\Delta x \Delta y \Delta h]^T \); \( \mathbf{u}(t) = a_1(t) - a_2(t) \) represents the dynamics of the probe tip with respect to the drogue center. Based on Eq. (3), define \( z(t) = \mathbf{B}(x_r, t) - \mathbf{B}(x_s, t) \), where \( z(t) \) is a Gaussian process with the covariance as

\[ E \left( \left( z(t_2) - z(t_1) \right) \left( z(t_2) - z(t_1) \right)^T \right) = 2 \left( 1 - \rho \left( \hat{x}_r \right) \left( t_2 - t_1 \right) \right) I_1, \quad t_1 < t_2. \]

(5)

The concrete proof of Eq. (5) is presented in the Appendix.

According to Eq. (5), \( z(t) = \sqrt{2} (1 - \rho (\hat{x}_r)) W(t) \) is obtained. Then, Eq. (4) is rewritten as

\[ \text{d}x_r(t) = \left( u(t) + r(t) \hat{x}_r(t) \right) dt + \sqrt{2} \left( 1 - \rho (\hat{x}_r) \right) \Gamma' dW(t) \]

(6)

\[ = a_1(t) dt + \beta (\hat{x}_r) \Gamma dW(t) . \]

Eq. (6) is a standard stochastic differential equation, where \( W(t) \) is a standard 3D Brownian motion, \( \alpha (\hat{x}_r, t) \) is the drift term, and \( \beta (\hat{x}_r) \) is the diffusion term. According to Eq. (6), the relative
motion model between the probe tip and the drogue center is expressed as
\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta h
\end{bmatrix} = \left[
\begin{array}{ccc}
-V_t + V_v(t) \cos \gamma_t(t) \cos \psi_t(t) \\
V_v(t) \cos \gamma_t(t) \sin \psi_t(t) \\
V_v(t) \sin \gamma_t(t)
\end{array}
\right] \, dt + r(t) \left[
\begin{array}{c}
\Delta x \\
\Delta y \\
\Delta h
\end{array}
\right] \, dt + \sqrt{2 [1 - \rho(\bar{X}_t)]} \begin{bmatrix}
\sigma_h \\
0 \\
0 
\end{bmatrix} \, dW(t). \tag{7}
\]

Here, \( \bar{X}_t \) is restricted by the state constraint \( \mathcal{X} \subset \mathbb{R}^3 \), where \( \mathcal{X} \) represents the range of the feasible relative positions between the probe tip and the drogue center in the docking phase.

The spatial correlation function \( \rho(\bar{X}_t) \) is expressed as:
\[
\rho(\bar{X}_t) = \exp \left( -c_h \| \bar{X}_t \|_h - c_v \| \bar{X}_t \|_v \right), \tag{8}
\]
where \( c_h > 0 \) and \( c_v > 0 \) represent the longitudinal correlation coefficient and latitudinal correlation coefficient, respectively. Their values reflect the correlation degree of the wind perturbations between the magnitude of wind perturbations and the distance between the receiver and tanker aircrafts in the vertical and horizontal direction, respectively. In Eq. (8), \( \| \bar{X}_t \|_h \) and \( \| \bar{X}_t \|_v \) can be expressed as
\[
\| \bar{X}_t \|_h = \sqrt{\Delta x^2 + \Delta y^2}, \quad \| \bar{X}_t \|_v = |\Delta h|. \tag{9}
\]

As described previously, a neighborhood region around the desired final states in the docking phase is defined as a successful docking region marked with target set \( \mathcal{D} \). For the relative motion described by Eq. (7), the target set \( \mathcal{D} \) is a cube as shown in Fig. 1. It is reasonable to regard relative longitudinal distance, latitudinal distance, and altitude of the probe tip with respect to the drogue center as the concerning states in the target set \( \mathcal{D} \), since the relative position between the receiver and tanker aircrafts in the docking phase largely determines whether the docking process is successful or not.

Given a target set \( \mathcal{D} \), the docking success probability for an initial state \( \bar{X}_t \) in the docking phase over a given time interval \([0, t_f]\) is represented as
\[
P \left( \bar{X}_t \left(t_f \right) \in \mathcal{D} | \bar{X}_t (0) \in \mathcal{X} \right), \tag{10}
\]
where \( t_f > 0 \) represents the end time of the docking phase.

3. Docking success probability computation

3.1. General idea

A stochastic approximation method is proposed to predict the docking success probability based on the Markov chain approximation, where the Cartesian grids are used to approximate the state space [21]. With properly chosen transition probabilities, the Markov chain converges weakly\(^1\) to the solution of the stochastic differential equations.

The general procedure of determining the docking success probability using the Markov chain stochastic approximation method is described as follows.

1. State space division. As shown in Fig. 2, the state space is divided into three sets, namely the target set \( \mathcal{D} \), the state constraint set \( \mathcal{X} \), and the state set which is outside the target set \( \mathcal{D} \) but inside the state constraint set \( \mathcal{X} \), denoted as \( \mathcal{L} = \mathcal{X} / \mathcal{D} \).

2. State space discretization. The state space \( \mathcal{L} \) is further divided into boundary grids \( \partial \mathcal{Q}_\mathcal{L} = \partial \mathcal{Q}_\mathcal{L}^+ \cup \partial \mathcal{Q}_\mathcal{L}^- \) and interior grids \( \mathcal{Q}_0 = \mathcal{Q} / \partial \mathcal{Q}_\mathcal{L} \), as presented in Fig. 2.

3. Markov chain establishment. The dynamics of grid transfer is described with a Markov chain. As shown in Fig. 3, for grid \( q \), it is only allowed to transfer to the adjacent grids and itself. The adjacent grid set \( \mathcal{N}_q \) includes \( q_{1+}, q_{1-}, q_{2+}, \ldots, q_{3+} \) and \( q_{3-} \), where \( q_{i+} = q_{i-} + \eta_i \delta \) and \( q_{i-} = q_{i-} - \eta_i \delta \). Here, \( \eta_i (i = 1, 2, 3) \) is related to element \( \sigma_i \) in the constant diagonal matrix \( \Gamma \) and specifically expressed as \( \eta_i = \sigma_i / \| \Gamma \| \), where \( \sigma_1 = \sigma_2 = \sigma_0, \sigma_3 = \sigma_v \) and \( \| \Gamma \| = \max \sigma_i \).

4. Backward propagation. The docking success probability of the grids in the state space \( \mathcal{L} \) is obtained by propagating the transition probabilities of the Markov chain backwards starting from the target set \( \mathcal{D} \).

3.2. Computation of transition probability of Markov chain

The transition probabilities of different kinds of grids in the state space \( \mathcal{L} \) are calculated as follows.

(1) If \( q \in \partial \mathcal{Q}_\mathcal{L} \), the transition probability is expressed as
\[
P \left( \mathcal{Q}_{k+1} = q' | \mathcal{Q}_k = q \right) = \begin{cases} 1 & q' = q \\ 0 & \text{otherwise} \end{cases}. \tag{11}
\]

Eq. (11) indicates that a grid \( q \) belonging to \( \partial \mathcal{Q}_\mathcal{L} \) will not transfer to other grids, since the set \( \partial \mathcal{Q}_\mathcal{L} \) is an absorbing region.

(2) If \( q \in \mathcal{Q}_0 \), the transition probability from a grid \( q \) to an adjacent grid \( q' \in \mathcal{N}_q \) is expressed as
\[
P \left( \mathcal{Q}_{k+1} = q' | \mathcal{Q}_k = q \right) = \begin{cases} p_q(q') & q' \in \mathcal{N}_q \cup \{q\} \\ 0 & \text{otherwise} \end{cases}. \tag{12}
\]

\(^1\) A sequence of points \( \{x_n\} \) in a Hilbert space \( H \) is said to be converge weakly if a point \( x \) in \( H \) satisfies \( \lim_{n \to \infty} f(x_n) = f(x) \).
where $p^k_q$ represents the transition probability from grid $q$ to grid $q'$ at the $k$th step. Let $\Delta t$ be time sampling interval. Then, $p^k_q$ can be expressed as

$$p^k_q = \begin{cases} \frac{\xi^k_q}{C^k_q} & q' = q \\ \frac{\exp(\delta\xi^k_q)}{C^k_q} & q' = q_{i+}, i = 1, 2, 3 \\ \frac{\exp(-\delta\xi^k_q)}{C^k_q} & q' = q_{i-}, i = 1, 2, 3 \end{cases}$$

(13)

where

$$\xi^k_q = \frac{2}{\lambda \sigma^2 \beta(q)^2} - 2n, \quad \xi^k_q = \frac{(\alpha(q, k\Delta t))_i}{n \sigma^2 \beta(q)^2}$$

$$C^k_q = \sum_{i=1}^{3} (\exp(\delta\xi^k_q)) + \exp(-\delta\xi^k_q)) + \xi^k_q,$$

and $\lambda$ is a positive constant which needs to be set small enough such that $\xi^k_q$ is positive. This is guaranteed by the condition $0 < \lambda < \frac{1}{\max\beta(q)^2}$. According to Eq. (7), the definitions of $\alpha(q, k\Delta t)$ and $\beta(q)$ are

$$\alpha(q, k\Delta t) = \begin{bmatrix} -V_t + V_r (k\Delta t) \cos \gamma_r (k\Delta t) \cos \phi_r (k\Delta t) \\ V_r (k\Delta t) \cos \gamma_r (k\Delta t) \sin \phi_r (k\Delta t) + V_t (k\Delta t) \sin \gamma_r (k\Delta t) \end{bmatrix}$$

$\beta(q) = \sqrt{2(1 - \rho \xi_q (k\Delta t))}$.

To ensure the accuracy of the calculation, the time sampling interval $\Delta t$ is determined by $\lambda$ and grid size $\delta$, namely $\Delta t = \lambda \delta^2$.

Previous studies have proved that the above Markov chain converges weakly to the solution of stochastic differential equation when the grid size $\delta \to 0$ [24].

### 3.3. Backward propagation

In this part, the backward propagation of the Markov chain is presented to calculate the docking success probability. The probability of entering the target set $D$ for each grid $q \in \mathcal{X}$ is computed by propagating the transition probabilities of the Markov chain, backwards starting from the target set during the specified time interval $[0, t_f]$. For step-by-step backward propagation, let $k_f = t_f / \Delta t$. Then, an initial condition is given as

$$p^{(0)}(q) = \begin{cases} 1, & q \in \partial Q_D \\ 0, & \text{otherwise} \end{cases}$$

(14)

Then, for $k \in [0, k_f - 1] \subseteq \mathbb{Z}$, the docking success probability of the grid point $q \in \mathcal{X}$ is recursively calculated as

$$p^{(k)}(q) = \begin{cases} p^k_q, & \text{if } q \in Q_f \\ p^{k+1}_q, & \text{if } q \in Q_0 \end{cases}$$

(15)

where $p^{k+1}_q$ is the docking success probability of the grid point $q \in \mathcal{X}$ at the $(k+1)$th step, $p^k_q$ is the transition probability from grid $q$ to $q'$ between the grid $q$ and the adjacent grid $q' \in \mathcal{X}$ at the $k$th step, respectively. Thus, the docking success probability is 1 when $q \in \partial Q_D$. On the other hand, the grid $q \in \partial Q_X$ represents the state reaching the boundary of the state space $\mathcal{X}$ and $\partial Q_X$ is also an absorbing region. Hence, the docking success probability is 0 when $q \in \partial Q_X$. According to Eqs. (14) and (15), the docking success probability of any grid point $q \in \mathcal{X}$ can be recursively obtained.

In summary, the computation procedure of the docking success probability is summarized as follows and its flow chart is shown in Fig. 4.

1. **Step 1:** Initialize the state space $\mathcal{X}$, the target set $D$, and the state space $\mathcal{L} = \mathcal{X}/D$.
2. **Step 2:** Set the grid size $\delta$ and discretize the state space.
3. **Step 3:** Establish the Markov chain for each grid point $q \in \mathcal{X}$, and compute its transition probabilities based on the relative motion model between the probe tip and the drogue center.
4. **Step 4:** Define the initial condition according to Eq. (14). Then, obtain $p^k_q$ by propagating backwards from the initial condition according to Eq. (15).
5. **Step 5:** If $k = 0$, output the docking success probability of the whole state space; otherwise, go back to Step 4.

### 4. Simulation analysis

#### 4.1. Simulation configuration

This subsection elaborates on the configuration of the parameters of the relative motion model and the approximated Markov chain, the state constraint set, and the target set.

The docking phase is divided into two stages. The time interval of each stage cannot be set too short as the control variables cannot be changed frequently, especially for manned vehicles, so we set its value to 1 s. Related parameters of the relative motion model in the two time intervals are shown in Table 1 The positive
constant \( \lambda \) is set to 0.015 which can guarantee \( \Delta_0^h(q) \) in Eq. (13) is positive. Note that the speeds of the receiver aircraft at the two stages are set a little faster than those of the tanker aircraft, because it can ensure the probe plunge into the drogue.

Considering the computational time requirements and the stability of the proposed method, the valid ranges of the state space and the grid division are shown in Table 2. If the states are out of the valid ranges, the docking is regarded as failed. According to Eq. (7), the target set is set with respect to \( \mathbf{x}_r \), as:

\[
\mathcal{D} = \{ \mathbf{x}_r \in \mathbb{R}^3 | -0.3 \leq \Delta x \leq 0, -0.5 \leq \Delta y \leq 0.5, -0.5 \leq \Delta h \leq 0.5 \}.
\] (16)

### 4.2. Results and analysis

The simulation is divided into two parts. One part does not consider the latitudinal deviation of the relative motion between the receiver and tanker aircrafts while the other part does. In the scenario where the latitudinal deviation is not considered (Section 4.2.1), the simulation is performed in a two-dimensional space which has a better visual display. In this scenario, three cases are provided to show the computation result of the docking success probability under the influence of wind perturbations, different values of correlation coefficients and the flight states of the receiver aircraft, respectively. In the scenario where the latitudinal deviation is considered (Section 4.2.2), the simulation is performed in a three-dimensional space. The docking success probability under the influence of the three-dimensional wind perturbations is presented.

#### 4.2.1. Result without considering the latitudinal deviation

In this scenario, the receiver aircraft only needs to adjust its longitudinal distance and altitude deviation between its probe and the drogue of the tanker to complete the docking. Then, Eq. (9) is rewritten as

\[
||\mathbf{x}_r|| = ||\Delta x||, ||\mathbf{x}_\theta|| = ||\Delta h||.
\] (17)

**Case 1:** In this case, the simulation results with and without wind perturbations are compared. If wind perturbations are not considered, it means that \( f(x, t) = 0 \). The longitudinal correlation coefficient \( c_0 \) and latitudinal correlation coefficient \( c_v \) are set to 0.2 and 0.05, respectively. If wind perturbations are considered, the function \( f(x, t) = r(t)x + 1^t \) is set to be

\[
f(x, t) = \begin{bmatrix} 1 & 0 \\ -\frac{1}{5} & 0 \end{bmatrix} \begin{bmatrix} x^r \\ r^h \end{bmatrix} + \begin{bmatrix} 3r \\ t^2 \end{bmatrix}.
\] (18)

The wind perturbations can be viewed as a clockwise swirling windstorm as shown in Fig. 5. It can be seen that the position center of wind perturbations changes with time. Note that the form of \( f(t) \) does not affect the docking success probability since this term will be removed in the relative motion model between the probe tip and the drogue center.

The computed docking success probability is depicted in Fig. 6. Figs. 6(a) and 6(b) show the result without considering wind perturbations, and Figs. 6(c) and 6(d) show the result considering wind perturbations, respectively. The innermost rectangle represents the target set, and the envelopes outside it are probability contours. Fig. 6 depicts the predicted docking success probabilities at different relative positions between the receiver and tanker aircrafts. It can be seen that the envelopes of the docking success probability bent counterclockwise due to the influence of clockwise wind perturbations.

**Case 2:** To evaluate the influence of the correlation coefficients on the docking success probability, the longitudinal correlation coefficient \( c_0 \) and the latitudinal correlation coefficient \( c_v \) are set to different values. In this case, wind perturbations are not considered, and the results are shown in Fig. 7. It can be seen...
that with the increase of correlation coefficients, the envelopes of docking success probability enlarge to a certain extent. As shown in Eq. (8), the longitudinal correlation coefficient $c_h$ and the latitudinal correlation coefficient $c_v$ affect the spatial correlation function. If $c_h$ and $c_v$ increase, the variable $\rho(\tilde{x}_{rt})$ which characterizes the strength of spatial correlation in the random field $B(\cdot)$ decreases and the term $\sqrt{2(1-\rho(\tilde{x}_{rt}(k\Delta t)))}$ increases.

Case 3: In this case, we study the influence of flight states in receiver aircraft on the docking success probability. The two parameters speed and flight path angle of the receiver aircraft are set to different values for the time interval $[0, 1]$. Besides the values in Table 1, $V_r$ is set to 123 m/s and $\gamma_r$ is set to $2^\circ$. The results are shown in Fig. 8. It can be observed that a fast docking speed and flight path angle correspond to a large envelop of docking success probability, which means that the speed and
flight path angle of the receiver aircraft are sensitive parameters for docking success. Furthermore, a faster docking speed and larger flight path angle indicate that docking is more likely to be successful in a short time interval.

4.2.2. Result considering the latitudinal deviation

In this part, the latitudinal deviation is considered, which means that the receiver aircraft needs to adjust the longitudinal distance, altitude deviation, and latitudinal deviation between its probe and the drogue of tanker to complete docking. The wind perturbations are set to be

$$r(k\Delta t)\bar{x}_r = \begin{bmatrix} \frac{1}{5}k\Delta t & 0 & 0 \\ 0 & -\frac{1}{5}k\Delta t & 0 \\ 0 & 0 & \frac{1}{5}k\Delta t \end{bmatrix} \begin{bmatrix} \Delta x_r \\ \Delta y_r \\ \Delta h_r \end{bmatrix}.$$ (19)

The time interval is set to [0, 1]. Figs. 9(a) and 9(b) show the result without considering wind perturbations, and Figs. 9(c) and 9(d) show the result considering wind perturbations, respectively. The green cube refers to the target set and the isosurfaces with different colors from inside to outside are docking success probabilities with values of 0.2, 0.4, 0.6 and 0.8. From Fig. 9, we can see that the docking success probability isosurfaces with wind perturbations are smaller than those without wind perturbations, which means that it is difficult to dock successfully with an influence of three-dimensional wind perturbations. The docking success probabilities of some representative grid points in the state space are presented in Table 3.

4.3. Result verification

In this part, the Monte Carlo method [25] is used to verify the results obtained in Section 4.2. The docking success probabilities are computed with our method and Monte Carlo method, respectively.

The procedure of using the Monte Carlo method to calculate the docking success probabilities is summarized as follows.
Step 1: Initialize the state space, target set and grid division according to Section 4.1.

Step 2: Select $N = 1000$ grids randomly in the state space. For each grid $q_j (j = 1, \ldots, 1000)$, the number of simulated trajectories is set to be $m = 100$.

Step 3: The state variables corresponding to the grid point $q_j$ are viewed as the initial state of Eq. (7). The variable $\mathbf{W}(t)$ in Eq. (7) is a standard 3D Brownian motion, with mean value 0 and covariance $\sqrt{\Delta t}$ for $d\mathbf{W}(t)$. If $q_j$ enters target set $\mathcal{D}$ within the time interval $[0, t_f]$, then $n_{\text{MC}, j} = n_{\text{MC}, j} + 1$.

Furthermore, the correlation coefficient adopted to evaluate the relevance of results with these two methods is built as

$$ r = \frac{\sum_{j=1}^{N} (P_j - \bar{P}) \sum_{j=1}^{N} (P_{\text{MC}, j} - \bar{P}_{\text{MC}})}{\sqrt{\sum_{j=1}^{N} (P_j - \bar{P})^2 \sum_{j=1}^{N} (P_{\text{MC}, j} - \bar{P}_{\text{MC}, j})^2}}. \tag{20} $$
where $N$ represents the number of selected grid points, $P_j$ and $P_{MC,j}$ denote the docking success probabilities with our proposed method and Monte Carlo method, respectively. $P_{MC,j}$ is calculated as $P_{MC,j} = N_{MC,j}/m$, where $N_{MC,j}$ refers to the number of trajectories entering the target set for the grid point $j$ within the time interval $[0, t]$ and $m$ denotes the number of simulated trajectories. $\bar{P}$ and $\bar{P}_{MC,j}$ represent the average probability of $P_j$ and $P_{MC,j}$, and can be denoted as $\bar{P} = \sum_{j=1}^{N} P_j/N$ and $\bar{P}_{MC,j} = \sum_{j=1}^{N} P_{MC,j}/N$.

The computed correlation coefficient is 0.96 for $t \in [0, 1]$ and 0.91 for $t \in [0, 2]$, which can demonstrate the correctness of our proposed method. Meanwhile, the comparison of simulation time between these two methods is listed in Table 4. Note that the simulation is performed with Matlab R2010a on a laptop with a 2.4 GHz processor and 2 GB RAM. From Table 4, it can be concluded that the proposed method is more efficient than the Monte Carlo method. The reason is that the docking success probability of each grid is directly calculated by the proposed method, while Monte Carlo method requires numerous simulations to obtain identical result.

### 4.4. Discussion

Although the proposed method is more efficient than the Monte Carlo method, it is still a kind of numerical method. The computational cost is proportional to grid denseness for each dimension, and the computational complexity grows exponentially with the number of state space dimensions. The proposed method can be used to easily analyze the system up to three dimensions in practice. For five-dimensional scenario, it is also feasible but requires long computation time. In this paper, the relative motion model between the drogue center and the probe tip is a three-dimensional model. Thus, the simulation results can be easily obtained. If more parameters are considered in the docking phase or more dimension of the relative motion model is used, the limitation of proposed method makes it cannot be used in online results computation. To overcome this limitation, more docking success probabilities can be established to publish an off-line database under the scenario of different parameter settings and more forms and intensity of wind perturbations. Then, based on the database, an intelligent recognition method is required to be studied to generate the docking success probability according to real-time wind perturbations and flight states of the receiver aircraft. This method could achieve real-time docking risk evaluation, which also makes the research results better applied to practical engineering applications.

### 5. Conclusion

In this paper, a Markov chain stochastic approximation method is proposed to predict the docking success probability at the docking phase of aerial refueling. The docking success probabilities of different grid points in the state space under influence of wind perturbations are predicted by the proposed method. The effectiveness and efficiency of our method are demonstrated with simulation. The studied method and corresponding results have significant meaning for aerial refueling safety. In practice, the research result of this paper can be used as a reference for pilots during aerial refueling. If the docking success probability is lower than a specified value, the receiver aircraft pilot needs to increase the relative distance between the receiver and tanker aircrafts to reduce the docking risk, and adjust the flight state to dock again. In future research, there are two aspects could be further studied. First, more parameters need to be considered to coincide with the practical docking phase of aerial refueling. In this case, new numerical methods should be studied to reduce the computational complexity. Second, autonomous control algorithms corresponding to different docking success probabilities should be studied to make the docking highly reliable and safe.

### CRediT authorship contribution statement

**Ying Liu:** Conceptualization, Methodology, Validation, Software, Writing - original draft.  **Zhiyao Zhao:** Formal analysis, Supervision, Writing - review & editing.  **Haibiao Ma:** Visualization, Investigation.  **Quan Quan:** Writing - review & editing.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61903008, in part by Young Talents Support Project of Beijing Association for Science and Technology, China, and in part by the Outstanding Youth Cultivation Project of Beijing Technology and Business University, China. The authors thank Jun Wang and Lei Yang of Air Force Command College, China, for their valuable suggestions and support for this research.

### Appendix

#### Proof of Eq. (5).

Since $\mathbf{z}(t) = \mathbf{B}(\mathbf{x}, t) - \mathbf{B}(\mathbf{x}_s, t)$, it has

$$E \left[ (\mathbf{z}(t_2) - \mathbf{z}(t_1))^T (\mathbf{z}(t_2) - \mathbf{z}(t_1)) \right] = E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2) - \mathbf{B}(\mathbf{x}, t_1) + \mathbf{B}(\mathbf{x}_s, t_1))^T \right] = E \left[ ((\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2)) - (\mathbf{B}(\mathbf{x}, t_1) - \mathbf{B}(\mathbf{x}_s, t_1)))^T \right].$$

Expanding the above equation, we have

$$E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2))^T (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2)) \right] - E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_1))^T (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_1)) \right]$$

$$= E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2))^T (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2)) \right]$$

$$- E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_1))^T (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_1)) \right] + E \left[ (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_1))^T (\mathbf{B}(\mathbf{x}, t_2) - \mathbf{B}(\mathbf{x}_s, t_2)) \right].$$

<table>
<thead>
<tr>
<th>Time interval (unit: s)</th>
<th>Simulation time of Monte Carlo method (unit: s)</th>
<th>Simulation time of the proposed method (unit: s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1]</td>
<td>22.70</td>
<td>0.22</td>
</tr>
<tr>
<td>[0, 2]</td>
<td>38.17</td>
<td>0.32</td>
</tr>
</tbody>
</table>
The proof is concluded.

According to Eq. (A.1), it is obtained

\[
E \left[ \mathbf{z}(t_2) - \mathbf{z}(t_1) \right] \left[ \mathbf{z}(t_2) - \mathbf{z}(t_1)^\top \right] = 2 \left[ 1 - \rho(\tilde{x}_t(t_2) - t_1) \right] I_3 \quad t_1 < t_2. \tag{A.5}
\]

The proof is concluded.

References


